Skeletonization of Noisy Images via the Method of Moment

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Abstract – *In this paper, we propose a novel approach to robust skeletonization, that is developed based on a statistical method using the Zernike moment theory controlled by Maximum Entropy Principal (MEP). This new concept of skeletonization is articuled into three steps. In the first one, estimation of the underlying probability density function (pdf) using Zernike moment is carried out. In the second, the estimation of optimal pdf is selected using MEP criterion. Finally, the subset of local maxima pixels of the optimal pdf are selected as belonging to the skeleton. This new method is applied on noisy and free noisy binary images. We have tested the proposed Zernike Moment Skeletonization Method (ZMSM) on a variety of real and simulated noisy images, it produces excellent and visually appealing results, with comparison to some well known traditional methods.*

1. Introduction

The skeleton is widely recognized as one of the most important descriptors in image processing and pattern recognition. Since the first study by Blum [1], the Skeletonization of shapes has attracted attentions from many researchers in various fields. Commonly used computational methods for skeleton extraction include topological thinning [2-4], approaches based on distance transform [5], [6], hierarchical methods based on Voronoi diagrams [7], voxel coding based methods [8], and some approaches based on physical simulations [9] or curve evolution [10]. These approaches present several advantages, however the main drawback of most of these methods is their high sensitivity to noise.

In this paper a novel skeletonization approach, is developed using a statistical method based on the estimation of probability density function pdf where the skeleton is defined as the local maxima of this pdf.

Our proposed approach is based on the expansion of a multivariate function pdf in terms of Zernike polynomials by means of Zernike moment. For this purpose the pdf is approximated by a truncated series of polynomials. As the determination of the expansion order is a difficult problem in the framework of unsupervised classification, we propose the determination of the optimal order for which the estimated pdf has maximum entropy.

As the solution to this problem is mathematically too complexe to be tractable, we introduce an exhaustive search for the optimal order. We propose to estimate the pdf for different orders and to select the optimal one as the one for which the entropy reaches a maximum according to the Maximum Entropy Principal MEP [11-15]. This latter has been used for clustering, see for example [16] and for image restoration [17]. Having the optimal pdf, the true points of the skeleton are the local maxima of the pdf. Extraction of the local maxima of the pdf is carried out using the last phase of the proposed algorithm.

As a summary, our proposed ZMSM skeletonization method based on the combination of the moment theory and MEP as a selection criterion, is composed of the three following steps:

1- Computation of the pdf using the Zernike moment.
2- Estimation of the optimal pdf using MEP method.
3- Extraction of the local maxima of the optimal pdf taken as the skeleton points.

The paper is organized as follows: the next section describes the basis of our statistical model, using Zernike moment. The maximum entropy principal is given in section 3. The details of our skeletonization algorithm is presented in section 4. Section 5 performs main results and performances of our skeletonization method. Finally section 6 deals with the summary of important results and conclusions of this work.

2. Statistical Modelisation using Zernike Moment

2.2 Zernike Moments Computation

The use of Zernike moments in image analysis was pioneered by Teague [18]. Since then, the Zernike moments have been frequently utilized for a number of image processing and computer vision tasks [19-25]. In order to define the Zernike moments, we need to introduce the concept of Zernike functions.

The (p,q) order Zernike function is defined as [22]:

$$V_{p,q}(x,y)=R_{p,q}(\rho)\exp(jq\phi), \quad x^2+y^2\le 1 \quad (1)$$

where

p: Positive integer or zero.

q: Positive and negative integers subject to constraints $p-|q|$ even and $|q|\le p$.

ρ : Length of the vector from origin to (x,y) pixel .

ϕ : Angle between vector ρ and x-axis in counterclockwise direction.

In (1), $R_{p,q}(\rho)$ is a polynomial in ρ of degree $p\ge 0$ containing no power of ρ less than $|q|$, the form of this polynomial is:

$$R_{p,q}(\rho)=\sum_{s=0}^{p-|q|/2}\frac{(-1)^s(p-s)!}{s!(\frac{p+|q|}{2}-s)!(\frac{p-|q|}{2}-s)}\rho^{p-2s} \quad (2)$$

These polynomials are orthogonal and satisfy:

$$\iint_D V^*_{p,q}(x,y)V_{p',q'}(x,y)\,dxdy=\frac{\pi}{(p+1)}\delta_{p,p'}\delta_{q,q'} \quad (3)$$

