An Analysis of Some Multiuser Detectors in Impulsive Noise

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RÉSUMÉ

L'object de cet article est l'étude de la détection de données à travers des canaux de communication non gaussiens, en séquence directe, spectre large et accès multiple. Ce problème apparaît dans des situations pratiques parce que la plupart des canaux physiques, dans lesquels les communications en multiple accès sont utilisées, sont connus pour être incontestablement non gaussiens. Concernant les systèmes synchrones, le détecteur multi-utilisateur optimal (au sens du maximum de vraisemblance) est obtenu et ses performances sont analysées et comparées avec plusieurs détecteurs sous optimaux.

ABSTRACT

The present paper deals with the problem of data detection in *direct-sequence spread-spectrum* multiple-access non-Gaussian channels. This issue arises in practical situations because many physical channels in which multiple-access communications is applied are known to be decidedly non-Gaussian. With reference to a synchronous system, the optimum (in the maximum likelihood sense) multiuser detector is derived, and its performance is analyzed and compared with that of several suboptimum detectors.

1 Introduction

Multiple-access communication channels are channels in which multiple transmitter/receiver pairs communicate through the same physical medium using non-orthogonal multiplexing. Such channels arise in a number of new and emerging applications, including third-generation mobile telephony, wireless personal communications, indoor communications, underwater acoustic communications, nomadic computing, and wireless tactical military communications.

Multiple-access channels are inherently non-Gaussian in nature due to the presence in the channel of highly structured multiple-access interference (MAI). Moreover, for many of the physical channels arising in the applications cited above, the ambient noise is known through experimental measurements to be decidedly non-Gaussian. This is particularly true of urban and indoor radio channels [5] and underwater acoustic channels [4].

In this paper we consider the problem of data detection in *direct-sequence spread-spectrum* multiple-access non-Gaussian channels. At first, in Section 2, the adopted system model is described. Then, in Section 3, several detectors apt to counteract the degrading effects of non-Gaussian noise are proposed. Specifically, the optimum (in the maximum-likelihood sense) multiuser detector is derived and its complexity is discussed. Moreover, three suboptimum detectors are proposed. Two of them are based on different approximations of the likelihood-function, whereas the third is a natural generalization of the well known decorrelating detector, taking into account the non-Gaussian nature of the noise environment. Finally, in Section 4, with reference to a synchronous system, the performance of the considered detectors is assessed via Monte Carlo computer simulations.

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2 System model

The waveform received by a given terminal in a multiple-access *direct-sequence spread-spectrum* system can be modeled as:

$$r(t) = S_t(\mathbf{b}) + N_t, \quad -\infty < t < \infty,$$

where $S_t(\mathbf{b})$ and N_t represent the useful signal and the ambient channel noise, respectively.

The useful signal $S_t(\mathbf{b})$ is comprised of the data signals of K active users in the channel, and can be written as

$$S_t(\mathbf{b}) = \sum_{k=1}^{K} A_k \sum_{i=-M}^{M} b_k(i) s_k(t - iT - \tau_k),$$

where $2M + 1$ is the number of symbols per user in the data frame of interest, T is the symbol interval, and where A_k, τ_k, $\{b_k(i)\}$, and $\{s_k(t); 0 \leqslant t \leqslant T\}$ denote, respectively, the received amplitude, delay, symbol stream, and normalized modulation waveform of the k^{th} user. The matrix $\mathbf{b}$ denotes the $K \times (2M + 1)$ matrix whose $k, i-$th element is $b_k(i)$. We will assume that the symbols are binary ± 1, and that the modulation waveforms are zero for $t \notin [0, T]$. The signaling constellation (i.e., $s_1, s_2, \ldots, s_K$) consists of a set of normalized non-orthogonal signals

$$s_k(t) = \begin{cases} \sqrt{\frac{2}{T}} a_k(t)\cos(\omega_c t + \phi_k) & , \quad t \in [0, T] \\ 0 & , \quad t \notin [0, T] \end{cases}$$

where ω_c is a common carrier frequency, ϕ_k is the phase of the k^{th} user relative to some reference, and the spreading waveforms $a_k(t)$ are of the form:

$$a_k(t) = \sum_{n=0}^{N-1} a_n^k p_{T_c}(t - nT_c).$$

Here, $a_0^k, a_1^k, \ldots, a_{N-1}^k$ is a signature sequence of +1's and -1's assigned to the k^{th} user, and p_{T_c} is a unit-amplitude pulse of duration T_c (where $NT_c = T$). For the sake of simplicity, we will assume that the received delays are integral numbers of chip intervals; i.e., $\tau_k = m_k T_c$, $k = 1, 2, \ldots, K$, where $m_1, m_2, \ldots, m_K$ are integers in the range 0 and $N - 1$.

