Cooperative Strategies for Simultaneous Relay Channels

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Abstract – The simultaneous relay channel is defined as a discrete memoryless relay channel where the source is unaware of the channel statistic controlling the communication but knows that it is one of two the possible channel statistics. The achievable rates and cooperative strategies are analyzed for this channel. Applications of these results arise when the source node is uncertain of the noise levels or the network topology (e.g. due to user mobility the positions of the relay and the destination nodes are unknown). This problem is recognized as being equivalent to that of sending common and private information to two destinations in presence of two helper relays. In this scenario, each possible relay channel becomes a branch of a broadcast relay channel. The coding for this problem is designed such that it is adaptable to each channel and can guarantee a minimum achievable rate for both channels while sending more information in each case. An achievable rate region along with a general upper bound are presented for the simultaneous relay channel. It is shown that these bounds are tight for the case of semi-degraded simultaneous relay channels and Gaussian degraded simultaneous relay channels. Moreover it is shown that the Block Markov code can be used for both Decode-and-Forward (DF) and Compress-and-Forward (CF) and thus it is an oblivious code to cooperative strategies.

1 Introduction

Cooperative networks have been of huge interest during recent years between researchers. Using the multiplicity of information in nodes, these networks can increase in capacity and reliability using the appropriate strategy. The simplest of these networks is the relay channel. A fundamental contribution was made by Cover and El Gamal [1], where the main strategies of Decode-and-Forward (DF) and Compress-and-Forward (CF), and an upper bound were developed for this channel along with capacity theorems for special classes of relay channels.

The specification of wireless networks undergoes the extensive changes due to a variety of factors (e.g. interference, fading and user mobility). As a consequence, even when the channels are quasi-static, it is often difficult for the source to know the noise level of the relay link. Hence, the encoder is unable to decide on the suitable coding strategy that would better exploit the presence of the relay. This scenario is frequently seen in *ad-hoc* networks where the source is often assumed to be unaware of the presence of relay users yet in most of the previous works the channel is assumed to be fixed and known to all the users. In practical networks like Ad hoc networks, the channel is not known before the communication and most of these cases can be conceived as a Simultaneous Relay Channel.

The Simultaneous Relay Channel (SRC) is defined as a relay aided communication between a source and a destination where the source is not aware of the channel however it knows that the probability distribution of the channel(W) belongs to the set $\mathscr{W}$. The source knowing all the possibilities sends the common information which can be decoded regardless of the actual channel and furthermore sends the private information corresponding to the actual case. The problem involves a straightforward relation with the cases of compound channel, broadcast channels and evidently relay channels. The problem offers a vast perspective of the practical application in the cooperative networks.

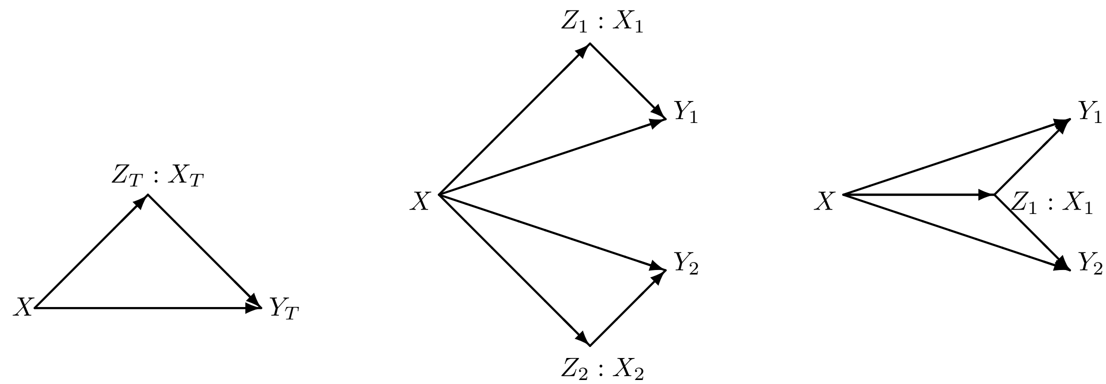
In this paper we investigate the simultaneous relay channel (SRC) with two possible channel outcomes. It can be shown that the problem of simultaneous channels can be turned into the problem of broadcast channels. Thus the problem of simultaneous relay channel can be seen as related to sending common and private informations over the broadcast relay channel (BRC) where each destination is aided by its own relay. This idea, usually called *broadcasting strategy* was used in [2, 3]. Particularly this idea was developed for the relay channels in [4].

