The Constrained Trilinear Decomposition With Application to MIMO Wireless Communication Systems

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Résumé – Dans cet article, nous présentons une nouvelle décomposition tensorielle qui consiste à décomposer un tenseur du troisième ordre en une somme triple de facteurs tensoriels de rang-1 avec des intéractions entre les différents facteurs. La structure d'intéraction est contrôlée par trois *matrices de contraintes* composées par des vecteurs canoniques. Nous présentons une application de cette décomposition à des systèmes de communication sans-fil MIMO (*Multiple-Input Multiple-Output*). Une nouvelle structure de transmission est proposée, où les matrices de contraintes de la décomposition sont exploitées pour configurer des *précodeurs canoniques*. La détection aveugle est possible grâce aux propriétés d'unicité partielle de la décomposition. Pour illustrer cette application, le taux d'erreur de bit est évalué pour quelques choix de précodeurs.

Abstract – In this paper, we present a new tensor decomposition that consists in decomposing a third-order tensor into a triple sum of rank-one tensor factors, where interactions involving the components of different factors are allowed. The interaction pattern is controlled by three *constraint matrices* composed of canonical vectors. An application of this decomposition to Multiple-Input Multiple-Output (MIMO) wireless communication systems is presented. A new multiple-antenna transmission structure is proposed, where the constraint matrices of the decomposition are exploited to design *canonical precoders*. Blind detection is possible thanks to the partial uniqueness properties of the decomposition. For illustrating this application, we evaluate the bit-error-rate performance for some precoder configurations.

1 Introduction

Tensor decompositions can be viewed as extensions of matrix decompositions to orders higher than two. One of the most popular tensor decompositions is the third-order Parallel Factor (PARAFAC) decomposition which decomposes a third-order tensor in a trilinear sum of rank-one third-order tensors [1]. This decomposition has been exploited and generalized in several works for solving different signal processing problems such as blind source separation using higher order statistics [2,3], multiuser detection/equalization [4–7], and space-time coding/spreading [8–11], just to mention a few.

In this paper, we formulate a new tensor decomposition with constraints. We consider the decomposition of a third-order tensor in a "constrained triple sum" of rank-one third-order tensors, where different interactions involving the factors of the decomposition are allowed. Such an interaction pattern is controlled by three *constraint matrices*, the columns of which are canonical vectors. Each constraint matrix is associated with a given dimension or *mode* of the tensor. An application of the new decomposition to MIMO wireless communications is presented. A new multiple-antenna transmission model is formulated exploring the constrained structure of the decomposition. The core of the proposed MIMO system is a precoder tensor that controls the joint coupling of multiple data streams, spreading codes and transmit antennas to generate the transmitted signal. Several transmit schemes can be designed by varying the structure of the three precoder constraint matrices. Based on the resulting tensor model for the received signal, we study the implication of the partial uniqueness property of this decomposition to blind symbol/code/channel recovery. For illustration purposes, we evaluate the bit-error-rate performance for some configurations of the precoder constraint matrices.

This paper is organized as follows. In Section 2, the trilinear decomposition with constraints is formulated. Partial uniqueness of this decomposition is investigated in Section 3. In Section 4, we present a multiple-antenna transmission system exploiting the constrained structure of the proposed decomposition. This section also discusses the blind symbol/code/channel recovery. Some simulation results are provided in Section 5 for bit-error-rate performance evaluation. Section 6 concludes the paper and perspectives are drawn.

2 Trilinear tensor decomposition with constraints

Let us consider a third-order tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$, three component matrices $\mathbf{A} \in \mathbb{C}^{I_1 \times R_1}$, $\mathbf{B} \in \mathbb{C}^{I_2 \times R_2}$, $\mathbf{C} \in \mathbb{C}^{I_3 \times R_3}$, and three *constraint matrices* $\mathbf{\Psi} \in \mathbb{C}^{R_1 \times F}$, $\mathbf{\Phi} \in \mathbb{C}^{R_2 \times F}$, $\mathbf{\Omega} \in \mathbb{C}^{R_3 \times F}$. The trilinear decomposition of $\mathcal{X}$ with F constrained factor combinations is given by:

$$x_{i_1,i_2,i_3} = \sum_{f=1}^{F} \sum_{r_1=1}^{R_1} \sum_{r_2=1}^{R_2} \sum_{r_3=1}^{R_3} a_{i_1,r_1} b_{i_2,r_2} c_{i_3,r_3} \psi_{r_1,f} \phi_{r_2,f} \omega_{r_3,f}, \tag{1}$$