The great majority of research on multiuser detection has ascribed the simplest possible model to the ambient channel noise; namely, that the only ambient channel noise is additive white Gaussian noise (AWGN) with fixed spectral height, say σ^2. In such a case, the general structure of optimum procedures for determining the data symbols from the received waveform consists of an analog front-end that extracts the matched filter outputs

$$y_k(i) = \sum_{n=0}^{N-1} a_n^k \, \mathbf{Re}\left\{ e^{-j\phi_k} \tilde{r}_{n+iN+m_k} \right\}, \tag{1}$$

with $j = \sqrt{-1}$, and

$$\tilde{r}_n = \sqrt{\frac{2}{T}} \int_{nT_c}^{(n+1)T_c} r(t) e^{-j\omega_c t} dt, \; n = 0, \pm 1, \pm 2, \ldots, \tag{2}$$

followed by a decision algorithm that infers optimum decisions from the collection of these outputs.

The AWGN model has been an appropriate model in previous studies, since the focus there has been on the mitigation of the most severe noise source - the MAI. However, as increasingly practical techniques for multiuser detection become available, the situation in which practical multiple-access channels will be ambient-noise limited can be realistically envisioned.

In the single-user context, it is well known that non-Gaussian noise can be quite detrimental to the performance of conventional systems based on the Gaussian assumption, whereas it can be beneficial to performance if appropriately modeled and ameliorated.

In this paper, we study the issue of non-Gaussian ambient noise in the multiuser context by adopting the discrete-time model obtained by considering the sequence $\{\tilde{r}_n\}$ in (2). This discrete-time model is convenient since non-Gaussian ambient noise can be studied by representing the noise in the sequence $\{\tilde{r}_n\}$ as a sequence of independent and identically distributed (i.i.d.) complex random variables with a non-Gaussian distribution.

3 Detector structures

In this section, we propose several detectors apt to ameliorate non-Gaussian ambient noise, whose performance will be successively analyzed.

A basic way of dealing with the impulsive noise is to replace the conventional linear correlator (1) with a nonlinear one:

$$\tilde{y}_k(i) = \sum_{n=0}^{N-1} a_n^k \; g\left(\mathbf{Re}\left\{ e^{j\phi_k} \tilde{r}_{n+iN+m_k} \right\}\right), \tag{3}$$

where g is an instantaneous nonlinearity. Detection structures based on (3) with different choices of the nonlinearity g have been extensively analyzed in [1] and [2].

Straightforward modifications of the nonlinear correlator (3) that can give superior performance in combined MAI and non-Gaussian noise, belong to the general family of nonlinear correlators of the form

$$\tilde{y}_k(i) = \sum_{n=0}^{N-1} [a_n^k + x_n^k] \; g\left(\mathbf{Re}\left\{ e^{j\phi_k} \tilde{r}_{n+iN+m_k} \right\}\right), \tag{4}$$

where the adjustment signals $x_0^k, x_1^k, \ldots, x_{N-1}^k$ are orthogonal to their corresponding spreading codes. Detectors of this form might be optimized over the nonlinearity g and the correlator weights $x_0^k, x_1^k, \ldots, x_{N-1}^k$. For linear g, and weights chosen to force the output MAI to zero, the decorrelating detector is obtained, whereas with weights chosen to minimize the mean-square value of the output MAI plus noise, the well known MMSE detector is obtained. Unfortunately, for nonlinear g, the correct optimization criterion is not clear.

Aside from choice of the correlator adjustment weights, the use of a single nonlinear correlator of the form (4) requires knowledge of the timing and spreading code of a single user only. Thus, this kind of detector can be used in a downlink situation if the adjustment weights can be chosen adaptively. In uplink situations, it is of interest to consider an approach incorporating all users' waveforms and timing information.

In order to consider such approach, under the assumption of zero-mean, i.i.d continuous-amplitude noise in the sampled signal $\{\tilde{r}_n\}$, we can write a likelihood function for this signal conditioned on the bit matrix $\mathbf{b}$. By assuming for simplicity that the users' received phases ϕ_k are all the same (say $\phi_1 = \phi_2 = \cdots = \phi_K = 0$), we can write the log-likelihood function as:

$$\ell\left(\tilde{r}_n; n = 0, \pm 1, \pm 2, \cdots\right) = \sum_{n=-\infty}^{\infty} \log \frac{f\left(r_n - s_n(\mathbf{b})\right)}{f\left(r_n\right)}, \tag{5}$$

where $r_n = \mathbf{Re}\{\tilde{r}_n\}$, f is the marginal probability density of the noise in r_n, and

$$s_n(\mathbf{b}) = \frac{T_c}{T} \sum_{k=1}^{K} A_k \sum_{i=-M}^{M} b_k(i) a_{n-iN-m_k}^k.$$

Note that direct maximization of (5) over the bit matrix $\mathbf{b}$ requires, in general, exhaustion over $2^{K(2M+1)}$ choices. Thus, some kind of approximation must be used in order to bring this complexity down.