2 Definitions and Main Results

2.1 Problem Definition

The simultaneous relay channel with discrete source, relay inputs $x \in \mathscr{X}$, $x_T \in \mathscr{X}_T$ and discrete outputs $y_T \in \mathscr{Y}_T$, $z_T \in \mathscr{Z}_T$, is characterized by two conditional probability distributions (PDs) $\left\{W_T : \mathscr{X} \times \mathscr{X}_T \longmapsto \mathscr{Y}_T \times \mathscr{Z}_T\right\}_{T=1,2}$, where T is the channel index. It is assumed that the encoder (source node) is unaware of the realization of T that governs the communication, but T should not change during the transmission. However, T is assumed to be known at the destination and the relay ends.

Definition 1 (Code) *A code for this relay channel consists of an encoder mapping* $\{\varphi : \mathcal{M}_{1n} \times \mathcal{M}_{2n} \longmapsto \mathscr{X}^n\}$*, two decoder mappings* $\{\psi_T : \mathscr{Y}_T^n \longmapsto \mathcal{M}_{Tn}\}$ *and a set of relay functions*



(a) The simultaneous relay channel (b) BRC with two relays (c) BRC with common relay

FIG. 1: The simultaneous relay channel

$\{f_{T,i}\}_{i=1}^n$ *such that* $\{f_{T,i} : \mathscr{Z}_T^{i-1} \longmapsto \mathscr{X}_T^n\}_{i=1}^n$, *for some finite sets of integers* $\mathcal{M}_{Tn} = \{1, \ldots, M_{Tn}\}$. *The code rates are* $n^{-1} \log M_{Tn}$ *and its maximum error probability*

$$e_{\max}^{(n)}\big(\varphi, \psi, \{f_{T,i}\}_{i=1}^n\big) \doteq \max_{T=1,2} \max_{w \in \mathcal{M}_{Tn}} \Pr\Big\{\psi(\mathbf{Y}_T) \neq w\Big\}.$$

Definition 2 (Achievable rate and capacity) *A pair of numbers* $(R_1, R_2) \in \mathbb{R}^{+2}$ *is an achievable rate pair for the simultaneous relay channel if for every* $0 \leq \epsilon, \gamma < 1$ *and for sufficiently large* n *there exist* n*-length block code whose error probability satisfies* $e_{\max}^{(n)}\big(\varphi, \psi, \{f_{T,i}\}_{i=1}^n\big) \leq \epsilon$ *and the rates* $n^{-1} \log M_{Tn} \geq R_T - \gamma$. *The set of all achievable rates is called the capacity region for the simultaneous relay channel.*

Since the relay and the receiver are aware of the realization of T, it is not difficult to see, as shown in Fig. 1(b), that the problem of coding for the simultaneous relay channel can be turned into that of the BRC. The BRC consists of two relay links, each one equivalent to the relay channel with $T = \{1, 2\}$. We allow the encoder to send common and private information to the receivers, where each relay serves the reliable transmission of its own information to the destination. The messages (W_0, W_i) are sent to the destination $T = i$ at the rates (R_0, R_i) with $i = \{1, 2\}$, all messages are assumed independent. The definitions of achievability for the rates (R_0, R_1, R_2) and that of capacity remain the same as for standard BCs (see [5–7]).

2.2 The bounds on the capacity of broadcast relay channel

Consider a broadcast relay channel with two relay. It is assumed that for one channel the better strategy is DF and for another one is CF. This is the common case when we don't know that the relay is close to the transmitter or to the receiver. The following theorem presents an inner bound for the general channel using such scenario.

