

Blind Equalization Using Split Adaptive Transversal Filtering

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Résumé – Cette communication considère la mise en oeuvre d’égaliseurs aveugle basée sur les techniques de Bussgang et une nouvelle structure de filtrage “split”. Les algorithmes de Sato, Stop-and-Go et Godard sont analysés. Les résultats de comparaison indiquent une meilleure performance de la nouvelle structure de filtrage par rapport à celles bien connues de la littérature.

Abstract – This paper considers the implementation of blind equalizers based on the Bussgang techniques and a novel split filtering structure. The cases of Sato, Stop-and-Go and Godard algorithms are analyzed. Comparison results indicate a significant improvement of the new filtering structure over the traditional technique taken from the literature.

1 Introduction

Since blind equalization became one of the main areas in communication systems many researchers have been working on new structures and algorithms to improve the performance of such scheme [1]. However, it still presents some disadvantages when compared with the traditional trained equalizer. The presence of local minima and a slower convergence rate can be considered main drawbacks of these self-learning structures.

In this particular work we pursue the effects of a novel transversal filtering structure together with some of the well-known Bussgang algorithms [2, 3, 4]. Thus, we are concerned with adaptive transversal equalizers updated with gradient techniques. In such cases the eigenvalue spread of the autocorrelation matrix dominates the convergence rate [5]. The performance can be strongly improved if we introduce an operation capable of reducing such a spread. One of these operations consists of splitting the transversal filter into its symmetric and antisymmetric parts [7, 8].

This new scheme was first proposed by [6]. Later on, [7, 8] proposed a new approach to the split transversal filtering combining it with a linearly constrained optimization scheme. This gave birth to a split structure based on the Generalized Sidelobe Canceller (GSC). It is this structure that we present in the next section.

2 Split Transversal Filtering

Let us consider the classical scheme of an adaptive transversal filter as shown in Figure 1, where $W(n)$ is a vector of N -by-1 coefficients. If we split up this filter into its symmetric and antisymmetric parts then:

$$W(n) = W_s(n) + W_a(n) \quad (1)$$

where $W_s(n) = \frac{1}{2}[W(n) + JW(n)]$, $W_a(n) = \frac{1}{2}[W(n) - JW(n)]$ and J is the reflection matrix.

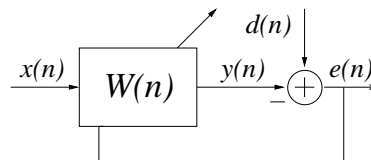


FIG. 1: Traditional adaptive transversal filter

The symmetry and antisymmetry conditions of $W_s(n)$ and $W_a(n)$ can be easily introduced through a linearly constrained approach [7]. It consists of making:

$$C_s = \begin{bmatrix} I_K \\ -J_K \end{bmatrix} \quad C_a = \begin{bmatrix} I_K \\ J_K \end{bmatrix} \quad (2)$$

$$F_s = F_a = 0_K$$

for N even and $K = \frac{N}{2}$, and of imposing:

$$C_s^t W_s(n) = F_s \quad \text{and} \quad C_a^t W_a(n) = F_a \quad (3)$$

in a constrained optimization process of the mean-square value of the estimation error $e(n)$, which is defined as the difference between the desired response $d(n)$ and the filter output $y(n)$.

Now, using the GSC structure with the symmetry and antisymmetry constraints, the split filtering scheme can be represented in the form of Figure 2 (N even). For N odd, please refer to [8].

As far as the adaptation process is concerned, the LMS algorithm can be applied independently in each branch in a normalized fashion. Thus, the algorithm for the symmetric filter can be defined as:

$$W_{\perp s}(n+1) = W_{\perp s}(n) + \frac{\mu}{r_s(n)} X_{\perp s}(n) e(n) \quad (4)$$

and for the antisymmetric filter as:

$$W_{\perp a}(n+1) = W_{\perp a}(n) + \frac{\mu}{r_a(n)} X_{\perp a}(n) e(n) \quad (5)$$

where:

$$r_i(n) = \gamma r_i(n-1) + \frac{1}{n}(X_{\perp i}^2(n) - \gamma r_i(n-1)) \quad i = a, s \quad (6)$$

γ is the forgetting factor and μ is the adaptation step-size.