With $\delta_{a,b}=\begin{cases}1 & a=b\\ 0 & \text{oterwise}\end{cases}$

Zernike moments are the projection of the image function onto these orthogonal functions, the Zernike moment of order p with repetition q for a continuous image function $f(x,y)$ that vanishes outside the unit circle is:

$$A_{p,q}=\frac{(p+1)}{\pi}\iint_{x^2+y^2\le 1} f(x,y)\,V^*_{p,q}(\rho,\phi)\,dx\,dy \quad (4)$$

The aforementioned favorable properties of the Zernike moments are valid as long as one use a true analog image function. In practice, the Zernike moments have to be computed from sampled data, i.e., the rectangular sampling of the original image function $f(x,y)$, producing the set of samples $f(x_i,y_j)$ with an (M,N) array of pixels, thus we define the discrete version of $A_{p,q}$ in terms of summation by the traditional commonly used formula [20].

$$\tilde{A}_{p,q}=\frac{p+1}{\pi}\sum_{x_i}\sum_{y_j} f(x_i,y_j)V^*_{p,q}(\rho,\phi), \quad x_i^2+y_j^2\le 1 \quad (5)$$

Liao and Pawlak in [21,22], proposed a modified version of the Zernike moment given by:

$$\hat{A}_{p,q}=\frac{p+1}{\pi}\sum_{x_i}\sum_{y_j} f(x_i,y_j)H_{p,q}(x_i,y_j), \quad x_i^2+y_j^2\le 1 \quad (6)$$

Where $H_{p,q}(x_i,y_j)=\int_{x_i-\Delta x/2}^{x_i+\Delta x/2}\int_{y_j-\Delta y/2}^{y_j+\Delta y/2} V^*_{p,q}(\rho,\phi)\,dx\,dy$ (7)

Represents the integration of $V^*_{p,q}(\rho,\phi)$ over (x_i,y_j) pixel.

2.2 Estimation of the Probability Density Function

By taking the orthogonality principle, of the Zernike polynomials, into consideration, the image function $f(x,y)$ can be written as an infinite series expansion in terms of the Zernike polynomials over the unit disk:

$$f(x,y)=\sum_{p=0}^{\infty}\sum_{q=-p}^{p}A_{p,q}V_{p,q}(\rho,\phi), \quad p-|q| \text{ even} \quad (8)$$

Suppose that one knows all moments $A_{p,q}$ of $f(x,y)$ up to a given order θ. It is desired to reconstruct a discrete function $f_\theta(x,y)$ where moments exactly match those of $f(x,y)$ up to given order θ:

$$f_\theta(x,y)=\sum_{p=0}^{\theta}\sum_{q=-p}^{p}A_{p,q}V_{p,q}(\rho,\phi), \quad p-|q| \text{ even} \quad (9)$$

Note that since $A_{p,-q}=A^*_{p,q}$, then $|A_{p,q}|=|A_{p,-q}|$, thus one can concentrate on $|A_{p,q}|$ with $q\ge 0$, as far as the defined Zernike moments are concerned in the reconstruction process. The reconstructed image function can be written as:

$$\hat{f}_\theta(x,y)=\sum_{p=0}^{\theta}\sum_{q=0}^{p}\hat{A}_{p,q}V_{p,q}(\rho,\phi), \quad p-|q| \text{ even} \quad (10)$$

Let $\hat{p}(x_i,y_j)$ be the estimated probability density function obtained by normalizing $\hat{f}(x_i,y_j)$ [13], [15]:

$$\hat{p}(x_i,y_j)=\frac{\hat{f}(x_i,y_j)}{\sum_{x_i,y_j\in\Omega}\hat{f}(x_i,y_j)} \quad (11)$$

where $\sum_{x_i,y_j\in\Omega}\hat{p}(x_i,y_j)=1$ (12)

and $0\le\hat{p}(x_i,y_j)\le 1$, Ω is the image plane.

The estimated pdf depends only on the expansion order θ, a criterion for choosing this order is explained in the next paragraph according to the maximum entropy principal.

3. Optimal Order Moments Selection using MEP

We introduce the maximum entropy principle MEP for the search of this optimal order, this automatic technique can estimate the optimal number of moments directly from the available data and does not require any a priori image information specially for noisy images.