Theorem 2.1 *An inner bound on the capacity region of the BRC with heterogeneous cooperative strategies is given by [4]*

$$\begin{aligned}
\mathscr{R}_I \doteq \bigcup_{P \in \mathscr{P}} \Big\{ &(R_0 \geq 0, R_1 \geq 0, R_2 \geq 0) : \\
&R_0 + R_1 \leq I_1 \\
&R_0 + R_2 \leq I_2 - I(U_2; X_1 | U_0 V_0) \\
&R_0 + R_1 + R_2 \leq I_1 + J_2 - I(U_1 X_1; U_2 | U_0 V_0) \\
&R_0 + R_1 + R_2 \leq J_1 + I_2 - I(U_1 X_1; U_2 | U_0 V_0) \\
&2R_0 + R_1 + R_2 \leq I_1 + I_2 - I(U_1 X_1; U_2 | U_0 V_0) \Big\},
\end{aligned}$$

where the quantities (I_i, J_i) *with* $i = \{1, 2\}$ *are given by*

$$\begin{aligned}
I_1 &\doteq \min\big\{I(U_0 U_1; Z_1 | X_1 V_0), I(U_1 U_0 X_1 V_0; Y_1)\big\}, \\
J_1 &\doteq \min\big\{I(U_1; Z_1 | X_1 U_0 V_0), I(U_1 X_1; Y_1 | U_0 V_0)\big\}, \\
I_2 &\doteq I(U_2 U_0 V_0; \hat{Z}_2 Y_2 | X_2), \; J_2 \doteq I(U_2; \hat{Z}_2 Y_2 | X_2 U_0 V_0),
\end{aligned}$$

and the set of all admissible PDs $\mathscr{P}$ *is defined as*

$$\begin{aligned}
\mathscr{P} \doteq \big\{&P_{V_0 U_0 U_1 U_2 X_1 X_2 X Y_1 Y_2 Z_1 Z_2 \hat{Z}_2} = P_{V_0} P_{X_2} P_{X_1 | V_0} \\
&P_{U_0 | V_0} P_{U_2 U_1 | X_1 U_0} P_{X | U_2 U_1} P_{Y_1 Y_2 Z_1 Z_2 | X X_1 X_2} P_{\hat{Z}_2 | X_2 Z_2}, \\
&I(X_2; Y_2) \geq I(Z_2; \hat{Z}_2 | X_2 Y_2), \\
&(V_0, U_0, U_1, U_2) \ominus (X_1, X_2, X) \ominus (Y_1, Z_1, Y_2, Z_2)\big\}.
\end{aligned}$$

In particular it is interesting to look at the case where we are interested only in common information R_0. Then the corollary below follows directly of previous theorem:

Corollary 1 (common-information) *A lower bound on the capacity of the compound (or common-message BRC) relay channel is given by*

$$R_0 \leq \max_{P_{X_1 X_2 X} \in \mathscr{P}} \min\big\{I(X; Z_1 | X_1), I(X, X_1; Y_1), \\ I(X; \hat{Z}_2 Y_2 | X_2)\big\}.$$

This corollary shows that the Block Markov coding using backward decoding can be used for CF case as well without the performance loss. So the same code can be used for both DF and CF which means that the code is oblivious toward the cooperative strategy. The oblivious codes are of high practical importance because they can be used in random cooperative networks where the source is not aware of the relay cooperative strategy.

Now consider the case of broadcast relay channel with common relay Fig. 1(c). Assume that the relay is using DF strategy. The following theorem proves an inner bound on this channel: [4]

Theorem 2.2 *An inner bound on the capacity region of the BRC-CR is given by the set of the rates (R_1, R_2) satisfying:*

$$\begin{aligned}
&\mathscr{R}_I^{(BRC)} \doteq \bigcup_{p_{UVX_1X} \in \mathscr{P}} \Big\{(R_0 \geq 0, R_1 \geq 0): \\
&R_0 \leq \ I(U,V;Y_2) - I(U;X_1|V), \\
&R_0 + R_1 \leq \min\big\{I(X;Z_1|X_1,V), I(X,X_1;Y_1)\big\} \\
&R_0 + R_1 \leq \min\big\{I(X;Z_1|X_1,U,V), I(X,X_1;Y_1|U,V)\big\} \\
&\qquad\qquad +I(U,V;Y_2) - I(U;X_1|V)\Big\}
\end{aligned}$$

for all PDs $\mathscr{P}$

$$\begin{aligned}
&P_{UVX_1X} = P_{X|UX_1}P_{X_1U|V}P_V, \\
&(U,V) \ominus (X_1,X) \ominus (Y_1,Z_1,Y_2).
\end{aligned}$$

