Robust MRE methods for blind multichannel image restoration

WIRAWAN, Karim ABED-MERAIM, Pierre DUHAMEL, Henri MAÎTRE

Dépt. Traitement du Signal et des Images, ENST Paris
46, rue Barrault, 75634 Paris Cedex 13
wirawan@tsi.enst.fr

Résumé – Nous étudions le problème de restauration des images multicanaux utilisant un banc de filtres RIF. Nous avons proposé antérieurement une méthode de restauration des images multicanaux basée sur la méthode d'égaliseurs mutuellement référencés (EMR). Cette méthode a d'abord été appliquée au problème d'égalisation d'un canal de transmission numérique, ensuite a été généralisée au problème de restauration des images multicanaux. Nous présentons dans cet article une nouvelle version de la méthode de EMR 2-D avec plus de robustesse vis-à-vis du bruit additif. Par rapport à d'autres méthodes multicanaux, le calcul de la méthode EMR 2-D est moins coûteux, étant donné que la taille de la matrice principale utilisée dans l'algorithme ne dépend que de celle des canaux et des filtres égaliseurs et non pas de celle de l'image. Finalement, nous illustrons les performances de l'agorithme EMR 2-D et les comparons à celles de la méthode des moindres carrées.

Abstract – We address the problem of blind multichannel image restoration using multiple FIR filters. Previously, we have extended the 1-D mutually referenced equalizers (MRE) method for blind equalization of multichannel FIR system to the multichannel image restoration problem. In this paper, we present further results of a new version of 2-D MRE method that is more robust against additive noise. Compared to other restoration methods using multiple FIR filters our method is computationally more efficient, since the size of the matrix in the algorithm depends only on the size of blur and restoration filters and not on the image size. Finally, simulation results are presented in order to demonstrate the restoration results using the 2-D MRE method, which we compare with the least-squares (LS) method.

1 Introduction

Multichannel image restoration using FIR restoration filters, especially in blind setting, has gained increasing interest recently. That is mainly due to its simplicity in relaxing assumption on input image model, i.e. deterministic or random, and in relying only on the diversity of the channels [3, 4, 5, 6, 7]. Moreover, multichannel image restoration problem is in general more stable than single channel restoration problem and can be performed without regularization for moderate SNR. In addition, by using small size FIR restoration filters we can gain in computational cost. Examples of possible applications where more than one differently blurred versions of a single image are available, include remote sensing, electron microscopy, or multi-band imaging.

Blind multichannel image restoration in general can be done in two ways. The two-step approach estimates first the blurs, then uses any conventional method to restore the original image based on the previous channel estimate [5, 3]. The one-step approach estimates directly the original image or the restoration filter/equalizer [3, 6, 7]. Previously, we have extended the 1-D mutually MRE method for blind equalization of multichannel FIR system to the multichannel image restoration problem [7]. In this paper, we present further results of a new version of 2-D MRE method, borrowing the idea presented in [1] for 1-D signals.

2 Problem Formulation

Denote $x(n_1,n_2)$ the original image, $\mathbf{y}(n_1,n_2) = [y_1(n_1,n_2), \ldots, y_K(n_1,n_2)]^T$ the outputs from K linearly spatially invariant FIR unknown blur functions $\mathbf{h}(n_1,n_2) = [h_1(n_1,n_2), \ldots, h_K(n_1,n_2)]^T$, and $\mathbf{b}(n_1,n_2) = [b_1(n_1,n_2), \ldots, b_K(n_1, n_2)]$ the observation AWGN noise. We model them as

$$\mathbf{y}(n_1,n_2) = \sum_{k_1,k_2} \mathbf{h}(k_1,k_2)x(n_1-k_1,n_2-k_2) + \mathbf{b}(n_1,n_2) \quad (1)$$

For the subsequent presentations we use the following matrix form expression

$$\mathbf{Y}(n_1,n_2) = \mathbf{H}\mathbf{X}(n_1,n_2) + \mathbf{B}(n_1,n_2) \quad (2)$$

where the system matrix $\mathbf{H}$ is of size $Km_gn_g \times (m_h + m_g - 1)(n_h + n_g - 1)$, whose expression is given in [7], $\mathbf{Y}(n_1,n_2) = [\mathbf{y}^T(n_1,n_2), \ldots, \mathbf{y}^T(n_1-m_g+1, n_2-n_g+1)]^T$ and $\mathbf{B}(n_1,n_2) = [\mathbf{b}^T(n_1,n_2), \ldots, \mathbf{b}^T(n_1-m_g+1), n_2-n_g+1))]^T$ are $Km_gn_g \times 1$ vectors, and $\mathbf{X}(n_1,n_2) = [x(n_1,n_2), \ldots, x(n_1-(m_h+m_g-2), n_2-(n_h+n_g-2))]^T$ is a $(m_h + m_g - 1)(n_h+n_g-1) \times 1$ vector. $(m_h \times n_h)$ and $(m_g \times n_g)$

are the size of the multichannel blur filters and restoration filters, respectively. We assume that the system matrix $\mathbf{H}$ is of full-column rank.