Let G_w be a set of estimated underlying probability density function for various Zernike moment orders θ:

$$G_w=\{\hat{p}_\theta \,/\, \theta=1........\omega\} \quad (13)$$

By applying the maximum entropy principle for noisy images, we deduce that among these estimates of the probability density function, there is one and only one probability density function denoted $\hat{p}^*_\theta(x_i,y_j)$ whose entropy is maximum [13],[17] and which represents the optimal probability density function, and then gives the optimal order of moments.

The Shannon entropy of $\hat{p}^*_\theta(x_i,y_j)$ is defined as:

$$S(\hat{p}_\theta) = -\sum_{x_i,y_j\in\Omega} \hat{p}_\theta(x_i,y_j)\log(\hat{p}_\theta(x_i,y_j)) \tag{14}$$

and the optimal $\hat{p}^*_\theta$ is such that

$$S(\hat{p}^*_\theta) = MAX\{S(\hat{p}_\theta)\ /\ \hat{p}_\theta \in GW\} \tag{15}$$

The process of determinating the optimal order θ consists in estimating the pdf for different orders and selecting the optimal one as the one for which the entropy reaches maximum. The following is basic algorithm which consists in an exhaustive search to determine the optimal order which maximises $S(\hat{p}^*_\theta)$:

1- Initialise θ

2- Compute the pdf $\hat{p}_\theta$ and its corresponding Shannon entropy $S(\hat{p}_\theta)$

3- If $S(\hat{p}_\theta)$ is maximum, then θ is optimal and $\hat{p}_\theta = \hat{p}^*_\theta$, else $\theta=\theta+1$ and go to 2.

Then, having $\hat{p}^*_\theta$, we assign to each point of the optimal pdf $\hat{p}^*_\theta(x_i,y_j)$ defined by (11). In this case, the "good data" are the set of points belonging to the mode of $\hat{p}^*_\theta$. By extracting the local maxima of $\hat{p}^*_\theta$, we can determine the exact points of the skeleton. In the next section the details of our skeleton extraction algorithm is presented.

4. Skeleton Extraction

We define the skeleton as the local maxima of the estimated probability density function selected in the previous section. The extraction of these local maxima allows us to determine the skeleton associated to the shape. The general idea of this algorithm consists of a successive points extraction presenting a local maxima of the selected optimal pdf.

The procedure consists in making a sweep mask of size 3x3 on all the image. The comparison of the pdf estimated for the central pixel of the mask with its close eight neighbours following the eight directions (Fig.1), allows to confirm if this central pixel is a point of the skeleton or no.

P(i-1,j-1)	P(i,j-1)	P(i+1,j-1)
P(i-1,j)	P(i,j)	P(i+1,j)
P(i-1,j+1)	P(i,j+1)	P(i+1,j+1)

Fig. 1. The pixel (i,j) and its eight close neighbour

Indeed two types of comparison are undertaken, a comparison following lines and columns and a comparison following the diagonal. A pixel is a point candidate if it presents a local maximum compared to its four neighbours following the lines and column direction (Fig.1c) or if it present a local maxima compared to its four neighbours following the diagonal direction (Fig.1d).

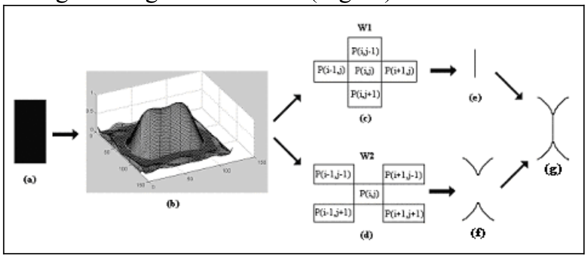


Fig. 2. Rectangle shape skeleton obtained by ZMSM approach: a) rectangle shape, b) optimal pdf corresponding to order 8, c) the mask w1 following lines and columns direction, d) the mask w2 following diagonal direction, e) extracted skeleton following lines and columns, f) extracted skeleton following the diagonal, g) resulting skeleton of the rectangle shape

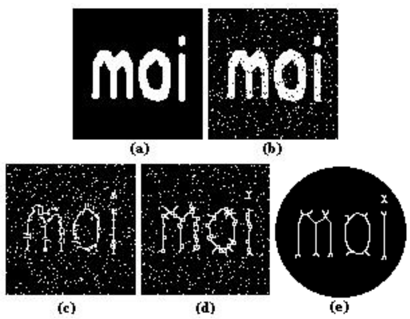
5. Experimental Results

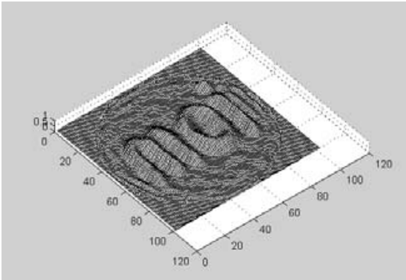
In this section, a comparison study is carried out on simulated and real images. The proposed ZMSM skeletonization method is compared to Distance Transform [5] and Parallel Thinning Algorithms [3].

