Decision Directed Algorithms for Blind Equalization Based on Constant Modulus Criteria

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Résumé – Le but de cet article est de proposer une famille de techniques d'égalisation aveugle de signaux du type QAM. Les algorithmes proposés sont dirigés par la décision et basés sur des fonctions de coût du type Module Constante modifiées. Deux approches sont considérées: l'approche du Gradient Stochastique et l'approche du *Normalized Constant Modulus Algorithm* (NCMA). Les simulations numériques confirment les résultats attendus et montrent que les techniques proposées améliorent les performances des algorithmes classiques basés sur la fonction de coût CM.

Abstract – A family of techniques aiming to perform blind equalization for Quadrature Amplitude Modulation (QAM) signals is presented. The proposed algorithms are decision directed and based on modified Constant Modulus (CM) criteria. Two approaches are used to develop the algorithms: the Stochastic Gradient Descent approach and the Normalized Constant Modulus Algorithm (NCMA) approach. Computer simulations confirm the expected results and show that the proposed algorithms outperform the conventional CM based algorithms.

1 Introduction

The Constant Modulus Algorithm (CMA), developed independently by Godard [1] and Treichler [2], is one of the most used techniques to perform blind equalization and it works very well for modulations in which all points of the signal constellation have the same radius, like Phase Shift Keying (PSK) modulations. However, when the constellation points are characterized by multiple radii, the estimation error obtained with algorithms based on the CM criterion does not reach zero, even if the channel is perfectly equalized. This is one of the reasons for the unsatisfactory performance of conventional CM algorithms with QAM signals. It can only achieve a moderate level of Steady-State Error (SSE). Moreover, we can say that the speed of convergence is another important drawback of the CM-type algorithms.

The main contribution of this work is to propose two new classes of algorithms inspired on the CMA to improve its performance for high level QAM signals. The considered cost functions can be viewed as modifications of the CM criterion. In fact, one of the proposed modified CM cost functions can be viewed as a generalization of the CM criterion for constellations with multiple radii. The others can be seen as generalizations of the Modified CM (MCM) criterion, which allows to jointly perform blind equalization and carrier recovery.

It is well-known that the NLMS algorithm outperforms the LMS algorithm. Thus, we will develop normalized versions of the algorithms trying to improve their performance, specially in the case of high level QAM signals. Actually, each family is composed of four algorithms with very desirable properties and advantages over the original CM algorithms (CMA and NCMA). Dual-mode versions of the algorithms are also studied. They start with a robust algorithm, i. e. not decision directed, and after switch to a novel one, aiming to avoid an excessive number of incorrect decisions during initial iterations.

2 Description of the Algorithms

Let us assume that: the transmitted i.i.d. sequence $\{a(n)\}$ can take the value of any constellation symbol with equal probability; the output sequence of the equalizer $\{y(n)\}$ is given by (1), where $\mathbf{h} = [h(0)\ h(1)\ .\ ...\ h(N-1)\ ]$ is the impulse response of the Moving-Average (MA) channel, N is the length of the channel and $v(n)$ is an additive white Gaussian noise (AWGN). The number of taps of the transversal equalizer is M and $\mathbf{w}(n)$ is its tap-weight vector. $\hat{a}(n)$ is the output of the decision device (estimated symbol).

$$y(n) = \sum_{i=1}^{M} w(i)x(n-i),\ where\ x(n) = \sum_{i=0}^{N} h(i)a(n-i)+v(n). \quad (1)$$

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2.1 CMA-type Algorithms

In [3] we proposed a technique inspired by the CMA for blind equalization of QAM signals: the Decision-Directed Modulus Algorithm (DDMA). The DDMA can be seen as a generalization of the CM cost function for multiple radii constellations and it uses the squared magnitude of the decided symbol. The DDM cost function is expressed in Table 1(line 2). Taking the stochastic gradient of the DDM cost function, we obtain the tap-weight vector update equation of the Decision-Directed Modulus Algorithm (DDMA):

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu e(n)\mathbf{x}^*(n). \tag{2}$$

where $e(n)$ is given in Table 1 (line 2). The derivative of $\hat{a}(n)$ relative to $\mathbf{w}(n)$ was assumed to be zero. As we can see, the DDM cost function is able to reach to zero for any symbol of the constellation. We may say that the DDM is a "constellation matched" cost function. For PSK modulations, the DDM and the CM cost functions are equivalent. The main advantage of the DDMA algorithm is its great performance with QAM constellations, in terms of convergence speed and steady-state error (SSE). When perfect equalization is achieved, the second term of the DDMA update equation (2) goes to zero while in CMA it never does so.

