Wide-Sense Polynomial Equalizers in Digital Communications

Ruolun Liu, Pierre Comon

Laboratoire I3S CNRS/UNSA
2000 Route des Lucioles, BP 121, 06903 Sophia-Antipolis Cedex, France
liu@i3s.unice.fr, comom@i3s.unice.fr

Résumé – On propose dans cet article un Egaliseur Polynômial au Sens Large (WSPE), dédié aux canaux spéculaires. En se basant sur la distribution QPSK des entrées, largement utilisée en communications numériques, on développe des WSPE d'ordre 3. On se penche sur des simplifications des WSPE généraux afin de réduire la complexité numérique. Les simulations mettent en évidence l'intérêt des termes cubiques lorsque le nombre de capteurs est faible.

Abstract – In this paper, a novel Wide-Sense Polynomial Equalizer (WSPE), dedicated to linear specular channels, is proposed. Based on the QPSK distribution of the inputs, widely used in digital communications, up to 3rd order WSPE are considered. Simplifications of the WSPE are investigated in order to reduce the numerical complexity. Simulations show that cubic terms give outstanding performance when the number of sensors is relatively small.

1 Introduction

In digital communications, transmitted signals suffer from distortions when passing through a fading multipath channel. These distortions cause adjacent pulses to interfere with each other, which is known as Inter-Symbol Interference (ISI). At the receiver, equalization is generally required to compensate for the effects of ISI. In multiuser communication systems, equalization is also required to be effective in reducing Multiple-Access Interference (MAI).

Most of the equalizers, either linear or nonlinear, need a known pilot sequence transmitted to the receiver for the purpose of adjusting the equalizer coefficients. However, there are some applications, where it is desirable for the receiver to synchronize to the received signal and to adjust the equalizer without a pilot sequence. Equalization techniques working without pilot sequences are referred to as self-recovering, or *blind*. Other advantages of blind techniques include: an increase in transmission rate, better performance for fast varying channels, easier use for long channel responses, and robustness to loss in calibration.

Linear equalizers are used in the applications where the channel distortion is not too severe. Nonlinear equalization of digital communication channels has received a great attention, but has been mainly iterative and based on a decision feedback.

The proposed Wide-Sense Polynomial Equalizer (WSPE) is of block type (non iterative), and is based on a least-mean-square-error (LMSE) scheme, which makes it easy to implement adaptively. From the original measurement, an "*augmented*" measurement is generated, and consists of a Wide-Sense Polynomial (WSP) function (hence non linear) of the actual measurement. Then, a linear LMSE equalizer is built with this augmented measurement, similarly to a Volterra filter. It can be blind when pilot sequence is not available.

Since there is a high degree of freedom to construct the WSP measurement, it is necessary to investigate how small its size can be and, for a fixed size, what kind of combination or simplification gives the best result. Based on the knowledge of the distribution of the source of interest (QPSK in this paper), simplifications of the proposed WSPE are studied in details in order to optimize the complexity.

This paper is organized as follows: the problem is stated and the WSPE is defined in sections 2 and 3, respectively. Section 4 investigates the improvement on performance made by WSPE. Performances of different simplifications are given in terms of Symbol Error Rate (SER) v.s. SNR in section 5. Section 6 concludes.

2 Problem Statement

The linear baseband observation model is assumed as

$$\boldsymbol{y}(n) = \boldsymbol{A}\boldsymbol{B}\boldsymbol{s}(n) + \boldsymbol{w}(n) = \boldsymbol{H}\boldsymbol{s}(n) + \boldsymbol{w}(n) \tag{1}$$

where $\boldsymbol{y}(n)$ is a K-dimensional measurement vector, $\boldsymbol{s}(n)$ is a L-dimensional QPSK signal vector containing delayed versions of the user of interest, and $\boldsymbol{w}(n)$ is a K-dimensional noise, a priori non Gaussian, standing for both background noise and interferences from other users; $\boldsymbol{A}$ is the K by P array response matrix, $\boldsymbol{B}$ is P by L matrix describing the specular channel, $\boldsymbol{H} = \boldsymbol{A}\boldsymbol{B}$ is a K by L matrix, K being the number of sensors, P the number of paths, and L the maximum length of the paths. More precisely