3 MRE Method

Instead of first identifying the channels and then finding the equalizers or the system inverse, the MRE method estimates directly the equalizers or the restoration filters. The basic idea of the method, originally developed in [2] for 1-D signals and extended to 2-D in [7], is that the outputs of different equalizers with different delays act as reference signals to each other.

We consider first the noiseless condition. Define two equalizers $\mathbf{g}_{(i,j)}$ and $\mathbf{g}_{(k,l)}$ of size $(Km_g n_g \times 1)$ satisfying

$$\begin{aligned} \mathbf{g}^T_{(i,j)}\mathbf{Y}(n_1,n_2) &= x(n_1-i,n_2-j) \\ \mathbf{g}^T_{(k,l)}\mathbf{Y}(n_1,n_2) &= x(n_1-k,n_2-l) \end{aligned} \quad (3)$$

where $(i,j),(k,l) \in (0,\ldots,m_h+m_g-2)\times(0,\ldots,n_h+n_g-2)$ are the restoration delays. Consequently we can write

$$\begin{aligned} \mathbf{g}^T_{(i,j)}\mathbf{Y}(n_1,n_2) &= \mathbf{g}^T_{(k,l)}\mathbf{Y}(n_1+k-i,n_2+j-l) \\ &= x(n_1-i,n_2-j) \end{aligned} \quad (4)$$

where $(i,j) \in (0,\ldots,m_h+m_g-3)\times(0,\ldots,n_h+n_g-3)$ and $(k,l) \in (i+1,\ldots,m_h+m_g-2)\times(j+1,\ldots,n_h+n_g-2)$. Using all equalizers corresponding to different delays as column vectors, we write an equalizer matrix $\mathbf{G}$ such that

$$\mathbf{G}^T\mathbf{H} = \alpha\mathbf{I},\ \alpha \in \mathbb{R} \quad (5)$$

In 1-D, the equalizers are estimated by minimizing the quadratic criterion [2]

$$\min J_1(\mathbf{G}) = \sum_n E\|[\mathbf{I}\,\mathbf{0}]\mathbf{G}^T\mathbf{Y}(n) - [\mathbf{0}\,\mathbf{I}]\mathbf{G}^T\mathbf{Y}(n+1)\|^2 \quad (6)$$

where $\mathbf{G}$ is a matrix of equalizers (i.e., the i-th column vector of $\mathbf{G}$ is an equalizer with a delay $(i-1)$) under a given constraint, i.e. trace$(\mathbf{G}^T\mathbf{G}) = 1$. This constraint is required to avoid the trivial solution $\mathbf{G} = 0$, as well as the non-zero blocking matrices giving $\mathbf{G}^T\mathbf{H} = 0$, which correspond to $\alpha = 0$.

Direct extension of the standard 1-D MRE method to 2-D leads to high computational cost since the number of restoration delays increases quadratically along with increasing filter size. However choosing any number of equalizers with different delays will in general give a solution that are mixtures of a given number of restored pixels of different delays, such that a separation step is still required. To alleviate this problem a modified version of 2-D MRE is proposed where we estimate only 4 equalizers corresponding to 4 different delays $(0,0),(0,n_h+n_g-2),(m_h+m_g-2,0)$, and (m_h+m_g-2,n_h+n_g-2), which are sufficient to restore the whole part of the original image [7]. In this way the number of common pixels of the original image contributing to a pair of observed images region is only one. Unfortunately, the restored image using the equalizers with delays of the edge part of the reconstruction area is in general more noise sensitive than using other delays of the recontruction area, i.e. the center part. Moreover, the estimated equalizers using linear or quadratic constraint generally contain also the components of the null space of $\mathbf{H}^T$ which may render the unknown restoration constant factor α close or even equal to zero.

