

Multi-Split Widely Linear LMS Algorithm for Channel Equalization

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Résumé – Dans cet article, nous présentons la version « multi-split » de l’algorithme LMS largement linéaire. Également au filtrage linéaire conventionnel, la transformé « multi-split » augmente le facteur de diagonalisation de la matrice composée d’autocorrélation et de pseudoautocorrélation du signal d’entrée impropre, et un algorithme LMS avec un pas d’adaptation variable et normalisé à la puissance est utilisé pour adapter les coefficients des filtres. Des résultats de simulation évaluent la performance de l’algorithme « multi-split » LMS largement linéaire pour l’égalisation adaptative d’un canal.

Abstract – In this paper, we present the multi-split version of the widely linear LMS algorithm. As in conventional linear filtering, the multi-split transform increases the diagonalization factor of the composed autocorrelation and pseudoautocorrelation matrix of the improper input signal, and a power normalized and time-varying step-size LMS algorithm is used for updating the filter parameters. Simulation results assess the performance of the multi-split widely linear LMS algorithm for adaptive channel equalization.

1. Introduction

Widely linear (WL) processing has been extensively used in multiuser detection, blind and non-blind equalization [1, 2], beamforming [3] and MIMO systems [4]. In the presence of improper complex-valued sequences, it provides significant performance improvements when compared to conventional linear filtering.

Typically, an improper process appears when a real-valued signal (e. g., from an M-PAM alphabet) is transmitted through a complex baseband channel. Several communication systems can be modeled according to this scenario, for instance, transmission with OQAM (Offset Quadrature Amplitude Modulations), the GSM system (Global System for Mobile Communication) and systems transmitting with binary CPM (Continuous-Phase Modulation) and modulation index $h = \frac{1}{2}$ [1].

In adaptive systems, due to its simplicity and robustness, the standard LMS algorithm is the most widely used algorithm for updating the WL filter parameters. However, the performance of the WL-LMS algorithm in terms of convergence rate and tracking capability depends inherently on the eigenvalue spread of the input signal correlation matrix [5].

Recently, a low computational burden multi-split (MS) preprocessing of the input signal has been proposed for improving the performance of the LMS algorithm [6, 7]. After preprocessing, the adaptive FIR filter is realized as a

set of zero-order filters connected in parallel, and with each single coefficient independently updated. Such a technique can be viewed as a transform domain filter, in which multi-split preprocessing is applied to the input data vector.

In this paper, we incorporate the multi-split transform into the WL-LMS algorithm in order to improve its performance. The resulting MS-WL-LMS algorithm presents a faster convergence rate than the WL-LMS and normalized WL-LMS (WL-NLMS [8]) algorithms. It is worth to stress that the application of the multi-split transform in widely linear adaptive filtering has not been yet considered in the literature.

The paper is organized as follows. Next section presents briefly the multi-split transform. The application of widely linear processing in channel equalization is discussed in Section 3. Section 4 is devoted to analyzing the incorporation of the MS transform into WL processing. Simulation results are shown in Section 5 and, finally, some final remarks are drawn in Section 6.

2. Multi-Split Transform

Figure 1 shows the classical scheme of transversal filtering. Consider initially that all the parameters are real-valued. When the number of tap-weights of the FIR filter is $N=2^L$, $L \geq 1$, the continued splitting process of the filter impulse response in symmetric and antisymmetric parts can be represented by the filtering scheme shown in Figure 2 [6], where

$$\mathbf{M}_N = \begin{bmatrix} \mathbf{M}_{N/2} & \mathbf{J}_{N/2} \mathbf{M}_{N/2} \\ \mathbf{J}_{N/2} \mathbf{M}_{N/2} & -\mathbf{M}_{N/2} \end{bmatrix}. \quad (1)$$

This work was supported by the Brazilian National Council for Scientific and Technological Development (CNPq), under grants X302568/2006-6, and the CAPES-COFECUB program under grants 544/07.

