

# A Finite Memory Non Stationary LMS Algorithm For Tracking Radio-Mobile Channels with Abrupt Jumps

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**Résumé** – Dans ce papier nous montrons que l'algorithme Finite Memory Non Stationary LMS (FM-NSLMS) est efficace pour la poursuite des canaux de transmission radio-mobile dont les variations temporelles sont à la fois caractérisées par un modèle markovien d'ordre  $P$  et affectées de ruptures brusques. La non stationnarité du canal de transmission est représentée par un modèle markovien d'ordre  $P$ .

L'algorithme FM-NSLMS est conçu pour la poursuite des canaux à variations temporelles markoviennes. Par ailleurs, la capacité de mémorisation introduite dans l'algorithme lui permet de tenir compte de la nature récursive de la non stationnarité. Ainsi, il est capable de poursuivre avec une grande vitesse de convergence les sauts des paramètres markovien de certaines valeurs à d'autres.

**Abstract** – In this paper we show that the Finite Memory Non Stationary algorithm (FM-NSLMS) is efficient to track a radio-mobile channel that not only varies through time but also undergoes abrupt jumps. The time variations are characterized by  $P$ -order non-stationarity markov model.

The proposed FM-NSLMS algorithm, is tailored for the tracking of markovian time varying channels. Besides, the memorization capabilities built in the algorithm allow it to take into account the recursive nature of the non-stationarity of the channel. Hence, it is able to track with a high convergence speed, the jump of the markov parameters from some values to others.

## 1 Introduction

In the context of mobile communications it is possible to find, quite often, phenomena that are best captured by hybrid models. For instance, in order to model the propagation of transmitted signals, it is necessary to take into consideration the encountered obstructions such as roadside trees, utility poles, buildings, and other obstacles. These obstructions will lead to communication outages or fading. Hence, this paper, addresses the realistic mobile communications problem where the channel model not only varies through time but also undergoes abrupt jumps. The channel changes may be caused for example, by the mobility of the receiver through a variable environment. This phenomena is also noted for land mobile satellite channel, [1]. A markov model represents the time variations of the radio-mobile channel impulse response. A variation of the transmission conditions may cause the jump of the markov parameters from some values to others.

The Finite Memory Non Stationary LMS (FM-NSLMS) adaptive algorithm is used for real time variation of the adaptive filter. This algorithm proposed in [3] is tailored for the tracking of markovian time varying channels. In fact, it is designed in a way to take into account the prior knowledge of the non-stationarity model. The Finite Memory NSLMS used in this paper to identify the channel impulse response, is a generalization of the NSLMS. It takes into account the recursive nature of the marko-

vian model. The memorization capabilities built into the FM-NSLMS algorithm, enhances its performances in some critical situations such as the abrupt jumps of the markov parameters.

To enhance the convergence speed of the markov parameters to their true values, the time evolution of the FM-NSLMS adaptive filter is modified. Hence, the Modified FM-NSLMS algorithm proposed here, presents better performance than the FM-NSLMS algorithm.

The paper is organized as follows. In section 2, we present the recursive filtering problem and a description of the FM-NSLMS algorithm. In section 3, we present a study of the performance of the FM-NSLMS algorithm. We will show the superiority of the FM-NSLMS algorithm over the LMS and its ability to identify the abrupt jumps of the channel model. In section IV, we introduce the Modified FM-NSLMS algorithm in order to accelerate the convergence speed of the markov parameters.

## 2 Background

### 2.1 Non stationary context

Here, we are interested in adaptive identification of markovian time-varying channel. The classical formulation of such filtering problem is depicted in Figure (1).