3.1 Modified MRE method

We propose here a robust 2-D MRE method, based on the work in [1] for the 1-D MRE method, in which the equalizers are estimated such that the output energy of the restored image is maximized. To choose the equalizer matrix corresponding to large α we use the following criterion

$$\max_{\tilde{\mathbf{G}}} E(\|\tilde{\mathbf{G}}^T\mathbf{H}\|^2) \quad (7)$$

where $\tilde{\mathbf{G}}$ is the equalizer matrix which consists of only 4 equalizer vectors $\mathbf{g}_i, i = 1,\ldots,4$ corresponding to 4 different delays discussed before. Specifically, the modified cost function of (6) for 2-D MRE method is given by

$$\begin{aligned} \min J_2(\mathbf{g}_1,\mathbf{g}_2,\mathbf{g}_3,\mathbf{g}_4) &= \sum_{n_1,n_2} |\mathbf{g}_1^T\mathbf{Y}_2 - \mathbf{g}_2^T\mathbf{Y}_1|^2 \\ &+ \sum_{n_1,n_2} |\mathbf{g}_3^T\mathbf{Y}_4 - \mathbf{g}_4^T\mathbf{Y}_3|^2 \end{aligned} \quad (8)$$

where the equalizers $\mathbf{g}_i, i = 1,\ldots,4$ correspond to the restoration delays $(0,0),(m_h+m_g-2,n_h+n_g-2),(0,n_h+n_g-2)$, and $(m_h+m_g-2,0)$, respectively, and $\mathbf{Y}_1 = \mathbf{Y}(n_1-(m_h+m_g-2),n_2-(n_h+n_g-2)), \mathbf{Y}_2 = \mathbf{Y}(n_1,n_2), \mathbf{Y}_3 = \mathbf{Y}(n_1-(m_h+m_g-2),n_2)$, and $\mathbf{Y}_4 = \mathbf{Y}(n1_1,n_2-(n_h+n_g-2))$. We then rewrite (8) in two separate forms

$$\arg\min_{\mathbf{g}_{1,2}} \quad \mathbf{g}^T_{1,2}\mathbf{Q}_1\mathbf{g}_{1,2} \quad (9)$$

$$\arg\min_{\mathbf{g}_{3,4}} \quad \mathbf{g}^T_{3,4}\mathbf{Q}_2\mathbf{g}_{3,4} \quad (10)$$

where $\mathbf{g}_{1,2} = [\mathbf{g}_1^T,\mathbf{g}_2^T]^T$, $\mathbf{g}_{3,4} = [\mathbf{g}_3^T,\mathbf{g}_3^T]^T$, and $\mathbf{Q}_i, i = 1,2$ are two positive quadratic forms. We present the computation for $i = 1$ only, since it is similar for $i = 2$. Define $\mathbf{V}_1$ an orthogonal basis of the kernel of $\mathbf{Q}_1$. In noisy condition, $\mathbf{V}_1$ is given by the $d = 1+2(Km_g n_g - ((m_h+m_g-1)(n_h+n_g-1))$ least eigenvectors of $\mathbf{Q}_1$. Using the constraint $\|\mathbf{g}_{1,2}\| = 1$, the solution of (9) is given by $\mathbf{g}_{1,2} = \mathbf{V}_1\mathbf{u}$ where $\mathbf{u}$ is a $d \times 1$ unit vector. Consequently, the solution of (7) is $\mathbf{g}_{1,2} = \mathbf{V}_1\mathbf{u}_1$ where $\mathbf{u}_1$ is the principal eigenvector of

$$\mathbf{V}_1^T(\mathbf{I}\otimes\mathbf{R})\mathbf{V}_1 \quad (11)$$

where $\otimes$ denotes the Kronecker product, $\mathbf{I}$ is the 2×2 identity, and $\mathbf{R} = E(\mathbf{Y}(n_1,n_2)\mathbf{Y}^T(n_1,n_2))$.

Remarks

1. We have chosen J_2 in such a way that we can decouple the estimation of $\mathbf{g}_{1,2}$ from that of $\mathbf{g}_{3,4}$. However, we can improve slightly the estimation accuracy as well as the complexity by referring the first equalizers with the two last ones. In fact, instead of J_2, we can use
$$\begin{aligned} J_3(\mathbf{g}_1,\ldots,\mathbf{g}_4) &= \sum_{1\le i<j\le 4} \|\mathbf{g}_i^T\mathbf{Y}_i - \mathbf{g}_j^T\mathbf{Y}_j\|^2 \\ &= [\mathbf{g}_1^T\mathbf{g}_2^T]\tilde{\mathbf{Q}}\begin{bmatrix}\mathbf{g}_1\\ \mathbf{g}_2\end{bmatrix}\end{aligned}$$
where $\tilde{\mathbf{Q}}$ is the modified quadratic form.