In [4] we did an analysis of the minima of the DDM cost function based on the CMA analysis in [6]. It was demonstrated, for the case of real signals, that the Wiener solution is, approximately, a solution of the DDM criterion minimization. It was also shown that the DDMA has convergence properties that are very similar to that of the CMA. However, the DDMA uses information of last decided symbol, which makes the performance of the DDMA worst if the number of incorrect decisions is too large. To solve this problem, the initial adjustment of the tap weight vector can be done by the CMA and when some switching criterion is achieved, the equalization algorithm switches to DDMA. In [3], this algorithm was called *dual-mode DDMA* (or CMA-DDMA). The dual-mode DDMA has a better performance than the DDMA, in terms of convergence speed and SSE.

In [5] it was proposed a very interesting algorithm based on a decomposition of the CM cost function: the Modified Constant Modulus Algorithm (MCMA). The MCM cost function has real and imaginary references instead of one modulus of reference in the CM cost function. These in-phase and in-quadrature references make the MCMA more adapted to QAM signals than the CMA. The MCM cost function is the CM one decomposed into the real and imaginary parts (Table 1 - line 3). The MCMA is able to remove ISI and perform the carrier recovery jointly. It implicitly corrects phase errors and may outperform the conventional CMA with almost the same computational cost.

It is possible to unify the modifications done in the CM cost function to create a new cost function that incorporates the improvements of the DDMA and MCMA together. This cost function will be similar to the MCM cost function, but with variable references to the real and imaginary parts of the signal. We will call this cost function of Modified DDM (MDDM) cost function (Table 1 - line 4). Taking the stochastic gradient of the MDDM cost function, we obtain the tap-weight vector update equation of the Modified DDMA (MDDMA) (eq. 2 and Table 1 - line 4). The MDDMA is also able to perform jointly blind equalization and carrier recovery and it has the advantages of both CMA and DDMA. However, the MDDMA is also decision directed, so it also suffers if the number of incorrect decisions is too large. In this case, we can use the same approach as before: a robust algorithm performs the initial adjustments (MCMA in this case) of the equalizer and after we switch the algorithm.

2.2 NCMA-type Algorithms

The Normalized CMA (NCMA), proposed by a number of authors, e.g. [7], is based on a particular choice of the step-size and does not work very well with high QAM constellations. At each iteration a step-size is chosen such that the updated filter coefficients achieve the desired modulus when applied to current data vector.

Based on the idea behind the DDMA, we can change the NCMA constraint (Table 2 - line 2) to develop an algorithm with good performances for blind equalization with high QAM signals. The solution to this optimization problem can be obtained in a similar way of the development of the NCMA and leads to the Normalized DDMA (NDDMA):

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu}{\|\mathbf{x}(n)\|^2} e(n)\mathbf{x}^*(n). \tag{3}$$

where $e(n)$ is shown in Table 2 (line 2). This new algorithm can be seen as a generalization of the NCMA for multiple radii constellations. The squared magnitude of the NDDMA constraint can assume any of the squared magnitude values of the constellation symbols, allowing more flexibility to the algorithm. So the NDDMA should provide a significant performance improvement in relation to the NCMA, when applied to high level QAM constellations. The NDDMA can also be used as a dual-mode algorithm, but, in this case, the initial algorithm is the NCMA (to have less computational complexity).

A next step in the improvement of this family of algorithms is to employ the idea behind the MCMA. For this we can break the NCMA constraint into two (Table 2 - line 3), and develop the Normalized MCMA (NMCMA) in a similar way of NCMA (eq. 3 and Tab. 2 - line3). It is important to remark that the NMCMA has almost identical complexity than the NCMA. The NMCMA implicitly corrects phase errors and is not decision directed, which means that it is not prejudiced by an incorrect decision.

One can think in using the modifications done in the NCMA by the NDDMA and NMCMA to create a new algorithm that incorporates these improvements together. But if we try to do this (eq. 3 and Table 2 - line 4), the resulting algorithm is equivalent to the well-known Normalized Least Mean Square - Decision Directed (NLMSDD) algorithm. So, we can conclude that the NMCMA is more robust and has a worst performance than the NLMSDD.

Tab. 1: The family of the CMA-type Algorithms

Algorithms	Cost Functions	Estimation Errors - $e(n)$
CMA	$\mathrm{E}\{(\lvert y(n)\rvert^2 - R)^2\}$	$y(n)(\lvert y(n)\rvert^2 - R)$
DDMA	$\mathrm{E}\{(\lvert y(n)\rvert^2 - \lvert\hat{a}(n)\rvert^2)^2\}$	$y(n)(\lvert y(n)\rvert^2 - \lvert\hat{a}(n)\rvert^2)$
MCMA	$\mathrm{E}\{(y_R^2(n) - R_R)^2 \; + (y_I^2(n) - R_I)^2\}$	$y_R(n)(y_R^2(n) - R_R) + jy_I(n)(y_I^2(n) - R_I)$
MDDMA	$\mathrm{E}\{(y_R^2(n) - \hat{a}_R^2(n))^2 \; + (y_I^2(n) - \hat{a}_I^2(n))^2\}$	$y_R(n)(y_R^2(n) - \hat{a}_R^2(n)) + jy_I(n)(y_I^2(n) - \hat{a}_I^2(n))$