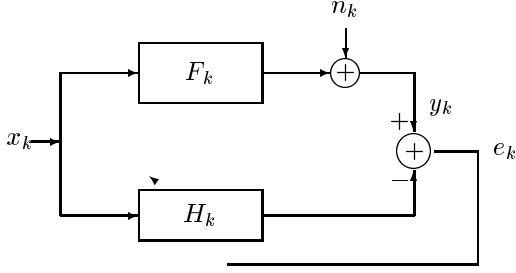


FIG. 1: Adaptive identification of time-varying channel

The noisy input/output equation of the channel is,

$$y_k = F_k^T X_k + n_k \quad (1)$$

where,  $X_k = (x_k, x_{k-1}, \dots, x_{k-N+1})^T$  is the known stationary input vector and  $n_k$  is an unknown i.i.d. observation noise. The filter parameter vector is assumed to be  $P$ -order markov time-varying,

$$F_k = \sum_{i=1}^P a_i F_{k-i} + \Omega_k \quad (2)$$

where the  $(a_i)_{i=1,P}$  ensure the stability of the channel, and  $\Omega_k$  called non-stationary noise, is an unknown zero-mean, i.i.d. process independent of  $\{X_k\}$  and  $\{n_k\}$ . We consider here, that the markov parameters  $(a_i)_{i=1,P}$  jump from some values to others.

This general model represents different types of non-stationarity such as variations of mobile transmission channels or underwater acoustic channels.

The evolution of the parameter vector  $H_k$  of the adaptive filter is governed by the estimate error,

$$e_k = y_k - H_k^T X_k$$

, in order to minimize a criterion, such as the mean square error  $E(e_k^2)$ , for the LMS.

The tracking capacity of the adaptive algorithm is measured by the normalized misadjustment,

$$M_\mu = \lim_{k \rightarrow \infty} (E(e_k^2) - P_n) / P_n$$

, where  $P_n$  is the power of  $n_k$ .

It is important to note that basically the LMS algorithm is not designed to track time-variations of a channel even though the FM-NSLMS is a tailored approach for tracking stochastic non-stationarities.

The Finite Memory Non Stationary LMS (FM-NSLMS) adaptive algorithm is used for real time variation of the adaptive filter  $H_k$ . This algorithm proposed in [3] is tailored for the tracking of markovian time varying channels. In fact, it is designed in a way to take into account the prior knowledge of the non-stationarity model (1).

## 2.2 The FM-NSLMS algorithm

We present, as proposed in [3], the equations of the algorithm for a first order markov Non-stationarity.

$$\begin{aligned} H_{k-m/k} &= (a_k - \mu X_{k-m-1} X_{k-m-1}^T) H_{k-m-1/k-1} \\ &\quad + \mu y_{k-m-1} X_{k-m-1} \\ &\vdots \\ &\vdots \\ H_{k-1/k} &= (a_k - \mu X_{k-2} X_{k-2}^T) H_{k-2/k} + \mu y_{k-2} X_{k-2} \quad (2) \\ H_{k/k} &= (a_k - \mu X_{k-1} X_{k-1}^T) H_{k-1/k} + \mu y_{k-1} X_{k-1} \quad (3) \\ a_{k+1} &= a_k + \mu_1 (y_k - H_{k/k}^T X_k) (H_{k-1/k}^T X_{k-1}) \quad (4a) \end{aligned}$$

where  $X_k = (x_k, \dots, x_{k-N+1})$  is the input sequence. As the real channel, the adaptive filter  $H_k$  is also characterized by a first order markov time evolution.

The latter algorithm can be generalized without any difficulty to a  $P$  order markov non-stationarity. These equations show a memory of length  $m$  in the computation of  $H_k$ . The increase of this memory is particularly attractive when the poles of the markov model are very close to the circle.

In [3], it was shown that the FM-NSLMS algorithm presents a much better tracking capacity than the classical LMS and RLS algorithms. Moreover, the FM-NSLMS algorithm is able to identify the order and the parameters of the channel markov model. Besides the memorization capabilities built in it, this algorithm takes into account the recursive nature of the non stationarity of the channel (1), and hence ensures a good quality of convergence of the markov parameters to their true values. All these good proprieties, give the algorithm the ability to detect and track in real time an abrupt change of the parameters of the channel markov model.

## 3 Performances of the FM-NSLMS algorithm.

The performance analyses of the FM-NSLMS are conducted by simulations. We consider an i.i.d. input with power  $P_x = 1$ . The power of the non-stationary noise  $P_\omega$  is fixed through  $\delta = \frac{NP_x P_\omega}{P_n}$  where  $P_n$  is the power of the observation noise.