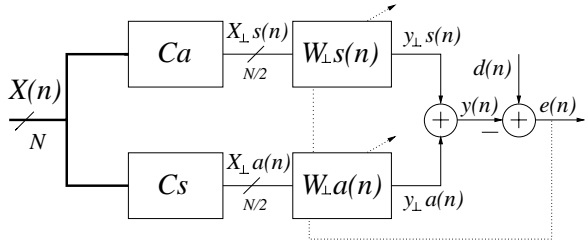


FIG. 2: GSC implementation of the split filter

Further details on split transversal filtering will be restricted to the literature [8], once our aim is to investigate the performance of the traditional blind algorithms when using this novel filtering structure.

3 Sato Algorithm

In [9] it was shown that an important generalization of the Sato algorithm [2], known as the BGR algorithm, possesses a desirable global convergence property under two idealized conditions. To meet these conditions the equalizer should be of infinite-length and/or the input data should be continuous. However, neither the first nor the second condition holds in actual digital communication systems. Thus, the Sato algorithm presents what is called Local Convergence.

Consider the channel having the following impulse response:

$$\text{Channel 1: } h(n) = 0.5(\delta(n) + \delta(n-2) + \delta(n-3))$$

and the equalizer having 21 coefficients (center-spike initialization). The input constellation is a binary 2-PAM, and the adaptation step-size is set to $\mu = 5 \cdot 10^{-4}$. In order to check if the system has converged to the global solution we will use the percentage Inter Symbol Interference (ISI) [9]:

$$P = \frac{\sum_i |t_i|}{\max |t_i|} - 1 \quad (7)$$

where

$$t_i = h_i * w_i \quad (8)$$

and h_i and w_i are the channel and the equalizer impulse responses. The overall channel-equalizer system has an open-eye if $P < 1$ and a closed-eye if $P \geq 1$.

In Figure 3 we present the simulation results for the system having the previous configuration and using the traditional transversal filtering. One can see that the equalizer was trapped in a local minimum. The final percentage ISI was about 200%, what guarantees that the eye is closed and no successful decisions can be made at the receiver.

Let us carry out the same simulations but with the split transversal filtering instead of the traditional scheme. The equalizer, $W(n)$, is implemented with 32 coefficients, so both $W_{\perp a}$ and $W_{\perp s}$ have 16 coefficients each. The step-size of the normalized LMS algorithm is set to $\mu = 1 \cdot 10^{-2}$.

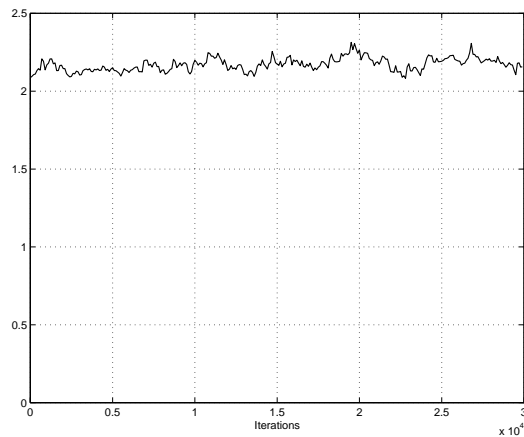


FIG. 3: Percentage ISI – Traditional transversal filtering

The results are shown in Figure 4, where the percentage ISI was reduced to about 10%, what means that the eye is open and correct decisions can be made in the receiver. Now the equalizer does converge to the global minimum.

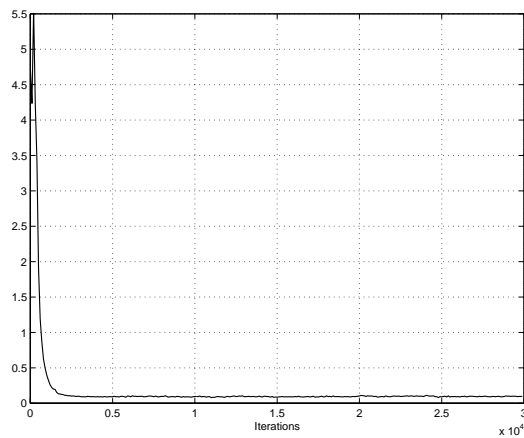


FIG. 4: Percentage ISI – Split transversal filtering

4 Stop-and-Go Algorithm

In [10] the problem of local convergence for equalizers implemented by finite length filters is addressed once more. In that work one of the examples showing this misbehavior is illustrated with the Stop-and-Go algorithm [3]. Let us apply the split transversal filtering to this other Busgang technique. Consider now the following channel:

$$\text{Channel 2: } h(n) = \frac{13}{31}(\delta(n) + \delta(n-1) + \delta(n-3))$$

with a 4-PAM input constellation [-3, -1, 1, 3]. For the traditional scheme the equalizer has 100 coefficients, center-spike initialization, step-size $\mu = 2 \cdot 10^{-4}$ and $\beta = 2$.