$\mathbf{J}_{N/2}$ is the $N/2$ -by- $N/2$ exchange matrix, which has unit elements along the cross diagonal and zeros elsewhere, $\mathbf{M}_1=[1]$ and $w_{\perp i}$, for $i = 0, 1, \dots, N-1$, are the single coefficients of the zero-order filters connected in parallel. It can be verified that \mathbf{M}_N is a matrix of $+1$'s and -1 's, in which the inner product of any two distinct columns is zero. In fact, \mathbf{M}_N is a nonsingular matrix and $\mathbf{M}_N^T \mathbf{M}_N = \mathbf{I}_N$, where \mathbf{I}_N is the N -by- N identity matrix.

The above multi-split scheme can be viewed as a linear transformation of the input data, which is given by

$$\mathbf{x}_{\perp}(n) = \mathbf{M}_N \mathbf{x}(n), \quad (2)$$

where $\mathbf{x}(n)=[x(n) \ x(n-1) \ \dots \ x(n-N+1)]^t$ denotes the tap-input vector and $\mathbf{x}_{\perp}(n)=[x_{\perp 0}(n) \ x_{\perp 1}(n) \ \dots \ x_{\perp N-1}(n)]^t$, with a butterfly structure that is very suitable for VLSI implementation. As it has been pointed out in [6], the multi-split transform does not reduce the eigenvalue spread of the input signal correlation matrix, but it does improve its diagonalization factor.

In the adaptive context, a power-normalized and time-varying step-size LMS algorithm, which exploits the nature of the transformed input correlation matrix, has been proposed for updating the single coefficients independently.

As far as a filter with complex parameters is concerned, it has been shown in [7] that the decomposition of the filter impulse response into conjugated symmetric and antisymmetric parts can be accomplished by means of the separated split decomposition of its real and imaginary parts.

The multi-split LMS (MS-LMS) algorithm is described by:

$$w_{\perp i}(n) = w_{\perp i}(n-1) + \frac{\mu_{MS}}{r_i(n)} x_{\perp i}(n) e(n) \quad (3)$$

and

$$r_i(n) = \frac{\sum_{j=1}^n \gamma^{n-j} |x_{\perp i}(j)|^2}{\sum_{j=1}^n \gamma^{n-j}} = \frac{p_i(n)}{q_i(n)}, \quad (4)$$

for $i=0, 1, \dots, N-1$, where $p_i(n)$ and $q_i(n)$ are recursively calculated by:

$$p_i(n) = |x_{\perp i}(n)|^2 + \gamma p_i(n-1) \quad (5)$$

and

$$q_i(n) = 1 + \gamma q_i(n-1), \quad (6)$$

$$e(n) = d(n) - y(n), \quad (7)$$

$$y(n) = \sum_{i=0}^{N-1} x_{\perp i}(n) w_{\perp i}(n), \quad (8)$$

μ_{MS} denotes the step size and γ ($0 << \gamma \leq 1$) is a forgetting factor. The case $\gamma=1$ applies to wide-sense stationary environments.

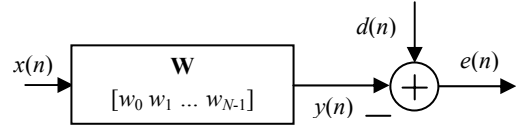


FIG. 1: Transversal filtering.

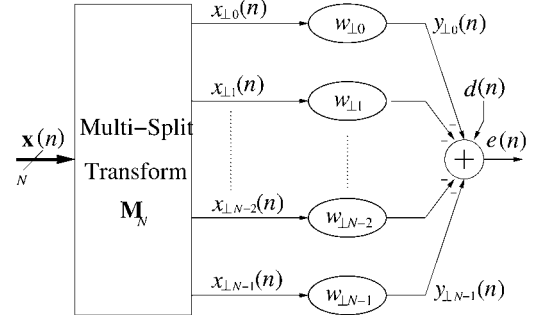


FIG. 2: Multi-split filtering.

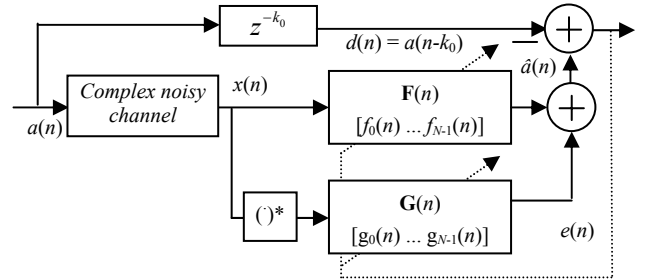


FIG. 3: Widely linear trained equalizer.