For the presented results, we use  $\delta = 1$ , and channel with 6 paths ( $N = 6$ ). This case is realistic for a radio mobile transmission. We consider a markovian non stationarity of order 1. The parameter  $a$  of the model changes through time from a value to an other at random ( $a = 0.7$ , or  $0.8$ , or  $0.9$ ). For the algorithm, we fix the memory  $m = 1$ .

### 3.1 Superiority of the FM-NSLMS over LMS

In figure 2, we plot the variation of the EMSE versus the step size  $\mu$ . This is done for the classical LMS algorithm (curve 1) and the FM-NSLMS algorithm (curve 2). It is clear here that the FM-NSLMS presents much better tracking capacity than the classical LMS. This result is foreseeable since contrarily to the FM-NSLMS, the LMS

adaptation is blind regarding to the non-stationarity of the channel.

### 3.2 Detection of the abrupt jumps of the channel.

Here the focus is on the convergence of the markov parameter to its true value and on the ability of the algorithm to detect the channel jumps. In figure 3, we plot the time variation of  $\hat{a}_k$  versus time (curve 2).

Comparing to the time variation of the true value of  $a$  (curve 1), we note that the algorithm detect the abrupt change of the parameter. However, when the change occur, the adaptive parameter  $\hat{a}_k$  takes a long time to converge to the new value. This phenomena is remarkable when the parameter jumps from a high value to a small one. In fact, this inconvenience is due to sensitivity of the equation (4a) to the modulus of  $H_{k-1/k}^T X_{k-1}$  that (is as smaller as the markov parameter is smaller).

Besides, equation (4a) shows that the modulus of  $H_{k-1/k}^T X_{k-1}$  is one of the parameters that control the convergence speed of  $\hat{a}_k$ . The speed is as lower as the modulus of  $H_{k-1/k}^T X_{k-1}$  is smaller.

## 4 Acceleration of the convergence speed of the markov parameters: Modified FM-NSLMS algorithm

To overcome this inconvenience of the FM-NSLMS algorithm, we bring a modification to the time evolution of adaptive markov parameter (4a). The quantity  $H_{k-1/k}^T X_{k-1}$  is replaced by its sign. Hence, for  $m = 1$  the Modified FM-NSLMS algorithm is described by,

$$H_{k-1/k} = (a_k - \mu X_{k-2} X_{k-2}^T) H_{k-2/k} + \mu y_{k-2} X_{k-2} \quad (2)$$

$$H_{k/k} = (a_k - \mu X_{k-1} X_{k-1}^T) H_{k-1/k} + \mu y_{k-1} X_{k-1} \quad (3)$$

$$a_{k+1} = a_k + \mu_1 (y_k - H_{k/k}^T X_k) \text{Sign} (H_{k-1/k}^T X_{k-1}) \quad (4b)$$

where *Sign* is the classical sign function. Curve 3 of figure 3, shows that with the Modified FM-NSLMS algorithm, not only the convergence speed is increased but also the quality of convergence is appreciably ameliorated (less bias). In another hand, curve 3 of figure 2 shows that the tracking capacity of the Modified FM-NSLMS algorithm is better than the FM-NSLMS algorithm. Moreover, the Modified FM-NSLMS is less sensitive to the choice of the step size  $\mu$  than the FM-NSLMS and the LMS algorithms.

## 5 Conclusion

This paper uses the Finite Memory NSLMS algorithm to identify and detect the abrupt jumps of the markovian radio-mobile channel. This algorithm takes into account the structure and especially the recursive structure of the stochastic time-variations of a channel.

The memorisation capabilities built in this algorithm, allows it to detect the abrupt jumps of the markov parameters. However to increase the convergence speed of the markov parameters to their true values, the time evolution of the FM-NSLMS adaptive filter was modified. The Modified FM-NSLMS algorithm has the same tracking capacity as the FM-NSLMS algorithm, but it has a higher convergence speed. Hence, it is more appropriate in real time situations.