In order to quantify the performance of the equalizer let us use the same ISI parameter defined in [10]:

$$ISI = \frac{\sum_i t_i^2 - \max_i t_i^2}{\max_i t_i^2} \quad (9)$$

where t_i is defined as in equation 8.

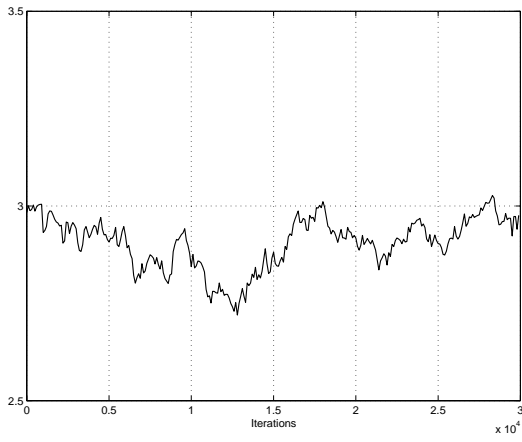


FIG. 5: ISI (dB) – Traditional transversal filtering

Based on Figure 5 we can conclude that the equalizer was trapped in a local minimum again. This also means that the eye is still closed and that no correct decisions can be made in the receiver.

Figure 6 shows the ISI evolution of the split equalizer with the same parameter specifications used in Section 3. It is clear that now the equalizer was able to reach the global minimum.

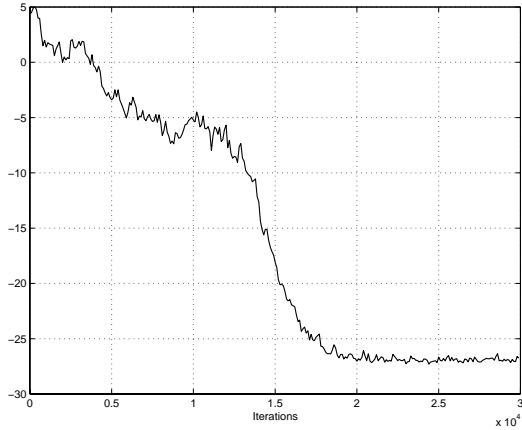


FIG. 6: ISI (dB) – Split transversal filtering

5 The Split Equalizer and The Godard Cost Function

Since the results from the last two sections had suggested that the split equalizer would have a better performance than the traditional transversal equalizer, we decided to investigate the effects of the split operation into a Godard-like cost function.

Consider then the following AR channel [11]:

$$\text{Channel 3: } x_n + \alpha x_{n-1} = a_n$$

where $\alpha = 0.6$, x_n are the output data and a_n the input data constrained to a binary 2-PAM constellation. The equalizer (CMA 2-2) has only two coefficients, $[w_0; w_1]$, such that:

$$y_n = w_0 x_n + w_1 x_{n-1} \quad (10)$$

and the cost function can be written as [11]:

$$F = \frac{1}{4}(w_0^4 E\{a_n^4\} + 6w_0^2(w_1 - \alpha w_0)^2 E\{a_n^2\} E\{x_{n-1}^2\} + (w_1 - \alpha w_0)^4 E\{x_{n-1}^4\} - 2R_2 w_0^2 E\{a_n^2\} + -2R_2(w_1 - \alpha w_0)^2 E\{x_{n-1}\} + R_2^2) \quad (11)$$

In Figure 7 it is possible to visualize F as a function of the equalizer coefficients $[w_0; w_1]$.