where $a_{i_1,r_1} = [\mathbf{A}]_{i_1,r_1}$, $b_{i_2,r_2} = [\mathbf{B}]_{i_2,r_2}$, $c_{i_3,r_3} = [\mathbf{C}]_{i_3,r_3}$ are entries of the component matrices $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$, respectively. Similarly, $\psi_{r_1,f} = [\mathbf{\Psi}]_{r_1,f}$, $\phi_{r_2,f} = [\mathbf{\Phi}]_{r_2,f}$, $\omega_{r_3,f} = [\mathbf{\Omega}]_{r_3,f}$ are entries of the constraint matrices $\mathbf{\Psi}, \mathbf{\Phi}$ and $\mathbf{\Omega}$, respectively. The structure of these constraint matrices follows two assumptions:

A.1 $\mathbf{\Psi}$, $\mathbf{\Phi}$, and $\mathbf{\Omega}$ are composed of canonical vectors[1] belonging, respectively, to the bases:

$$\{\mathbf{e}_1^{(R_1)}, \ldots, \mathbf{e}_F^{(R_1)}\} \in \mathbb{R}^{R_1}, \quad \{\mathbf{e}_1^{(R_2)}, \ldots, \mathbf{e}_F^{(R_2)}\} \in \mathbb{R}^{R_2}, \quad \{\mathbf{e}_1^{(R_3)}, \ldots, \mathbf{e}_F^{(R_3)}\} \in \mathbb{R}^{R_3};$$

A.2 $\mathbf{\Psi}$, $\mathbf{\Phi}$ and $\mathbf{\Omega}$ are full row-rank matrices.

As a consequence of these assumptions, we have:

$$\mathbf{\Psi}\mathbf{\Psi}^T = \mathrm{diag}(\boldsymbol{\mu}_A), \quad \mathbf{\Phi}\mathbf{\Phi}^T = \mathrm{diag}(\boldsymbol{\mu}_B), \quad \mathbf{\Omega}\mathbf{\Omega}^T = \mathrm{diag}(\boldsymbol{\mu}_C),$$

where $\boldsymbol{\mu}_A$, $\boldsymbol{\mu}_B$ and $\boldsymbol{\mu}_C$ are the *generating vectors* of $\mathbf{\Psi}$, $\mathbf{\Phi}$ and $\mathbf{\Omega}$, respectively. These vectors parameterize the three constraint matrices. They determine the *recombination factor* of the columns of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, respectively.

Alternative writing: This decomposition can be stated in a different manner, which sheds light on a different way of interpreting its constrained structure. By exchanging summations in (1), we obtain:

$$x_{i_1,i_2,i_3} = \sum_{r_1=1}^{R_1}\sum_{r_2=1}^{R_2}\sum_{r_3=1}^{R_3} a_{i_1,r_1} b_{i_2,r_2} c_{i_3,r_3} g_{r_1,r_2,r_3}(\mathbf{\Psi},\mathbf{\Phi},\mathbf{\Omega}),$$

where

$$g_{r_1,r_2,r_3}(\mathbf{\Psi},\mathbf{\Phi},\mathbf{\Omega}) = \sum_{f=1}^{F} \psi_{r_1,f}\phi_{r_2,f}\omega_{r_3,f} \tag{2}$$

is an element of a tensor $\mathcal{G}(\mathbf{\Psi},\mathbf{\Phi},\mathbf{\Omega}) \in \mathbb{C}^{R_1\times R_2\times R_3}$ that follows an F-factor PARAFAC decomposition in terms of $\mathbf{\Psi}$, $\mathbf{\Phi}$ and $\mathbf{\Omega}$. We call $\mathcal{G}(\mathbf{\Psi},\mathbf{\Phi},\mathbf{\Omega}) \in \mathbb{C}^{R_1\times R_2\times R_3}$, or simply $\mathcal{G}$, the *constrained core tensor* of the decomposition.

3 Uniqueness issues

The uniqueness (up to permutation and scaling) of the factor matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ in (1) depends on the particular structure of the constraint matrices $\mathbf{\Psi}$, $\mathbf{\Phi}$, $\mathbf{\Omega}$. In general, the degrees of freedom introduced in the decomposition by the three constraint matrices can induce a transformational ambiguity over (at least a subset of) the columns of the factor matrices.