2. Although using only the 4 restoration delays discussed before is sufficient to restore the original image, it does not exploit fully the channel structure. Specifically, the 4 estimated equalizers $\mathbf{g}_i, i = 1,\ldots,4$ rely on particular realization of $\mathbf{h}(0,0), \mathbf{h}(0, n_h+n_g-2), \mathbf{h}(m_h+m_g-2, 0)$, and $\mathbf{h}(m_h+m_g-2, n_h+n_g-2)$. Moreover, it is known from simulation that the noise-sensivity of FIR restoration filters would be different for different parts of the image [4]. For example, the parts of image reconstructed using the 4 'corner' restoration delays would be worse than the other parts. We can improve the noise sensivity by adding more equalizers corresponding to different delays other than the 4 previous delays. In particular, we can fully exploit the channel structure by using the equalizer with delay $(\lceil(m_h+m_g+2)/2\rceil, \lceil(n_h+n_g+2)/2\rceil)$. Let $\mathbf{g}_5$ and $\mathbf{Y}_5$ denote this equalizer and the corresponding observation vector. Then, we can estimate the 5 equalizers using
$$\min J_4(\mathbf{g}_1,\ldots,\mathbf{g}_5) = \sum_{1\le i<j\le 5} \|\mathbf{g}_i^T\mathbf{Y}_i - \mathbf{g}_j^T\mathbf{Y}_j\|^2$$

3. In this paper, we have implicitly assumed that the blur size (m_h, n_h) is exactly known. In fact, if the latter is unknown or incorrectly estimated the MRE method as presented above would fail to provide the desired result. A simple solution would be to test a set of filter size and chose the one corresponding to the best result. Another solution would be to chose a set of sizes $\{(m_{h,i}, n_{h,i})\}_{i=1,\ldots,I}$ in such a way it contains the exact channel size. Let $J_2(\tilde{\mathbf{G}}_i)$, where $\tilde{\mathbf{G}}_i = [\mathbf{g}_1^i,\ldots,\mathbf{g}_4^i]$, be defined as in (8) using the size $(m_{h,i}, n_{h,i})$. We can then estimate the equalizer from
$$\min J_5(\tilde{\mathbf{G}}) = \sum_{i=1}^{I} J_2(\tilde{\mathbf{G}}_i)$$

4 Simulation

In this section, we present simulation results of our proposed method tested on a photographics image of size (100×100). We measure the performance of the proposed 2-D MRE method against the LS method. Specifically for performance comparison using objective test, we use the improvement in SNR (ISNR) and normalized mean squared error (NMSE), given by:
$$\text{ISNR} = 10.\log_{10}\frac{1/K\sum_{k=1}^{K}\sum_{n_1,n_2}[x(n_1,n_2)-y_k(n_1,n_2)]^2}{\sum_{n_1,n_2}[x(n_1,n_2)-\hat{x}(n_1,n_2)]^2}$$
$$\text{NMSE} = \frac{\|\mathbf{X}-\hat{\mathbf{X}}\|^2}{\|\mathbf{X}\|^2}$$
where $\hat{x}(n_1, n_2)$ is the restored image and the average over k is to take multichannel setup into account.

The original *chapel* image is passed through 4 FIR channels of $(m_h\times n_h) = (3\times 3)$, whose values are chosen randomly using uniform distribution within the range 0 and 1. Energy preservation is assumed, since the imperfections in an image formation system normally act as passive operations on the data. White Gaussian noise with equal variance was then added for each channel. Using exact convolutional model, the observed images are consequently of size (98×98). The size of restoration filters is taken to be $(m_g\times n_g) = (3\times 3)$. To cope with the unknown multiplication constant in the solution of the MRE method, we assume that the range of the image value in each pixel is known. As mentioned before, here we compute 4 multichannel equalizers corresponding to 4 different delays.

Simulated SNR of 30 dB is used and the results of restoration using MRE method and LS filters shown in Fig. 1. The restored image using MRE method in Fig. 1d is selected for the best visual result among 4 restored images associated with different restoration delays. We notice that visually the quality of MRE restored images are slightly inferior from LS restored images, which is supported by quantitative comparisons using ISNR and NMSE illustrated in Fig. 2 and Fig. 3, respectively. The ISNR curves are averaged values of many runs for each SNR. The difference of around 5 dB is observed between the two ISNR curves. The delay (i, j) used for LS restored images is $((m_h+m_g)/2, (n_h+n_g)/2)$, which in general gives smaller restoration error than the delays on the edge of the reconstruction region, like the ones we use for the MRE method. This is to be expected, since using LS restoration filters, the multichannel blurs are completely known, whereas MRE algorithm operates in blind condition.