Tab. 2: The family of the NCMA-type Algorithms

Algorithms	Constraints	Estimation Errors - $e(n)$
NCMA	$\lvert\mathbf{w}^T(n+1)\mathbf{x}(n)\rvert^2 = R$	$y(n)(\frac{R}{\lvert y(n)\rvert} - 1)$
NDDMA	$\lvert\mathbf{w}^T(n+1)\mathbf{x}(n)\rvert^2 = \lvert\hat{a}(n)\rvert^2$	$y(n)(\frac{\lvert\hat{a}(n)\rvert}{\lvert y(n)\rvert} - 1)$
NMCMA	$Re\{\mathbf{w}^T(n+1)\mathbf{x}(n)\}^2 = R_R$ $Im\{\mathbf{w}^T(n+1)\mathbf{x}(n)\}^2 = R_I$	$y_R(n)(1 - \frac{\sqrt{R_R}}{\lvert y_R(n)\rvert}) + jy_I(n)(1 - \frac{\sqrt{R_I}}{\lvert y_I(n)\rvert})$
NLMSDD	$Re\{\mathbf{w}^T(n+1)\mathbf{x}(n)\}^2 = \hat{a}_R^2(n)$ $Im\{\mathbf{w}^T(n+1)\mathbf{x}(n)\}^2 = \hat{a}_I^2(n)$	$y_R(n)(1 - \frac{\lvert\hat{a}_R(n)\rvert}{\lvert y_R(n)\rvert}) + jy_I(n)(1 - \frac{\lvert\hat{a}_I(n)\rvert}{\lvert y_I(n)\rvert})$

In this case, we can use the same dual-mode approach as before. To make the complexity lower, we can start with the NMCMA and make the switch to the NLMS-DD by only switching the "reference radii": $\sqrt{R_R}$ to $\lvert\hat{a}_R\rvert$ and $\sqrt{R_I}$ to $\lvert\hat{a}_I\rvert$.

3 Simulation results

The proposed family of algorithms was tested by means of computational simulations. The simulation scenario consists in a transmitted signal with a 16QAM or 64QAM modulation, an equalizer with 9 taps, a 30dB SNR and a discrete-time channel with impulse response given by: $h(n) = 0.2798\delta(n) + 1\delta(n-1) + 0.2798\delta(n-2)$. A PLL is used to correct phase shift at the output of the equalizer, except for the algorithms of the "Modified" type, which perform the carrier recovery by themselves. All the Mean Squared Error (MSE) curves were obtained via Monte Carlo simulations using 50 independent data realizations and the horizontal line shows the optimum Wiener MSE.

Fig. (1) shows the learning curve of the CMA-type family of algorithms for a 16QAM signal. First, we remark that the DDMA has a SSE approximately 7dB smaller than the CMA. As regards the modified algorithms, it is clear from fig. (1) that they have provided improvements on the CMA and on the DDMA (MDDMA converges after 1800 iterations and DDMA after 4500 iterations approximately). In this case, again it was not necessary to employ the dual-mode algorithms. However, for a 64QAM signal the dual-mode approach provides very good improvements. We can see in fig. (2) that the MCMA has the better performance among all the single-mode algorithms, since decision directed algorithms were penalized by a high modulation level. Moreover, the DDMA has a higher convergence speed than the MDDMA and the Decision Directed Algorithm (DDA), which does not have an acceptable level of MSE even after 40000 iterations. However, working in a dual-mode version, the MDDMA has a very good improvement of its performance, outperforming even the dual-mode DDMA and the dual-mode DDA (MCMA-MDDMA converges after 1500 iterations, CMA-DDMA after 3200 iterations and the CMA-DDA after 2700 iteration, approximately).

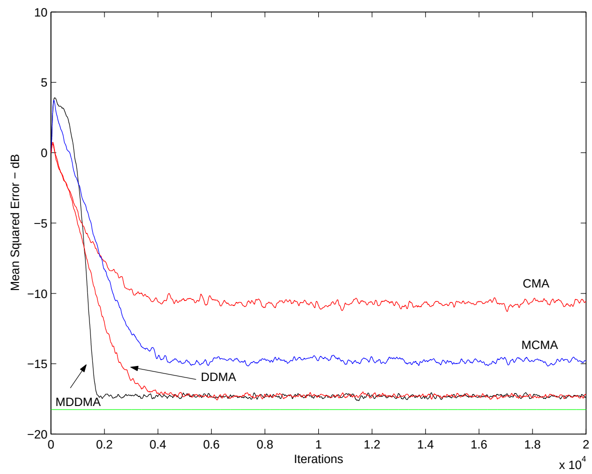

Fig. 1: MSE curves for the LMS-type algorithms - 16QAM.