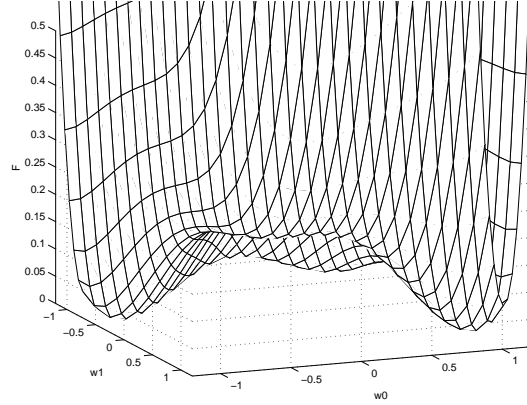


FIG. 7: Cost function for channel 3, 2-PAM, CMA 2-2 and traditional transversal filtering

Now, using the split transversal structure:

$$y_n = \begin{bmatrix} x_n & x_{n-1} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \end{bmatrix} \quad (12)$$

$$y_n = x_n(w_0 + w_1) + x_{n-1}(w_0 - w_1)$$

and making:

$$v_0 = (w_0 + w_1) \quad \text{and} \quad v_1 = (w_0 - w_1) \quad (13)$$

the cost function turns to be:

$$F = \frac{1}{4}(v_0^4 E\{a_n^4\} + 6v_0^2(v_1 - \alpha v_0)^2 E\{a_n^2\} E\{v_{n-1}^2\} + (v_1 - \alpha v_0)^4 E\{x_{n-1}^4\} - 2R_2 v_0^2 E\{a_n^2\} + -2R_2(v_1 - \alpha v_0)^2 E\{x_{n-1}\} + R_2^2) \quad (14)$$

which is plotted in Figure 8 as a function of the equalizer coefficients $[w_0; w_1]$.

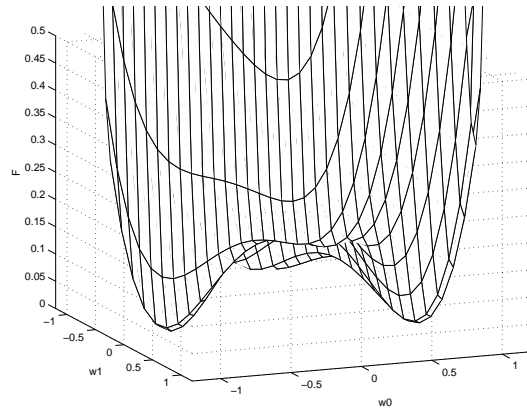


FIG. 8: Cost function for channel 3, 2-PAM, CMA 2-2 and split transversal filtering

Comparing Figures 7 and 8 we can verify that there is a slight difference in the form of the cost function, however, it is not such a change that turns it to be convex.

The cost function in equation 14 still has local minima. Somehow the split operation permits the local minima to be avoided during the adaptation process. How it may happen is what we intend to address in the next section.

6 The Zero-Forcing Equalizer

Consider then the case of a split zero-forcing equalizer. Defining as in [12]:

$$t_k = \sum_{j=-\infty}^{\infty} h_j w_{ak-j} + \sum_{j=-\infty}^{\infty} h_j w_{sk-j} \quad (15)$$

where

$$W_s = C_a W_{\perp s} \quad W_a = C_s W_{\perp a} \quad (16)$$

and then applying the zero-forcing condition, it follows that:

$$t_k = \sum_{j=-\infty}^{\infty} h_j w_{ak-j} + \sum_{j=-\infty}^{\infty} h_j w_{sk-j} = \delta_k \quad (17)$$

or in its Z-transform:

$$T(z) = H(z)W_a(z) + H(z)W_s(z) = 1 \quad (18)$$

and:

$$W_a(z) + W_s(z) = \frac{1}{H(z)} \quad (19)$$

It is very important to note that during the adaptation process the left-hand side of equation 19 can not be simply substituted by $W(z) = W_a(z) + W_s(z)$ once the symmetric and antisymmetric equalizers are updated independently by equations 4 and 5.

Equation 19 suggests that the split operation increases the degree of freedom of the system during the minimization of equation 14, which permits the equalizer to avoid the local minima.

7 Conclusions

In this work we presented the application of split transversal filtering into blind equalization. The proposed split equalizer performed much better than the traditional one, as we verified by simulations using some classical examples of the literature. Global convergence for both the Sato and Stop-and-Go algorithms were obtained when using the split equalizer, while the traditional equalizer was trapped in a local minimum.

It was also verified that the split equalizer cost function is not convex. So, even though the split operation does not vanishes the local minima, it is somehow able to avoid these undesirable equilibria in order to lead the equalizer to the global solution.

When analyzing the zero-forcing equalizer we concluded that the independent adaptation of the symmetric and the antisymmetric parts should be the cause of this better performance. Finally, it is important to improve the analytical analysis of the split equalizer. The studies concerning this aspect are in progress.

Finally, based on our research results, we can affirm that the split filtering structure should be viewed as a promising alternative to the traditional transversal scheme, specially when applied to blind equalization.

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