3. Widely Linear Equalizer

Widely linear processing (WLP) is used to estimate a desired random signal $d(n)$ based on the observation of a random signal $x(n)$ that is complex and also improper. A signal is said to be improper if its pseudoautocorrelation, given by

$$C(m) = E\{x(n+m)x(n)\}, \quad (9)$$

is nonzero ($C(m) \neq 0$). In other words, an improper process exhibits a nonvanishing pseudoautocorrelation.

It has been shown that WL processing gives a better estimate, in the MMSE (minimum mean square error) sense, than strictly linear processing, when the observation signal is improper. This processing uses both $x(n)$ and its conjugate $x^*(n)$ to estimate $d(k)$, as shown in Figure 3 for adaptive channel equalization [1, 9]. The optimum filters \mathbf{F}_0 and \mathbf{G}_0 are obtained in order to minimize $E\{|e(n)|^2\}$, where k_0 denotes a delay that should be chosen carefully and $e(n) = a(n-k_0) - \hat{a}(n)$ is the estimation error.

The scheme in Figure 3 can be viewed as shown in Figure 4, which corresponds to the polyphase representation of a fractionally spaced equalization (FSE) with sample rate $2/T$, where T is the symbol interval. Furthermore, it is well known that, for zero-forcing (ZF) equalization of polyphase channels, the ZF FIR equalizer

needs to have the same subchannel order, since the subchannels have disjoint roots, e.g., it has no real roots and no conjugate pair of roots [10].

Since the transmitted sequence $a(n)$ in Figure 3 is real-valued, it is easy to demonstrate that $\mathbf{G}_0 = \mathbf{F}_0^*$, which means that only one filter needs to be update with about the same computational complexity required for conventional linear equalization. In fact, since the WL equalizer can have the same channel order to satisfy the open-eye condition [1,6], the computational complexity can be greatly reduced.

Throughout this paper, we assume that the channel is truly complex, e.g., $\text{Re}\{\mathbf{C}\} \neq \mathbf{0}$ and $\text{Im}\{\mathbf{C}\} \neq \mathbf{0}$, where $\text{Re}\{\mathbf{C}\}$ and $\text{Im}\{\mathbf{C}\}$ denote the real and imaginary parts of \mathbf{C} , respectively. Such a condition usually holds for several applications [1]. The complex *wide-sense stationary* (WSS) additive Gaussian noise $\eta(n)$ is assumed to be proper, with variance σ_η^2 .

Finally, the WL-LMS algorithm with fix step-size is described by [1, 8]:

$$\begin{aligned} \hat{a}(n) &= \mathbf{F}^H(n) \mathbf{x}(n) + \mathbf{G}^H(n) \mathbf{x}^*(n) \\ e(n) &= a(n - k_0) - \hat{a}(n) \\ \mathbf{F}(n+1) &= \mathbf{F}(n) + \mu_{wl} e^*(n) \mathbf{x}(n) \\ \mathbf{G}(n+1) &= \mathbf{F}^*(n+1). \end{aligned} \quad (10)$$

4. Analysis of the Widely Linear Correlation Matrix

In this section, we analyze the effect of applying the multi-split transform to the WL processing, concerning the diagonalization factor and the eigenvalue spread of the widely linear correlation matrix, composed of the autocorrelation and pseudoautocorrelation of the improper input signal.

The WL correlation matrix \mathbf{R} is given by [2, 3]:

$$\mathbf{R} = \begin{bmatrix} \mathbf{\Gamma} & \mathbf{C} \\ \mathbf{C}^* & \mathbf{\Gamma}^* \end{bmatrix}, \quad (11)$$

where $\mathbf{\Gamma} = E\{\mathbf{x}(n)\mathbf{x}(n)^H\}$ and $\mathbf{C} = E\{\mathbf{x}(n)\mathbf{x}(n)^t\}$. The applications of the multi-split transform to the input data yields:

$$\mathbf{\Gamma}_\perp = E\{\mathbf{M}_N \mathbf{x}(n) \mathbf{x}^H(n) \mathbf{M}_N^H\} = 2^N \mathbf{M}_N^{-1} \mathbf{\Gamma} \mathbf{M}_N, \quad (12)$$

$$\mathbf{C}_\perp = E\{\mathbf{M}_N \mathbf{x}(n) \mathbf{x}^t(n) \mathbf{M}_N^H\} = 2^N \mathbf{M}_N^{-1} \mathbf{C} \mathbf{M}_N, \quad (13)$$

and the transformed WL correlation matrix is given by

$$\mathbf{R}_\perp = \begin{bmatrix} \mathbf{\Gamma}_\perp & \mathbf{C}_\perp \\ \mathbf{C}_\perp^* & \mathbf{\Gamma}_\perp^* \end{bmatrix}. \quad (14)$$

Considering that each WL filter has only two coefficients, we have:

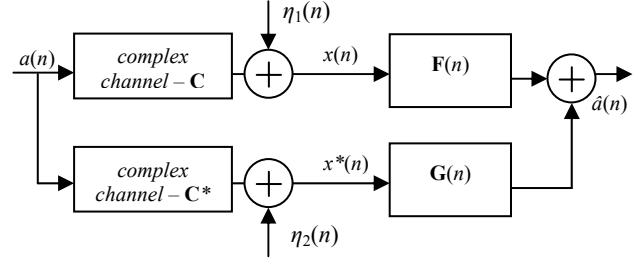


FIG. 4: Equivalent polyphase model of WL-equalizer.

$$\begin{aligned} \mathbf{\Gamma}_\perp &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} r_{00} & r_{01} \\ r_{10} & r_{11} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ &= \begin{bmatrix} r_{00} + r_{01} + r_{10} + r_{11} & r_{00} - r_{01} + r_{10} - r_{11} \\ r_{00} + r_{01} - r_{10} - r_{11} & r_{00} - r_{01} - r_{10} + r_{11} \end{bmatrix}. \end{aligned} \quad (15)$$

Since $r_{00} = r_{11}$ and $r_{01} = r_{10}^*$, we can rewrite $\mathbf{\Gamma}_\perp$ as

$$\mathbf{\Gamma}_\perp = 2 \begin{bmatrix} r_{00} + \text{Re}(r_{01}) & -j\text{Im}(r_{01}) \\ j\text{Im}(r_{01}) & r_{00} - \text{Re}(r_{01}) \end{bmatrix}. \quad (16)$$

In the same way, \mathbf{C}_\perp is given by

$$\mathbf{C}_\perp = 2 \begin{bmatrix} c_{00} + c_{01} & 0 \\ 0 & c_{00} - c_{01} \end{bmatrix}, \quad (17)$$

and, therefore,

$$\mathbf{R}_\perp = 2 \begin{bmatrix} r_{00} + \text{Re}(r_{01}) & -j\text{Im}(r_{01}) & c_{00} + c_{01} & 0 \\ j\text{Im}(r_{01}) & r_{00} - \text{Re}(r_{01}) & 0 & c_{00} - c_{01} \\ c_{00}^* + c_{01}^* & 0 & r_{00} + \text{Re}(r_{01}) & j\text{Im}(r_{01}) \\ 0 & c_{00}^* - c_{01}^* & -j\text{Im}(r_{01}) & r_{00} - \text{Re}(r_{01}) \end{bmatrix}. \quad (18)$$

So, it can be verified that the diagonalization factor of \mathbf{R}_\perp , defined by

$$\gamma(\mathbf{R}_\perp) = \frac{\text{trace}(\mathbf{R}_\perp)}{\sum_i \sum_j |\mathbf{R}_\perp(i, j)| - \text{trace}(\mathbf{R}_\perp)}, \quad (19)$$

is increased when compared to the diagonalization factor of \mathbf{R} . Moreover, we can verify that $\mathbf{R}_\perp = 2^N \mathbf{K}^{-1} \mathbf{R} \mathbf{K}$, where

$$\mathbf{K} = \begin{bmatrix} \mathbf{M}_N & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_N \end{bmatrix}, \quad (20)$$

which means that the matrices \mathbf{R} and $\mathbf{K}^{-1} \mathbf{R} \mathbf{K}$ are similar and, consequentially, \mathbf{R}_\perp and \mathbf{R} have the same eigenvalue spread.