$$\begin{aligned}
\boldsymbol{A} &= [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), ..., \boldsymbol{a}(\theta_P)] \\
\boldsymbol{B} &= [\boldsymbol{b}_1^{\mathrm{T}}, \boldsymbol{b}_2^{\mathrm{T}}, ..., \boldsymbol{b}_P^{\mathrm{T}}]^{\mathrm{T}} \\
\boldsymbol{a}(\theta_p) &= [\phi_1(\theta_p), \phi_2(\theta_p), ..., \phi_K(\theta_p),]^{\mathrm{T}} \\
\boldsymbol{b}_p &= [b_p(0), b_p(1), ..., b_p(L-1)] \\
\boldsymbol{s}(n) &= [s(n), s(n-1), ..., s(n-L+1)]^{\mathrm{T}}
\end{aligned}$$

where $\phi_k(\theta_p)$ is the response of the kth sensor to the pth path, $\boldsymbol{b}_p$ is the unit impulse response of the channel to the pth path, $p = 1, 2, ..., P$, $k = 1, 2, ..., K$, $n = 1, 2, ..., N$, N is the number of samples. Since L is the maximum length of the paths, $b_p(L_p + 1) = b_p(L_p + 2) = b_p(L - 1) = 0$ for the pth path if its length $L_p < L$. Note that vectors are denoted with **bold** face letters, as opposed to scalars.

Model (1) can always be re-expressed as

$$\boldsymbol{y}(n) = \boldsymbol{h}s(n) + \boldsymbol{v}(n) \tag{2}$$

where $\boldsymbol{h}$ is the first column of $\boldsymbol{H}$ and where

$$\boldsymbol{v}(n) = \boldsymbol{G}\boldsymbol{t}(n) + \boldsymbol{w}(n) \tag{3}$$

contains the other symbols of $s(n)$. When $s(n)$ is a *i.i.d.*, $\mathrm{E}\langle s(n)s^*(n-\tau)\rangle = 0$, and $\boldsymbol{v}(n)$ turns out to be independent of $s(n)$. Next, $\boldsymbol{G}\boldsymbol{t}(n)$ is the so-called Inter-Symbol Interference (ISI), where

$$\boldsymbol{H} = [\boldsymbol{h}, \boldsymbol{G}]$$
$$\boldsymbol{s}(n) = [s(n), \boldsymbol{t}(n)^{\mathrm{T}}]^{\mathrm{T}}$$

Now $\boldsymbol{v}(n)$ is the sum of background noise, interferers from other users (MAI) and ISI.

Traditional LMSE approaches consist of finding a weight vector, $\boldsymbol{f}_{mse}$, which leads to an estimate

$$\hat{s}(n) = \boldsymbol{f}^{\mathrm{H}}_{mse}\,\boldsymbol{y}(n) \tag{4}$$

yielding the Least Mean Square Error (LMSE), that is

$$\boldsymbol{f}_{mse} = \operatorname*{Min}_{\boldsymbol{f}} \mathrm{E}\langle|\boldsymbol{f}^{\mathrm{H}}\,\boldsymbol{y}(n) - s(n)|^2\rangle. \tag{5}$$

WSP Equalizers, proposed in this paper, are based on an stacked observation in the form of a well chosen polynomial of both $\boldsymbol{y}(n)$ and $\boldsymbol{y}^*(n)$, hence the name of *wide-sense* polynomial. It gives a better equalization than traditional linear equalizers, because of their nonlinear inputs.

3 WSPE

To build the WSP observation, up to 3rd order polynomials in $\boldsymbol{y}(n)$ and $\boldsymbol{y}^*(n)$ are considered. Denote $\boldsymbol{u} \otimes \boldsymbol{v}$ the Kronecker product between two vectors, and $\boldsymbol{u} \oslash \boldsymbol{u}$ the Kronecker product without redundancy of a vector with itself (also called symmetric Kronecker product); by extension, also denote $\boldsymbol{y}^{i\oslash} = \boldsymbol{y} \oslash ... \oslash \boldsymbol{y}$ the ith iterated symmetric Kronecker product [1]. Now let

$$\boldsymbol{Y}(n) = \operatorname*{\mathbf{stack}}_{1\leq i+j\leq 3}[\boldsymbol{y}^{i\oslash}(n) \otimes \boldsymbol{y}^{j\oslash *}(n)] \tag{6}$$

where "**stack**" denotes the operator stacking vectors one below the other. (6) produces up to third order WSP observations. Thus, the WSPE is defined as:

$$\boldsymbol{f}_{mse} = \operatorname*{Min}_{\boldsymbol{f}} \mathrm{E}\langle|\boldsymbol{f}^{\mathrm{H}}\,\boldsymbol{Y}(n) - s(n)|^2\rangle \tag{7}$$

$$\hat{s}(n) = \boldsymbol{f}^{\mathrm{H}}_{mse}\,\boldsymbol{Y}(n) \tag{8}$$

Remark 1. Note that, on purpose, no cross terms involving different time lags have been introduced. It is thus a **purely spatial** WSP function that has been used. More precisely, one may think of replacing (6) by

$$\boldsymbol{Y}(n;\tau_1,\tau_2) = \operatorname*{\mathbf{stack}}_{1\leq i+j\leq 3}[\boldsymbol{y}^{\oslash i}(n-\tau_1) \otimes \boldsymbol{y}^{\oslash j *}(n-\tau_2)] \tag{9}$$

$$\boldsymbol{Y}(n) = \operatorname*{\mathbf{stack}}_{\tau_1,\tau_2}[\boldsymbol{Y}(n;\tau_1,\tau_2)] \tag{10}$$

in order to extract more information from original measurements, where $(\tau_1, \tau_2 = 0, 1, ..., \mathcal{T})$, $\mathcal{T}$ being an adjustable parameter. It turns out that this is in general rather complicated and not of practical value, because of the huge size of the matrices involved. In addition, matrix $\boldsymbol{R}_Y$ can also be ill conditioned or even rank deficient, as it would be the case for *i.i.d.* QPSK signals.

Remark 2. Furthermore, it can also be shown that if the noise covariance matrix, in the spectral domain, satisfies:

$$\boldsymbol{R}_v(\nu) = \sigma_v^2(\nu)\boldsymbol{J} \tag{11}$$

where $\boldsymbol{J}$ is a constant matrix, space and time decouple in the expression of the Space-Time Matched Filter (STMF). In fact, using the inversion lemma, one gets for the linear case from

$$\boldsymbol{y}(\nu) = \boldsymbol{h}(\nu)s(\nu) + \boldsymbol{v}(\nu) \tag{12}$$

that $\hat{s}(\nu)$ is proportional to $\boldsymbol{h}^{\mathrm{H}}(\nu)\boldsymbol{R}_v^{-1}(\nu)\boldsymbol{y}(\nu)$. For a noise covariance of the above type, $\hat{s}(\nu)$ is thus proportional to $\boldsymbol{h}^{\mathrm{H}}(\nu)\boldsymbol{J}^{-1}\boldsymbol{y}(\nu)$. This shows that the spatial equalizer $\boldsymbol{J}^{-1}$ and the time equalizers $\boldsymbol{h}_k^*(\nu)$ can be applied separately. However, to make this simplification applicable for WSP equalizers, we need the above property to be satisfied for WSP observations (19). If this is the case, there is no need for cross space-time terms as suggested in Remark 1.

3.1 Selection of pertinent terms

For the sake of clarity, consider temporarily a simpler (scalar) case below, without loosing the generality of the reasoning:

$$y(n) = hs(n) + v(n) \tag{13}$$

where y, h, v are now all scalar. One possible $\boldsymbol{Y}(n)$ is the 9×1 augmented observation vector:

$$\boldsymbol{Y}(n) = \begin{bmatrix} [y(n), y^{3*}(n), y^*(n)y^2(n)]^{\mathrm{T}} \\ [y^2(n), y^{2*}(n), y(n)y^*(n)]^{\mathrm{T}} \\ [y^*(n), y(n)y^{2*}(n), y^3(n)]^{\mathrm{T}} \end{bmatrix} \tag{14}$$

For this WSP observation, based on the QPSK property of both signal $s(n)$ and noise $v(n)$, we have the relations given in table 1.