Definition 1: *The decomposition (1) is said to be partially unique (or restrictively nonunique), when a subset of the columns belonging to the set* $\{\mathbf{A},\mathbf{B},\mathbf{C}\}$ *are essentially unique while the remaining columns are affected by a linear transformation.*

Partial uniqueness was first observed in [12]. In [14], a partial uniqueness proof is presented for the PARALIND decomposition. In our case, partial uniqueness is directly linked to the joint structure of the three constraint matrices. We study sufficient (but not necessary) conditions for the partial uniqueness of (1) implying essential uniqueness in two modes.

[1]A canonical vector $\mathbf{e}_n^{(N)} \in \mathbb{R}^N$ is a unitary vector containing an element equal to 1 in its n-th position and 0's elsewhere.

Definition 2: *When $R_1 = R_2$ (resp. $R_2 = R_3$ and $R_1 = R_3$), the matrix set $\{\mathbf{\Psi},\mathbf{\Phi}\}$ (resp. $\{\mathbf{\Phi},\mathbf{\Omega}\}$ and $\{\mathbf{\Omega},\mathbf{\Psi}\}$) is said to be* equivalent *if:*

$$\mathbf{\Psi} = \mathbf{\Pi}_1\mathbf{\Phi}, \quad \Big(\textit{resp. } \mathbf{\Phi} = \mathbf{\Pi}_2\mathbf{\Omega}, \quad \mathbf{\Omega} = \mathbf{\Pi}_3\mathbf{\Psi}\Big), \tag{3}$$

where $\mathbf{\Pi}_{i=1,2,3}$ are arbitrary permutation matrices.

The equivalence of two constraint matrices means that there is *no* interaction (or factor recombinations) involving the columns of the associated factor matrices. For instance, when $\mathbf{\Psi} = \mathbf{\Pi}_1\mathbf{\Phi}$, we have:

$$\mathbf{G}^{(1,2)} = \mathbf{\Psi}\mathbf{\Phi}^T = \mathbf{\Pi}_1\mathbf{\Phi}\mathbf{\Phi}^T = \mathbf{\Pi}_1\mathrm{diag}(\boldsymbol{\mu}_B), \tag{4}$$

where $\mathbf{G}^{(1,2)}$ is the interaction matrix between the first and second modes. Similarly, when $\mathbf{\Phi} = \mathbf{\Pi}_2\mathbf{\Omega}$ and $\mathbf{\Omega} = \mathbf{\Pi}_3\mathbf{\Psi}$, we have:

$$\mathbf{G}^{(2,3)} = \mathbf{\Pi}_2\mathrm{diag}(\boldsymbol{\mu}_C), \quad \mathbf{G}^{(3,1)} = \mathbf{\Pi}_3\mathrm{diag}(\boldsymbol{\mu}_A), \tag{5}$$

$\mathbf{G}^{(2,3)}$ and $\mathbf{G}^{(3,1)}$ being the two other interaction matrices. Such an equivalence between pairs of constraint matrices implies the essential uniqueness of the corresponding factor matrices, since any transformational freedom involving the columns of these matrices is eliminated [15].

Partial uniqueness results: Based on such a concept of equivalence between pairs of constraint matrices, we can deduce the following partial uniqueness results:

R.1 If $R_1 = R_2 < R_3$ and $\{\mathbf{\Psi},\mathbf{\Phi}\}$ is an equivalent set, then $\mathbf{A}$ and $\mathbf{B}$ are essentially unique;

R.2 If $R_2 = R_3 < R_1$ and $\{\mathbf{\Phi},\mathbf{\Omega}\}$ is an equivalent set, then $\mathbf{B}$ and $\mathbf{C}$ are essentially unique;

R.3 If $R_1 = R_3 < R_2$ and $\{\mathbf{\Psi},\mathbf{\Omega}\}$ is an equivalent set, then $\mathbf{A}$ and $\mathbf{C}$ are essentially unique.

It is worth mentioning that the essential uniqueness in two modes comes at the expense of a restrictive nonuniqueness in the remaining mode. Such a "uniqueness tradeoff" is inherent to the trilinear decomposition with constraints.