The first example demonstrates the performance of the proposed skeletonization method with respect to noise. The experiment is performed on a hand-written word "moi" scanned and binarised on (100x100) image matrice (Fig.3a), then corrupted by an impulsive noise affected 10% of pixels (Fig.3b). The comparison of the skeletons generated by our algorithm (Fig.3e) with skeletons obtained using distance transform (Fig.3c) and parallel thinning algorithm (Fig.3d) illustrates clearly the insensitivity of the proposed method to noise.

Figure 4 shows the performance and the potential of the proposed approach and its insensitivity to different noise values. The ZMSM approach is applied to hand-written digit "6" With different noise values. One notices that the greater the noise level, the greater is the optimal order obtained by our ZMSM method, the presented figures show that our method performs well even with high noise levels.

Another example investigating the behaviour of the ZMSM method is presented applied to a 'plane' scanned and binarized on 100x100 image matrix (Fig.5a) corrupted by an gaussian noise having a (Signal to Noise Ratio) SNR=10db (Fig.5b). The pdf obtained by MEP corresponding to optimal moment order is presented in (Fig.5d). The skeleton obtained in Fig.5c by our approach ZMSM demonstrates the consistency of our algorithm against high level gaussian noise.





(f)

Fig. 3. Skeletonization of Hand-written word 'moi' by the proposed approach. a) original image. b) input noisy image with impulsive noise affecting 10% of pixels. c) skeleton obtained using the Distance Transform. d) skeleton obtained using the Parallel Thinning Algorithms. e) skeleton obtained using the proposed ZMSM. f) estimated pdf for optimal order 36.

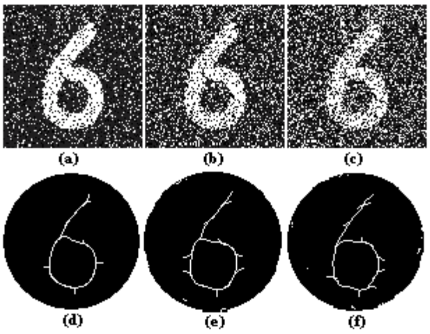
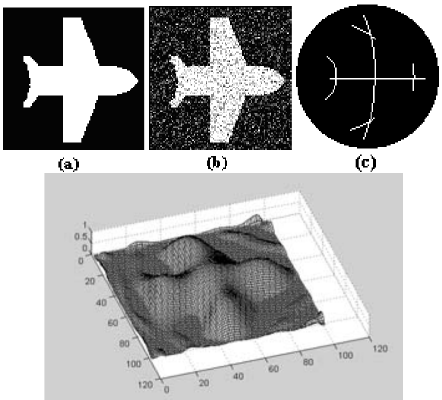


Fig. 4. Skeletonization of Hand-written digit '6' by the proposed approach. Input noisy image, with impulsive noise affecting 15% of pixels (a), 25% of pixels (b), 40% of pixels (c), corresponding skeletons obtained with ZMSM method for orders 21 (d), 22 (e), 24 (f).



(d)

Fig. 5. Skeletonization of 'plane' shape by the proposed ZMSM. a) original image and b) corresponding gaussian noisy image with SNR=10db. c) Skeleton for the noisy 'plane' shape by the proposed ZMSM. e) estimated pdf for optimal order 9.

6. Conclusion

In this work, we have proposed a statistical technique for skeletonization noisy images using the Zernike moment theory and the Maximum Entropy Principal. The advantages of our algorithm is that no a priori information about the original image is required. Consequently, the practical implementation of the approach does not require any parameter setting. Through a comparative study with conventional methods, it performed quite well in experimental tests and the skeletonization has been greatly improved which demonstrates the robustness of the proposed approach against different and high noise levels.

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