From the experiments, we found that underestimation of the channel filters support still provides better restored image than the blurred ones, whereas overestimation leads certainly to erroneous restoration. This can be easily explained by the fact that for the second case the channel matrix looses its fullrankness. In the real condition with blind setting, a strategy of augmenting gradually the estimated filter support, until most visually satisfying result is obtained, could then be employed.

5 Conclusion

We have presented a new version of the 2-D MRE method with more robust performance against additive noise. The proposed method overcomes a drawback of the standard 2-D MRE method by maximising the power of the restored image. Compared to other restoration methods using multiple FIR filters our method is computationally more efficient, since the size of the matrix in the algorithm depends only on the size of blur and restoration filters and not on the image size. Simulations result show that the performances of the robust MRE method are closed to the performances of the LS solution.

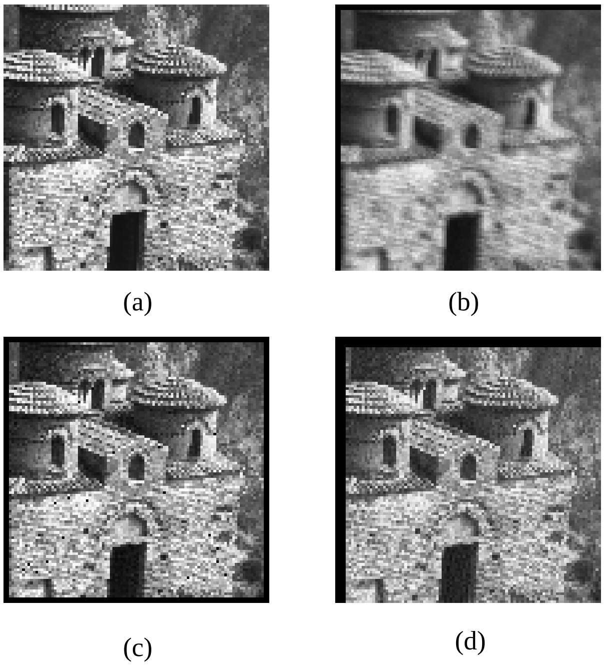

Figure 1: Simulation (a) original *chapel* image, (b) 1 of 4 blurred and noisy images, with SNR of 30dB (c) restored image using LS solution (d) restored image using the MRE method.

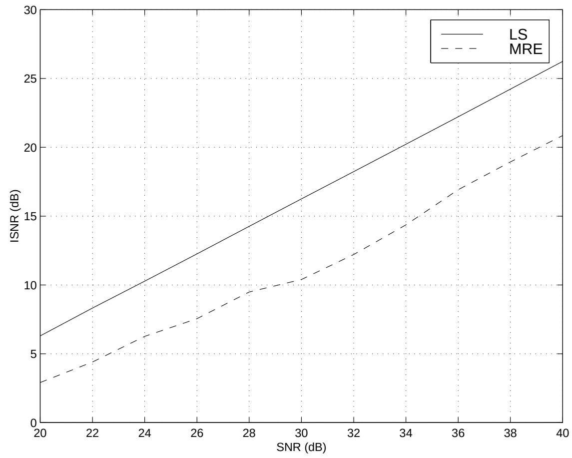

Figure 2: SNR improvement as a function of the SNR of the blurred images using the LS method (solid line) and using the MRE method (dashed line), for the *chapel* image.

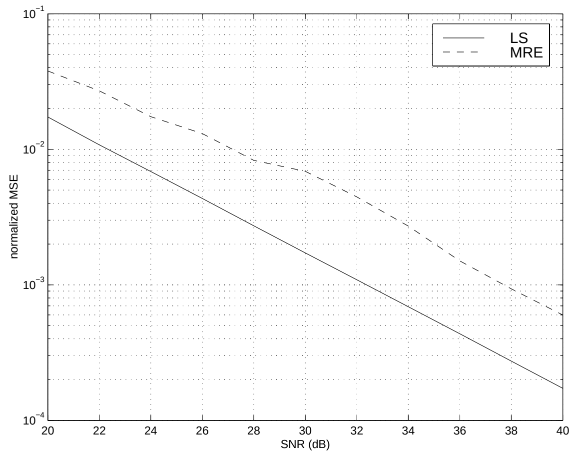

Figure 3: Normalized MSE as a function of the SNR of the blurred images using the LS method (solid line) and using the MRE method (dashed line), for the *chapel* image.

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