4 Application to MIMO wireless systems: canonical precoding

We consider a point-to-point MIMO system with M transmit antennas and K receive antennas. At the transmitter, R input data streams are transmitted using J spreading codes and M transmit antennas. The proposed transmission model consists in: i) generating F output signals to be transmitted by spreading R input data streams with the aid of J spreading codes and then ii) associating these F output signals with the M transmit antennas. Let P denote the spreading factor of the system. Each input data stream is a packet of N symbols. Let us define $s_{n,r} \doteq s\big((r-1)N+n\big)$ as the n-th symbol of the r-th data stream, $c_{p,j}$ be the p-th chip of the j-th symbol periodic spreading code and $h_{k,m}$ be the spatial channel gain between the m-th transmit antenna and the k-th receive antenna. Each transmit/receive antenna response is characterized by an independent Rayleigh flat fading. The matrices $\mathbf{S} \in \mathbb{C}^{N\times R}, \mathbf{C} \in \mathbb{C}^{P\times J}$, and $\mathbf{H} \in \mathbb{C}^{K\times M}$ define, respectively, the *symbol*, *code* and *channel* matrices, $s_{n,r} \doteq [\mathbf{S}]_{n,r}$, $\quad c_{p,j} \doteq [\mathbf{C}]_{p,j}$, $\quad h_{k,m} \doteq [\mathbf{H}]_{k,m}$.

4.1 Canonical precoding model

The signal to be transmitted is modeled by the sum of F *precoded* signal components. Let $g_{r,j,m}$ be the (r,j,m)-th element of the *precoder tensor* $\mathcal{G} \in \mathbb{C}^{R\times J\times M}$. This tensor determines the allocation of the r-th data stream and the j-th spreading code to the m-th transmit antenna. The F-factor decomposition of the precoder tensor is given, in scalar form, by the following "constrained" decomposition:

$$g_{r,j,m}(\mathbf{\Psi},\mathbf{\Phi},\mathbf{\Omega}) = \sum_{f=1}^{F} \psi_{r,f}\phi_{j,f}\omega_{m,f}. \quad (6)$$

$\mathbf{\Psi} \in \mathbb{C}^{R\times F}$, $\mathbf{\Phi} \in \mathbb{C}^{J\times F}$, and $\mathbf{\Omega} \in \mathbb{C}^{M\times F}$ are *stream reuse*, *code reuse* and *antenna reuse* matrices, respectively. The structure of these matrices follows assumptions **A.1** – **A.2**. For instance, $\psi_{r,f}\phi_{j,f}\omega_{m,f} = 1$ means that the r-th data stream is spread by the j-th spreading code and then transmitted by the m-th transmit antenna.

4.2 Transmitted/received signal models

The precoded signal tensor is represented by the third-order tensor $\mathcal{U} \in \mathbb{C}^{N\times P\times M}$ with typical element $u_{n,p,m}$. The precoded signal associated with the n-th symbol, p-th chip, and m-th transmit antenna is defined as $u_{n,p,m} \doteq u_m\big((n-1)P+p\big)$. It is given by:

$$u_{n,p,m} = \sum_{r=1}^{R}\sum_{j=1}^{J} s_{n,r}c_{p,j}\, g_{r,j,m}(\mathbf{\Psi},\mathbf{\Phi},\mathbf{\Omega}). \quad (7)$$

Example: Consider a precoding scheme with $F = 4$ precoded signals, $R = 2$ data streams, $M = 3$ transmit antennas, and $J = 2$, 3 or 4 orthogonal spreading codes. The structure of $\mathbf{\Psi}$ and $\mathbf{\Omega}$ is as follows:

$$\mathbf{\Psi} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad \mathbf{\Omega} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (8)$$

From the structure of $\mathbf{\Psi}$ and $\mathbf{\Omega}$, we can see that each data stream is simultaneously transmitted by two transmit antennas. We consider three code reuse patterns and the three choices for $\mathbf{\Phi}$ are:

$$\mathbf{\Phi} = \mathbf{\Psi}\ (J=2), \quad \mathbf{\Phi} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\ (J=3),$$
$$\mathbf{\Phi} = \mathbf{I}_4\ (J=4). \quad (9)$$

Note that the first scheme reuses twice both spreading codes. The second one reuses only the first spreading code, while the third one uses different spreading codes.