In this simple scalar example, $\boldsymbol{f}$ has 9 components, and satisfies the normal equation:

$$\begin{bmatrix} \Gamma_1 & \mathcal{Z}_{3,3} & \mathcal{Z}_{3,3} \\ \mathcal{Z}_{3,3} & \Gamma_2 & \mathcal{Z}_{3,3} \\ \mathcal{Z}_{3,3} & \mathcal{Z}_{3,3} & \Gamma_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \\ \boldsymbol{g}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \\ \boldsymbol{r}_3 \end{bmatrix} \tag{15}$$

where $\mathcal{Z}_{m,n}$ denotes the $m \times n$ zero matrix, and where

$$\Gamma_1 = \begin{bmatrix} A_{11}\,A_{12}\,A_{13} \\ A_{12}^*\,A_{22}\,A_{32}^* \\ A_{13}\,A_{32}\,A_{22} \end{bmatrix}, \boldsymbol{g}_1 = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, \boldsymbol{r}_1 = \begin{bmatrix} h \\ h^{3*} \\ h^2h^* \end{bmatrix},$$

$\mathrm{E}\langle s(n)s^*(n)\rangle = 1$	$\mathrm{E}\langle v(n)v^*(n)\rangle = \Gamma_v$
$\mathrm{E}\langle s^2(n)\rangle = 0$	$\mathrm{E}\langle v^2(n)\rangle = 0$
$\mathrm{E}\langle s^3(n)\rangle = 0$	$\mathrm{E}\langle v^3(n)\rangle = 0$
$\mathrm{E}\langle s^2(n)s^*(n)\rangle = 0$	$\mathrm{E}\langle v^2(n)v^*(n)\rangle = 0$
$\mathrm{E}\langle s^4(n)\rangle = 1$	$\mathrm{E}\langle v^4(n)\rangle = M_v$
$\mathrm{E}\langle s^3(n)s^*(n)\rangle = 0$	$\mathrm{E}\langle v^3(n)v^*(n)\rangle = 0$
$\mathrm{E}\langle s^2(n)s^{2*}(n)\rangle = 1$	$\mathrm{E}\langle v^2(n)v^{2*}(n)\rangle = K_v$
$\mathrm{E}\langle s^5(n)\rangle = 0$	$\mathrm{E}\langle v^5(n)\rangle = 0$
$\mathrm{E}\langle s^4(n)s^*(n)\rangle = 0$	$\mathrm{E}\langle v^4(n)v^*(n)\rangle = 0$
$\mathrm{E}\langle s^3(n)s^{2*}(n)\rangle = 0$	$\mathrm{E}\langle v^3(n)v^{2*}(n)\rangle = 0$
$\mathrm{E}\langle s^6(n)\rangle = 0$	$\mathrm{E}\langle v^6(n)\rangle = 0$
$\mathrm{E}\langle s^5(n)s^*(n)\rangle = 1$	$\mathrm{E}\langle v^5(n)v^*(n)\rangle = Q_v$
$\mathrm{E}\langle s^4(n)s^{2*}(n)\rangle = 0$	$\mathrm{E}\langle v^4(n)v^{2*}(n)\rangle = 0$
$\mathrm{E}\langle s^3(n)s^{3*}(n)\rangle = 1$	$\mathrm{E}\langle v^3(n)v^{3*}(n)\rangle = N_v$

TAB. 1: Relations satisfied when user of interest and interferers are QPSK-distributed.

$$\Gamma_2 = \begin{bmatrix} A_{13}\, A_{12}\, 0 \\ A_{12}^*\, A_{13}\, 0 \\ 0 \quad 0 \;\, A_{13} \end{bmatrix}, \; \boldsymbol{g}_2 = \begin{bmatrix} f_4 \\ f_5 \\ f_6 \end{bmatrix}, \; \boldsymbol{r}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\Gamma_3 = \begin{bmatrix} A_{11}\, A_{13}\, A_{12}^* \\ A_{13}\, A_{22}\, A_{32}^* \\ A_{12}\, A_{32}\, A_{22} \end{bmatrix}, \; \boldsymbol{g}_3 = \begin{bmatrix} f_7 \\ f_8 \\ f_9 \end{bmatrix}, \; \boldsymbol{r}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

and $A_{11} = hh^* + \Gamma_v$, $A_{12} = h^4 + M_v$, $A_{13} = h^2h^{2*} + K_v$, $A_{22} = h^3h^{3*} + N_v$, $A_{32} = h^*h^5 + Q_v$.

It is clear that only the first three entries, namely the original measurement $y(n)$ and WSP observations $y^{3*}(n)$ and $y^*(n)y^2(n)$ in (14), are worth considering.