As regards the normalized algorithms, fig. (3) shows the MSE for NCMA class of algorithms for a 16QAM signal. One can see that the NMCMA has a worst performance than the conventional NCMA. However the NDDMA has improved the performance a lot of the NCMA and the NLMSDD even more. Once again, it was not necessary to employ the dual-mode algorithms. However, for a 64QAM signal (figure (4)), we see that the NDDMA, the NCMA and the NLMSDD have not achieved yet a reasonable MSE

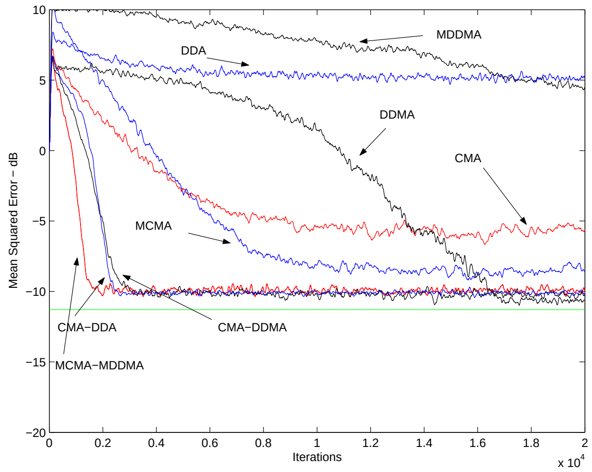

Fig. 2: MSE curves for the LMS-type algorithms - 64QAM.

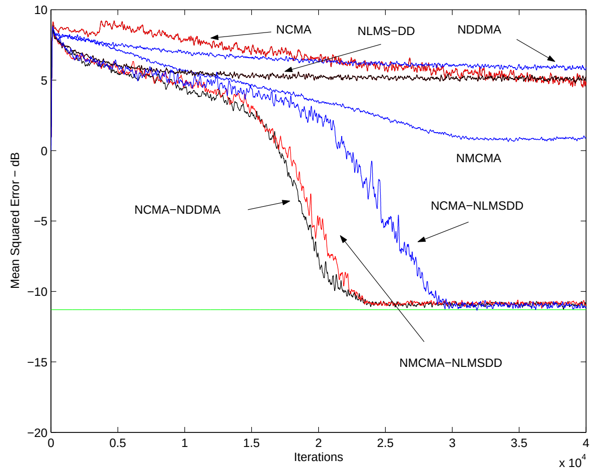

Fig. 4: MSE curves for the NLMS-type algorithms - 64QAM.

after 40000 iterations. Moreover the NMCMA does not suffer so much in this case, as it is not decision directed. Moreover, by working in a dual-mode version, the NDDMA and the NLMSDD have a very good improvement in their performance. It should be highlighted that the NLMSDD had a better performance when working with the NMCMA than with the NCMA.

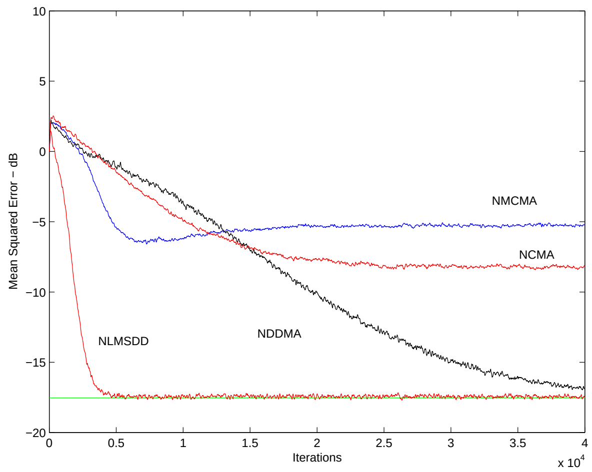

Fig. 3: MSE curves for the NLMS-type algorithms - 16QAM.

4 Conclusions

In this paper, we have presented a new family of algorithms characterized by some very desirable properties when performing blind equalization for QAM signals. Simulation results confirms the behavior expected, showing a great improvement of the proposed algorithms in relation to the conventional ones. Their gain in performance is very significative, with a very reasonable computational cost (in relation to the original CM algorithms). The SSE of the proposed algorithms are, in general, very close to the Wiener MSE. The MDDM based algorithms can perform jointly blind equalization and carrier recovery, and have shown to outperform the DDM based algorithms. Perspectives of this work include a study of convergence and generalization to nonlinear filter structures.

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