Following a chip-matched filter, the discrete-time complex baseband received signal at the n-th symbol period, p-th chip and k-th receive antenna is defined as $x_{n,p,k} \doteq x_k\big((n-1)P+p\big)$, $x_{n,p,k}$ being the (n,p,k)-th element of the *received signal tensor* $\mathcal{X} \in \mathbb{C}^{N\times P\times K}$ collecting the received samples associated with N symbols, P chips and K receive antennas. Using (7), $x_{n,p,k} = [\mathcal{X}]_{n,p,k}$ can be written, in absence of noise, as:

$$x_{n,p,k} = \sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{m=1}^{M} s_{n,r}c_{p,j}h_{k,m}\, g_{r,j,m}(\mathbf{\Psi},\mathbf{\Phi},\mathbf{\Omega}) \quad (10)$$

The following correspondences can be deduced by comparing (2) with (10):

$$(I_1,I_2,I_3,R_1,R_2,R_3,F) \leftrightarrow (N,P,K,R,J,M,F),$$
$$(\mathbf{A},\mathbf{B},\mathbf{C}) \leftrightarrow (\mathbf{S},\mathbf{C},\mathbf{H}). \quad (11)$$

The received signal model (10) can be represented in matrix form. The full information contained in the tensor $\mathcal{X} \in \mathbb{C}^{N\times P\times K}$ can be organized in three *unfolded matrices* $\mathbf{X}_1 \in \mathbb{C}^{KN\times P}$, $\mathbf{X}_2 \in \mathbb{C}^{NP\times K}$, and $\mathbf{X}_3 \in \mathbb{C}^{PK\times N}$. Their construction is similar to that of the standard PARAFAC decomposition [1, 4]. These matrices are defined as:

$$[\mathbf{X}_1]_{(k-1)N+n,p} = [\mathbf{X}_2]_{(n-1)P+p,k} = [\mathbf{X}_3]_{(p-1)K+k,n} = x_{n,p,k}.$$

$\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$ admit the following factorization [15]:

$$\mathbf{X}_1 = \big(\mathbf{H\Omega}\diamond\mathbf{S\Psi}\big)\big(\mathbf{C\Phi}\big)^T, \quad \mathbf{X}_2 = \big(\mathbf{S\Psi}\diamond\mathbf{C\Phi}\big)\big(\mathbf{H\Omega}\big)^T,$$
$$\mathbf{X}_3 = \big(\mathbf{C\Phi}\diamond\mathbf{H\Omega}\big)\big(\mathbf{S\Psi}\big)^T, \quad (12)$$

where $\diamond$ is the Khatri-Rao product.

4.3 Blind detection

The final goal of the proposed MIMO wireless system is the blind recovery of the transmitted data without training sequences or *a priori* explicit channel knowledge/estimation. The partial uniqueness results of Section 3 establish equivalences between pairs of constraint matrices that lead to different blind symbol/code/channel recovery properties. In this work, we assume $\max(R,J,M) < F$ (i.e data streams, spreading codes and transmit antennas are reused more than once). We consider two different precoder configurations:

1. $R = J \le M \le F$: Equal number of data streams and codes;
2. $M = R \le J \le F$: Equal number of data streams and transmit antennas.

Resorting to the partial uniqueness results **R.1** and **R.3** given in Section 3, we can obtain the following results:

- Configuration 1: If $\mathbf{\Psi}$ and $\mathbf{\Phi}$ are equivalent (data streams and spreading codes have equivalent reuse factors), then both $\mathbf{S}$ and $\mathbf{C}$ are essentially unique, i.e. *blind joint symbol-code* recovery is achieved;
- Configuration 2: If $\mathbf{\Psi}$ and $\mathbf{\Omega}$ are equivalent (data streams and transmit antennas have equivalent reuse factors), then both $\mathbf{S}$ and $\mathbf{H}$ are essentially unique, i.e. *blind joint symbol-channel* recovery is achieved.