3.2 A simpler form of the WSPE for QPSK inputs

Since the above conclusion holds true when y, h and v are changed into vectors or matrices, we can assume from now on that (6) reduces to the vector below, of size $K + K(K+1)(K+2)/6 + K^2(K+1)/2$:

$$\boldsymbol{Y}(n) = [\boldsymbol{y}^{\mathrm{T}}(n), [\boldsymbol{y}^{3\oslash *}(n)]^{\mathrm{T}}, [\boldsymbol{y}^*(n) \otimes \boldsymbol{y}^{2\oslash}(n)]^{\mathrm{T}}]^{\mathrm{T}} \quad (16)$$

Because of the property $s^{3*}(n) = s^*(n)s^2(n) = s(n)$, satisfied for QPSK distributions, any possible WSP observation $\boldsymbol{Y}(n)$ arranged as (16) can be expressed as

$$\boldsymbol{Y}(n) = T(\boldsymbol{h})s(n) + T[\boldsymbol{v}(n)] \quad (17)$$

where $T(\cdot)$ is a known vector function which returns a big vector consistent with (16).

Based on (17), the minimization of the MSE criterion leads to the augmented normal equation:

$$\boldsymbol{f}_{mse} = \boldsymbol{R}_Y^{-1}\, \boldsymbol{R}_{Ys} \quad (18)$$

where

$$\boldsymbol{R}_Y = \mathrm{E}\langle \boldsymbol{Y}(n)\, \boldsymbol{Y}^{\mathrm{H}}(n)\rangle$$
$$\boldsymbol{R}_{Ys} = \mathrm{E}\langle \boldsymbol{Y}(n)\, s(n)\rangle$$

When a pilot sequence is available, (18) can be used directly. But in most mobile communication systems, channel identification has been performed in a first stage, so that $\boldsymbol{H}$ and hence $\boldsymbol{h}$ can be considered as known. Then $\boldsymbol{R}_{Ys} = \sigma_s^2\, T(\boldsymbol{h})$ where σ_s^2 is the variance of s, so

$$\boldsymbol{f}_{mse} = \sigma_s^2\, \boldsymbol{R}_Y^{-1}\, T(\boldsymbol{h}) \quad (19)$$

4 Performance Improvement

As previously seen, the stacked WSP observation aimed at decreasing the LMSE. In this section, we try to quantify this improvement. Denote the linear MSE estimation given by (2) as

$$\hat{s}_1(n) = \boldsymbol{R}_{sy}\boldsymbol{R}_y^{-1}\boldsymbol{y}(n) \quad (20)$$

and the other stacked WSP MSE estimation as

$$\hat{s}_2(n) = [\boldsymbol{R}_{sy}\boldsymbol{R}_{sz}]\boldsymbol{\Gamma}_{yz}^{-1}[\boldsymbol{y}^{\mathrm{T}}(n)\boldsymbol{z}^{\mathrm{T}}(n)]^{\mathrm{T}} \quad (21)$$

where $\boldsymbol{\Gamma}_{yz} = \begin{bmatrix} \boldsymbol{R}_y & \boldsymbol{R}_{zy} \\ \boldsymbol{R}_{yz} & \boldsymbol{R}_z \end{bmatrix}$, $\boldsymbol{R}_{sy} = \boldsymbol{R}_{ys}^{\mathrm{H}}$, $\boldsymbol{R}_{zy} = \boldsymbol{R}_{yz}^{\mathrm{H}}$ and $\boldsymbol{z}(n)$ is either one of the two WSP observations in (16), or both. Their MSEs are

$$\varepsilon_1 = \boldsymbol{R}_s - \boldsymbol{R}_{sy}\boldsymbol{R}_y^{-1}\boldsymbol{R}_{ys}$$
$$\varepsilon_2 = \boldsymbol{R}_s - [\boldsymbol{R}_{sy}\boldsymbol{R}_{sz}]\boldsymbol{\Gamma}_{yz}^{-1}[\boldsymbol{R}_{ys}^{\mathrm{T}}\boldsymbol{R}_{zs}^{\mathrm{T}}]^{\mathrm{T}}$$