Based on the above characteristics of the transformed WL correlation matrix, a power normalized and time-varying step-size LMS algorithm (Table 1) is proposed for updating the WL filter parameters, as in conventional adaptive linear filtering [6].

TAB. 1 : MS-WL-LMS Algorithm

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- 1) *Linear transform:* $\mathbf{x}_\perp(n) = \mathbf{M}_N \mathbf{x}$
 - 2) *Updating:*

$$y(n) = \mathbf{F}_\perp^H(n) \mathbf{x}_\perp(n) + \mathbf{G}_\perp^H(n) \mathbf{x}^*_{\perp}(n)$$

$$e(n) = d(n) - y(n)$$

$$r_i(n) = p_i(n) / q_i(n) \quad (\text{eqs. (5) and (6)})$$

$$f_{\perp i}(n) = f_{\perp i}(n-1) + \mu_{MS} e(n) x^*_{\perp i}(n) / r_i(n)$$

$$\mathbf{G}_\perp(n) = \mathbf{F}_\perp^*(n)$$
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5. Simulation Results

In order to illustrate the performance of the MS-WL-LMS algorithm for channel equalization, we consider a complex discrete-time channel with impulse response $h_l(z) = (0,3921+0,3921j) + (0,0392+0,745j)z^{-1} + (0,051+0,2548j)z^{-2} + (0,132+0,136j)z^{-3} + (0,1068+0,1129j)z^{-4} + (0,0423+0,0819j)z^{-5}$. Such a channel corresponds to a non-minimum phase channel that possesses a severe distortion.

The transmitted sequence was taken from a 4-PAM constellation. The signal-to-noise ratio, defined by

$$SNR = 10 \log_{10} \left((\sigma_n^2 + \sigma_a^2 \sum_{m=0}^L |h(m)|^2) / \sigma_n^2 \right), \quad (21)$$

was set to 30 dB. The equalizer filters have $N=8$ taps. The simulation results were obtained from an average of 400 independent trials. The performance of the MS-WL-LMS, WL-NLMS [6] and WL-LMS algorithms are compared.

The learning curves are presented in Figure 5. It can be observed that the MS-WL-LMS algorithm presents the fastest convergence rate. The chosen delay was $k_0=2$ (Figure 3). We used a step size $\mu_{MS}=1/32$ and a forgetting factor $\gamma=0.9995$ for the MS-WL-LMS algorithm. The step sizes for the WL-NLMS and WL-LMS algorithms were $\mu_{NWL}=0.25$ and $\mu_{WL}=0.005$, respectively.

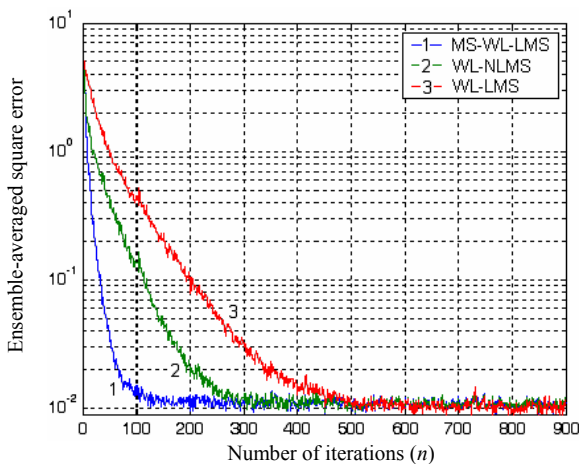


FIG. 5: Averaging learning curves for the algorithms.

6. Conclusion

In the present paper, we have applied the multi-split transform into widely linear processing. As in conventional linear filtering, it has been shown that the multi-split transform also increases the diagonalization factor of the composed autocorrelation and pseudoautocorrelation matrix of the improper input signal. So, a power normalized and time-varying step-size LMS algorithm has been proposed for updating the filter parameters, in order to improve the convergence rate. The better performance of the MS-WL-LMS algorithm in terms of convergence rate for adaptive channel equalization has been verified by simulation.

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