Receiver algorithm: The receiver algorithm capitalizes on model (12) to blindly estimate the symbol ($\mathbf{S}$) and channel ($\mathbf{H}$) matrices in presence of an Additive White Gaussian (AWG) noise by means of the Alternating Least Squares (ALS) algorithm [4]. Define $\tilde{\mathbf{X}}_i = \mathbf{X}_i + \mathbf{V}_i$, $i = 1,2,3$, as the noisy versions of $\mathbf{X}_i$, where $\mathbf{V}_i$ is an additive complex-valued AWG matrix. The constraint matrices as well as the spreading code matrix are known at the receiver, and they are fixed during the whole estimation process. In this case, the ALS algorithm consists of the following estimation steps:

1. Set $i = 0$; Randomly initialize $\widehat{\mathbf{S}}_{(i=0)}$;
2. $i = i + 1$;

3. Using $\tilde{\mathbf{X}}_2$, find an LS estimate of $\mathbf{H}_{(i)}$:
   $\widehat{\mathbf{H}}_{(i)}^T = \left[(\widehat{\mathbf{S}}_{(i)}\mathbf{\Psi} \diamond \mathbf{C\Phi})\mathbf{\Omega}^T\right]^\dagger \tilde{\mathbf{X}}_2;$
4. Using $\tilde{\mathbf{X}}_3$, find an LS estimate of $\mathbf{S}_{(i)}$:
   $\widehat{\mathbf{S}}_{(i)}^T = \left[(\mathbf{C\Phi} \diamond \widehat{\mathbf{H}}_{(i-1)}\mathbf{\Omega})\mathbf{\Psi}^T\right]^\dagger \tilde{\mathbf{X}}_3;$
5. Repeat steps 2-4 until convergence.

The convergence at the i-th iteration is declared when the error between the received signal tensor and its reconstructed version from the estimated factor matrices does not significantly change between iterations i and $i+1$.

5 Simulation results

We present some simulation results for illustrating the Bit-Error-Rate (BER) performance of the proposed MIMO system. The BER results are an average over 100 Monte Carlo runs. At each run, the additive noise power is generated according to the sample SNR value given by:

$$\text{SNR} = 10\log_{10}\frac{\|\mathbf{X}_1\|_F^2}{\|\mathbf{V}_1\|_F^2}\ \text{dB}.$$

The spatial channel gains are redrawn from an i.i.d. complex-valued Gaussian generator while the transmitted symbols are redrawn from a pseudo-random Quaternary Phase Shift Keying (QPSK) sequence. The BER curves represent the performance averaged on the R transmitted data streams. We assume $K = 2$, $N = 50$, and $P = 4$ (Hadamard(4) codes are used). Figure 1 depicts the Bit-Error-Rate (BER) performance of a precoding scheme with $F = 4$. The precoder constraint matrices are those given in the example of Sec. 4.2, with $J = 2$, 3 or 4 spreading codes (c.f. (8)-(9)). As expected, the performance improves at the expense of using more spreading codes. The schemes with $J = 2$ or 3 spreading codes have essentially the same performance, except for high SNR values, where the scheme with $J = 3$ codes is better. From the slope of the BER curves, we can also observe that a higher diversity gain is obtained with the third scheme using different spreading codes (no spreading code reuse). Such a gain comes at the expense of using more spreading codes.

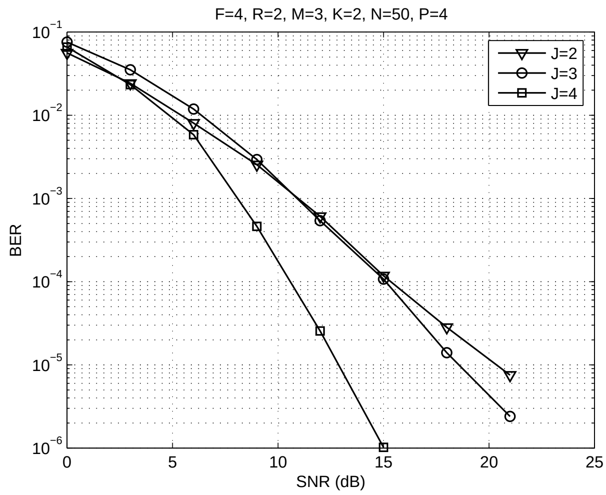

FIG. 1: BER vs. SNR performance ($F = 4$).

6 Conclusion and perspectives

This paper has presented a new decomposition of third-order tensor with constraints. This decomposition is capable of characterizing any kind of interaction pattern between factors associated with the first, second and third modes of the tensor. An application of the constrained trilinear decomposition to MIMO wireless communications has been presented. We have proposed a canonical precoding tensor model for designing multiple-antenna transmission schemes for MIMO systems. Different transmission schemes can be designed from proper choices of the three constraint matrices of the decomposition. Perspectives of this work include the study of uniqueness and the development of efficient algorithms. The determination of the number F of factor combinations when it is not known is an open issue that deserves more investigation.

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