It will be shown later that this rate is capacity achieving for a class of broadcast relay channels. The following theorem states an upper bound over the capacity of BRC:

Theorem 2.3 (outer bound BRC) *The capacity region $\mathscr{C}_{BRC}$ of the BRC is included in the set $\mathscr{C}_{BRC}^{out}$ of all rates (R_0, R_1, R_2) satisfying*

$$\begin{aligned}
\mathscr{C}_{BRC}^{out} = co &\bigcup_{P_{VV_1U_1U_2X_1} \in \mathscr{Q}} \Big\{(R_0 \geq 0, R_1 \geq 0, R_2 \geq 0): \\
R_0 &\leq \min\big\{I(V;Y_2), I(V;Y_1)\big\}, \\
R_0 + R_1 &\leq \min\big\{I(V;Y_1), I(V;Y_2)\big\} + I(U_1;Y_1|V), \\
R_0 + R_2 &\leq \min\big\{I(V;Y_1), I(V;Y_2)\big\} + I(U_2;Y_2|V), \\
R_0 + R_1 &\leq \min\big\{I(V,V_1;Y_1,Z_1|X_1), I(V,V_1;Y_2,Z_2)\big\} \\
&\qquad + I(U_1;Y_1,Z_1|V,V_1,X_1), \\
R_0 + R_2 &\leq \min\big\{I(V,V_1;Y_1,Z_1|X_1), I(V,V_1;Y_2,Z_2)\big\} \\
&\qquad + I(U_2;Y_2,Z_2|V,V_1,X_1), \\
R_0 + R_1 + R_2 &\leq I(V;Y_1) + I(U_2;Y_2|V) + I(U_1;Y_1|U_2,V), \\
R_0 + R_1 + R_2 &\leq I(V;Y_2) + I(U_1;Y_1|V) + I(U_2;Y_2|U_1,V), \\
&\qquad + I(U_1;Y_1,Z_1|X_1,U_2,V,V_1), \\
R_0 + R_1 + R_2 &\leq I(V,V_1;Y_2,Z_2) + I(U_1;Y_1,Z_1|V,V_1,X_1) \\
&\qquad + I(U_2;Y_2,Z_2|X_1,U_1,V,V_1)\Big\},
\end{aligned}$$

where $co\{\cdot\}$ denotes the convex hull and $\mathscr{Q}$ is the set of all joint PDs $P_{VV_1U_1U_2X}$ satisfying $X_1 \ominus V_1 \ominus (V,U_1,U_2,X)$.

If relays are not present, i.e., $Z_1 = Z_2 = X_1 = X_2 = V_1 = \emptyset$, it is not difficult to see that the previous bound reduces to the outer bound for general broadcast channels refers to as UVW-outer bound [8]. Furthermore, it was recently shown that such bound is at least as good as all the currently developed outer bounds for the capacity region of broadcast channels

2.3 Capacity Results

In this section capacity results are presented. We define first two class of a broadcast relay channels with common relay.

Definition 3 (degraded BRC) *A broadcast relay channel with common relay (BRC-CR) (as is shown in Fig. 1(c)) is said to be (or semi) degraded if the stochastic mapping $\big\{W : \mathscr{X} \times \mathscr{X}_1 \times \mathscr{X}_2 \longmapsto \mathscr{Y}_1 \times \mathscr{Z}_1 \times \mathscr{Y}_2 \times \mathscr{Z}_2\big\}$ satisfies one of the following Markov chains:*

(I) $X \ominus (X_1, Z_1) \ominus (Y_1, Y_2)$ *and* $(X, X_1) \ominus Y_1 \ominus Y_2$,

(II) $X \ominus (X_1, Z_1) \ominus Y_2$ *and* $X \ominus (Y_1, X_1) \ominus Z_1$,

where conditions (I) and (II) are referred to as degraded and semi-degraded BRC-CR, respectively.