Now the improvement brought by using WSP observations can be defined as the decrease in MSE, namely $\Delta\varepsilon = \varepsilon_1 - \varepsilon_2$. It is always positive or zero and takes the following general form:

$$\Delta\varepsilon = \boldsymbol{\Gamma}_{zys}^{\mathrm{H}}\boldsymbol{\Gamma}_{zyz}^{-1}\boldsymbol{\Gamma}_{zys} \quad (22)$$

where

$$\boldsymbol{\Gamma}_{zys} = [\boldsymbol{R}_{zs} + \boldsymbol{R}_{zy}\boldsymbol{R}_y^{-1}\boldsymbol{R}_{ys}]$$
$$\boldsymbol{\Gamma}_{zyz} = [\boldsymbol{R}_z - \boldsymbol{R}_{zy}\boldsymbol{R}_y^{-1}\boldsymbol{R}_{yz}]$$

The proof is not reproduced for reasons of space. This result extends that of [2]. We can observe that the improvement is strictly positive if: (i) $\boldsymbol{z}$ is not too much correlated with $\boldsymbol{y}$ (otherwise the MLSE is ill-conditioned), and (ii) $\boldsymbol{z}$ is enough correlated with either s or $\boldsymbol{y}$.

5 Simulations

Denote the three kinds of terms of (16) as L01, C30 and C12, respectively. Consider four different types of equalizers with increasing complexity:

- T1: based on L01 terms only (traditional LMSE);
- T2: based on L01 and C12 terms;
- T3: based L01 and C30 terms;
- T4: based on the three terms above together.

In order to access some ultimate performance evaluation, the actual source sequence $s(n)$ is utilized when computing the equalizer vector, $\boldsymbol{f}_{mse}$, in (18). In a second stage, the Symbol Error Rate (SER) between the actual and estimated source sequences, $s(n)$ and $\hat{s}(n)$, is estimated as a function of SNR. This way of computing the performance is *ultimate* in the sense that it is optimistic.

The length Q, Q', and Q'', of equalizers L01, C30, and C12, are chosen in such a way that the number of equations is the same (hence the same complexity). The number of equations is thus KQ for L01, $Q'K(K+1)(K+2)/6$ for C30, and $Q''K^2(K+1)/2$ for C12.

The transmit filter (pulse shape) is the raised cosine:

$$f(t) = \mathrm{sinc}(\pi t/T_s)\, \frac{\cos(\pi\beta t/T_s)}{1 - 4\beta^2 t^2/T_s^2} \quad (23)$$

where T_s is the symbol period, and the roll-off factor is $\beta = 0.22$.

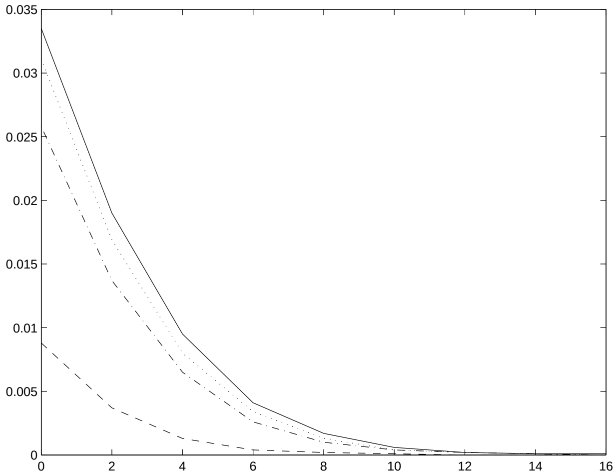

Fig.1 55 pure random channels 5 sensors with 255 parameter

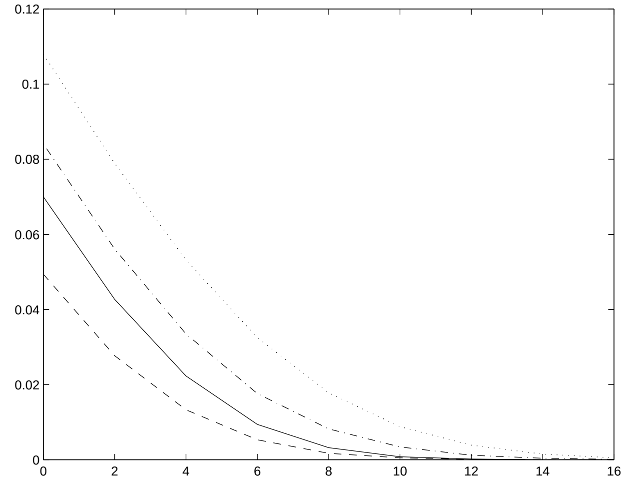

Fig.3 200 Clarke random channels 3 sensors with 69 parameters

Three channels are generated: 1 purely random, and 2 specular according to an urban model.