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## 6 References

- [1] H. Besbes, J. Ezzine, M. Jaidane-Saidane, "Exact Performances Analysis of Land Mobile Satellite Channel", Global Telecommunications Conference- Globecom'99, pp2768-2772.
- [2] M.K. Tsatsanis, G.B. Giannakis, G. Zhou, "Estimation and Equalization of Fading Channels with Random Coefficients", Signal Processing, 53, 1996, pp211-229.
- [3] M.Turki, M. Jaidane-Saidane, " A Finite Memory Non Stationary LMS for Adaptive tracking markovian time varying channel", ICECS'98, pp 457-460.

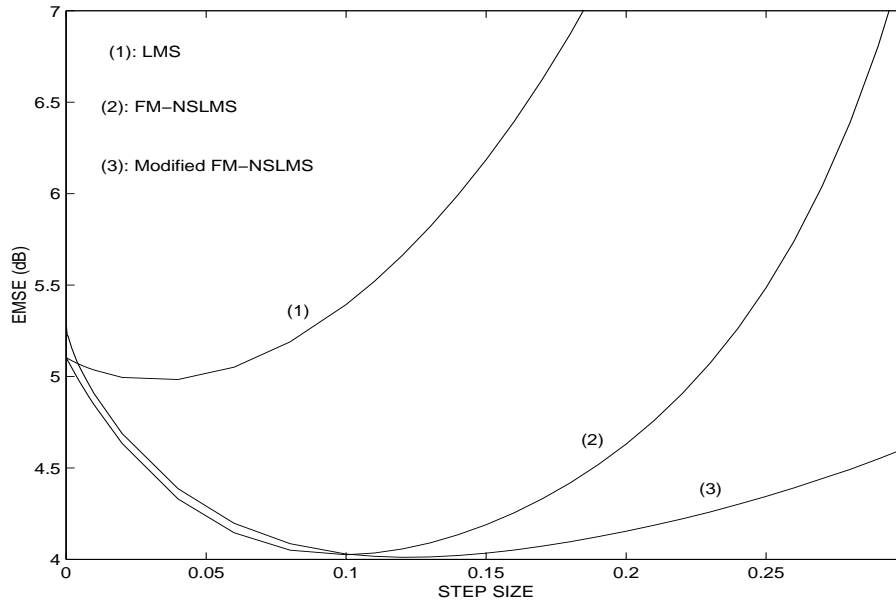


FIG. 2: Tracking capacity ( $N = 6, \delta = 1.0, \mu_1 = 0.001$ )

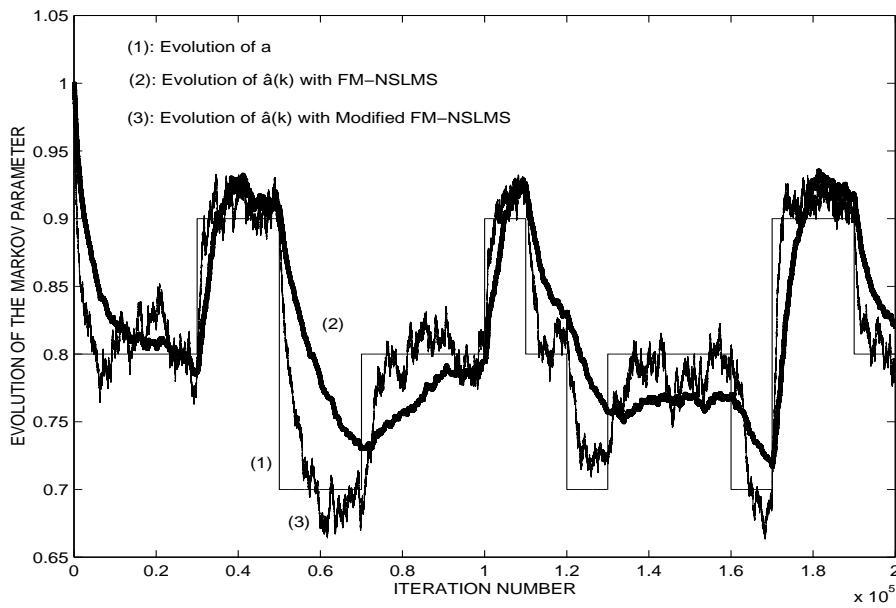


FIG. 3: Convergence of the markov parameter ( $N = 6, \delta = 1.0, \mu_1 = 0.001$ )