The degraded BRC-CR is combination of the degraded relay channel with degraded broadcast channel. On the other hand, semi-degraded case is the combination of the degraded and reversely degraded relay channel.

Now we are ready to give the following capacity results.

Theorem 2.4 *The capacity region of the semi-degraded BRC-CR is given by the following rate region*

$$\begin{aligned}
\mathscr{C}_{II} \doteq &\bigcup_{P_{UX_1X} \in \mathscr{P}} \Big\{(R_0 \geq 0, R_1 \geq 0): \\
R_0 &\leq \min\{I(U,X_1;Y_2), I(U;Z_1|X_1)\} \\
R_0 + R_1 &\leq \min\{I(U,X_1;Y_2), \\
&\qquad I(U;Z_1|X_1)\} + I(X;Y_1|X_1,U)\Big\},
\end{aligned}$$

where $\mathscr{P}$ is the set of all joint PDs P_{UX_1X} satisfying that $U \ominus (X_1, X) \ominus (Y_1, Z_1, Y_2)$ where $|\mathscr{U}| \leq |\mathscr{X}|\,|\mathscr{X}_1| + 2$.

Finally we give the capacity result of Gaussian channel. First we define the degraded Gaussian BRC-CR as

$$\begin{aligned}
Y_1 = X + X_1 + \mathcal{N}_1, \quad Y_2 = X + X_1 + \mathcal{N}_2, \\
Z_1 = X + \tilde{\mathcal{N}}_1
\end{aligned}$$

where the source and the relay have power constraints P, P_1, and $\mathcal{N}_1, \mathcal{N}_2, \tilde{\mathcal{N}}_1$ are independent Gaussian noises with variances $N_1, N_2, \tilde{N}_1$, respectively, such that the noises $\mathcal{N}_1, \mathcal{N}_2, \tilde{\mathcal{N}}_1$ satisfy the necessary Markov conditions in definition 3. Note that it is enough to suppose the physical degradedness of receivers respect to the relay and the stochastic degradedness of one receiver respect to another. It means that there exist $\mathcal{N}, \mathcal{N}'$ such that:

$$\mathcal{N}_1 = \tilde{\mathcal{N}}_1 + \mathcal{N}, \quad \mathcal{N}_2 = \tilde{\mathcal{N}}_1 + \mathcal{N}'.$$

and also $N_1 < N_2$. The following theorem states the capacity of degraded Gaussian BRC with common relay similar to [9].

Theorem 2.5 *The capacity region of the degraded Gaussian BRC-CR is*

$$\begin{aligned}
R_0 &\leq C\left(\frac{\alpha(P + P_1 + 2\sqrt{\overline{\beta}PP_1})}{\overline{\alpha}(P + P_1 + 2\sqrt{\overline{\beta}PP_1}) + N_2}\right), \\
R_1 &\leq C\left(\frac{\overline{\alpha}(P + P_1 + 2\sqrt{\overline{\beta}PP_1})}{N_1}\right), R_1 \leq C\left(\frac{\beta\gamma P}{\tilde{N}_1}\right)
\end{aligned}$$

$$R_0 + R_1 \leq C\left(\frac{\beta P}{\tilde{N}_1}\right)$$

where $0 \leq \beta, \alpha, \gamma \leq 1$.

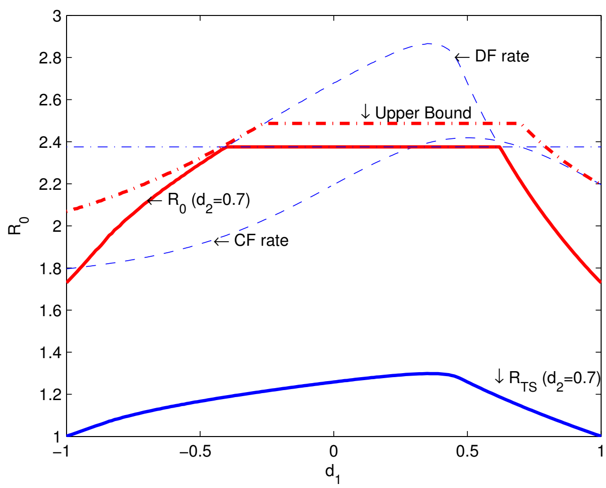


FIG. 2: Gaussian BRC with DF-CF strategies

3 Gaussian Example: Oblivious Coding

In this section we show the practical application of oblivious coding in the theorem 1. Consider first lower and upper bounds on the common-rate for the DF-CF region. The definition of the channels are as follows:

$$Y_{1i} = \frac{X_i}{\sqrt{d_{y_1}^{\delta}}} + \frac{X_{1i}}{\sqrt{d_{z_1y_1}^{\delta}}} + \mathcal{N}_{1i}, \quad \text{and} \quad Z_{1i} = \frac{X_i}{\sqrt{d_{z_1}^{\delta}}} + \tilde{\mathcal{N}}_{1i},$$
$$Y_{2i} = \frac{X_i}{\sqrt{d_{y_2}^{\delta}}} + \frac{X_{2i}}{\sqrt{d_{z_2y_2}^{\delta}}} + \mathcal{N}_{2i}, \quad \text{and} \quad Z_{2i} = \frac{X_i}{\sqrt{d_{z_2}^{\delta}}} + \tilde{\mathcal{N}}_{2i}.$$

The channel inputs $\{X_i\}$ and the relay inputs $\{X_{1i}\}$ and $\{X_{2i}\}$ must satisfy the power constraints $\sum_{i=1}^{n} X_i^2 \leq nP$ and $\sum_{i=1}^{n} X_{ki}^2 \leq nP_k$ for $k = \{1, 2\}$.

We set $X = U + \sqrt{\frac{\overline{\beta}P}{P_1}}X_1$ and evaluate Corollary 1. The goal is to send common-information at rate R_0. It is easy to verify that the two DF rates result in $R_{DF} \leq$

$$\min\left\{C\left(\frac{\beta P}{d_{z_1}^{\delta}\tilde{N}_1}\right), C\left(\frac{\frac{P}{d_{y_1}^{\delta}} + \frac{P_1}{d_{z_1y_1}^{\delta}} + 2\sqrt{\frac{\overline{\beta}PP_1}{d_{y_1}^{\delta}d_{z_1y_1}^{\delta}}}}{N_1}\right)\right\},$$

where the CF rate $I(U, X_1; Y_2, \hat{Z}_2|X_2)$ follows as

$$R_{CF} \leq C\left(\frac{P}{d_{y_2}^{\delta}N_2} + \frac{P}{d_{z_2}^{\delta}(\widehat{N}_2 + \tilde{N}_2)}\right). \tag{1}$$

Observe that the rate (1) is exactly the same as the Gaussian CF [6]. This means that DF regular encoding can also be decoded with the CF strategy, as well for the case with collocated relay and receiver. By using the proposed coding it is possible to send common information at the minimum rate between CF and DF schemes $R_0 = \min\{R_{DF}, R_{CF}\}$.

Fig. 2 shows numerical evaluation of R_0 for the common-rate case. All channel noises are set to the unit variance and $P = P_1 = P_2 = 10$. The distance between X and (Y_1, Y_2) is 1, while $d_{z_1} = d_1$, $d_{z_1y_1} = 1 - d_1$, $d_{z_2} = d_2$, $d_{z_2y_2} = 1 - d_2$. The position of the relay 2 is assumed to be fixed to $d_2 = 0.7$ but the relay 1 moves with $d_1 \in [-1, 1]$. This setting serves to compare the performances of our coding schemes regarding the position of the relay. It can be seen that one can achieves the minimum between the two possible CF and DF rates. These rates are also compared with a naive time-sharing strategy which consists in using DF scheme τ% of the time and CF scheme $(1-\tau)$% of the time[1]. Time-sharing yields the achievable rate

$$R_{TS} = \max_{0 \leq \tau \leq 1} \min\{\tau R_{DF}, (1-\tau)R_{CF}\}.$$

Notice that with the proposed coding scheme significant gains can be achieved when the relay is close to the source (i.e. DF scheme is more suitable), comparing to the worst case.

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[1] One should not confuse time-sharing in compound settings with conventional time-sharing which yields convex combination of rates.