The pure random channel consists of mere Gaussian coefficients:

$$\boldsymbol{B} = \mathrm{randn}(K, L) + \jmath * \mathrm{randn}(K, L); \tag{24}$$

The 6-path urban Clarke model has time delays $[0, 0, 1, 2, 2, 5]$ in symbol period unit, and amplitudes $[-3, 0, -2, -6, -8, -10]$ in dB.

The antenna array is chosen to be a linear and equispaced, with the sensor spacing of half a wavelength of the carrier. Directions of the other users are drawn randomly in the range [45, 135] degrees from the end fire.

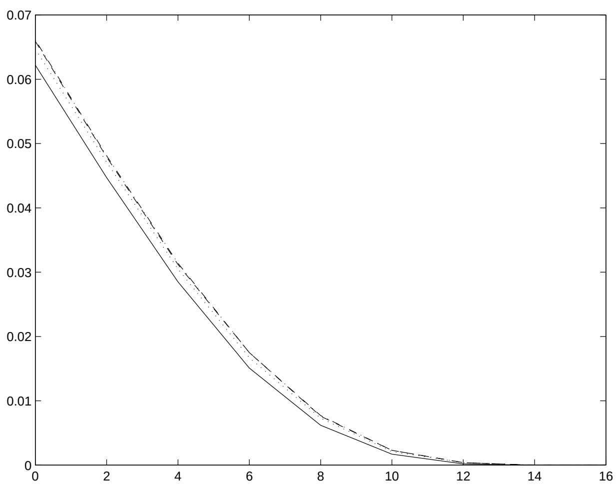

Fig.2 50 Clarke random channels 5 sensors with 255 parameters

For all the 3 figures below, performances (in terms of SER) of equalizers T1, T2, T3 and T4 are plotted in solid, dotted, dash-dotted, and dashed lines, respectively.

In Fig.1, $K = 5$, and channel taps are drawn randomly according to (24). For equalizer T1, $Q = 51$. For combination T2, $Q = 9$ and $Q' = 6$. For combination T3, $Q = 6$ and $Q'' = 3$. For combination T4, $Q = 7$, $Q' = 2$ and $Q'' = 2$. All equalizers have a system matrix of the same size, namely 255.

In Fig.2, Q, Q' and Q'' are the same as those of Fig.1, so as to the size of system matrix, but the channel follows the urban Clarke model above mentioned.

In Fig.3, $K = 3$. $Q = 23$ for T1; $Q = 3$ and $Q' = 6$ for T2; $Q = 5$ and $Q'' = 3$ for T3; and $Q = 7$, $Q' = 3$ and $Q'' = 1$ for T4. In this last example, the matrix of the linear system to solve is smaller (size 69 for all equalizers).

These 3 figures show that for pure random channels, cubic terms always yield an improvement: T1 performs the worse, and T4 performs the best. On the other hand, for Clarke channels, performances of the four equalizers are almost the same for $K = 5$ sensors; but with fewer sensors, namely $K = 3$, combination T4 performs again the best.

6 Conclusions

This paper proposes a novel WSPE dedicated to linear specular channels. Under the assumption that all users are *i.i.d.* QPSK, we have proved that only L01, C30 and C12 terms count, among all the WSP functions of the observations of at most 3rd order. We have also shown that cross space-time terms are negligible. Note that if the signal is not an *i.i.d.* process, we can still decouple the equalization in space and time domains separately, when the noise satisfies (11).

Our quantitative analysis proves that WSPE yields a reduced MSE, compared to traditional linear MSE equalizers, both analytically and by computer experiments. Simulations indeed show that for both pure random or Clarke channels, the proposed WSPE always outperforms the linear MSE equalizer when the number of sensors is relatively small.

References

[1] O. GRELLIER, P. COMON, "Egalisation aveugle en présence de sources discretes", in *Conférence Gretsi*, Vannes, France, 13-17 sept. 1999, pp. 1093–1096.

[2] B. PICINBONO, P. CHEVALIER, "Widely linear estimation with complex data", *IEEE Trans. on Sig. Proc.*, vol. 43, no. 8, pp. 2030–2033, Aug. 1995.

This work has been supported in part by the RNRT project *Paestum*.