A basic technique for approximating optimal detection procedures for non-Gaussian noise is to use an *asymptotic* *local* approach (cf., [3]), in which performance is optimized for the situation in which the per-sample signal-to-noise ratio is small and the integration time is long. These basic conditions are present in wideband systems, such as direct-sequence spread spectrum sampled at the chip rate.

Within regularity, this approach yields the following approximation to the log-likelihood function (5):

$$\ell \sim \frac{T_c}{T}\sum_{k=1}^{K} A_k \sum_{i=-M}^{M} b_k(i)\overline{y}_k(i)$$

$$-\left(\frac{T_c}{\sqrt{2}T}\right)^2 \sum_{k,k'=1}^{K} A_k A_{k'} \sum_{i,i'=-M}^{M} b_k(i) b_{k'}(i') \Xi(k,k';i,i') \quad (6)$$

where

$$\overline{y}_k(i) = \sum_{n=0}^{N-1} a_n^k \, g_{\ell o}\left(r_{n+iN+m_k}\right)$$

with $g_{\ell o}(x) = -f'(x)/f(x)$; and where

$$\Xi(k,k';i,i') = \sum_{n=-\infty}^{\infty} a^k_{n-iN-m_k} a^{k'}_{n-i'N-m_{k'}} \, \overline{g}(r_n) \quad (7)$$

with $\overline{g}(x) = [g_{\ell o}(x)]^2 - f''(x)/f(x)$.

In the Gaussian-noise, $g_{\ell o}(x) = Kx$ and $\overline{g}(x) = K$ for a positive constant K, so that this approximation gives the exact likelihood function [6]. More generally, however, the fact that

$$\Xi(k,k';i,i') = 0, \; |i-i'| > 1,$$

allows the likelihood function approximation (6) to be maximized by an $O(2^K)$-complexity-per-bit dynamic program. The only differences between this dynamic program and that of the Gaussian-channel optimal multiuser detector are the substitution of the nonlinear correlations $\overline{y}_k(i)$ for the linear correlations $y_k(i)$, and the appearance of the received waveform (in the form of $\overline{g}(r_n)$) in the quadratic term. If we further approximate this latter quantity with a constant, then this detector is identical to the maximum likelihood multiuser detector for the Gaussian multiple-access channel except that the nonlinear function $g_{\ell o}$ is inserted after the chip-rate sampler and before the correlator. A natural constant to use in such an approximation would be the mean value of $\overline{g}(r_n)$ under the noise distribution, which is easily seen to be equal to Fisher's information for location:

$$I(f) = \int (f')^2/f.$$

That is, a natural data-independent approximation to (7) is

$$\Xi(k,k';i,i') = I(f) \sum_{n=-\infty}^{\infty} a^k_{n-iN-m_k} a^{k'}_{n-i'N-m_{k'}}, \quad (8)$$

which is determined by the aperiodic cross-correlation properties of the spreading sequences.

4 Numerical results

In this section, with reference to a synchronous CDMA system, the performance of the detectors considered above is assessed via Monte Carlo computer simulation.

In the simulations, the $\varepsilon-$mixture model for the first-order probability density function of the noise samples is adopted; i.e.,

$$f(x) = (1-\varepsilon) f_n(x) + \varepsilon f_I(x). \quad (9)$$

Both the nominal density function $f_n(x)$ and the impulsive (or contaminating) component $f_I(x)$ are taken to be Gaussian. In such a model two parameters control the shape of the noise: $\varepsilon \in [0, 1]$ and the ratio of the variance of the impulsive component to the variance of the nominal one, defined as $\gamma^2 = \sigma_I^2/\sigma_n^2$. In the simulations the average noise samples variance has been held constant and equal to 1, whereas the power of all users has been varied to achieve the desired values of the signal-to-noise ratio (defined as in [1]).

The considered synchronous system uses $m-$sequences of length $N = 31$. The generating polynomial for the first sequence has octal representation 45, and moreover, the subsequent $m-$sequences are generated by shifting the original sequence.

For synchronous CDMA, the received signal on each bit interval does not depend on the bits sent during past or future time intervals. Consequently, it is sufficient to consider a one-shot system. Starting from (5) and taking into account (9), the log-likelihood function becomes

$$\ell^{opt-ng} = \sum_{n=0}^{N-1} r_n s_n(\mathbf{b}) - \frac{1}{2}\sum_{n=0}^{N-1} s_n^2(\mathbf{b})$$

$$+\sigma_I^2 \sum_{n=0}^{N-1} \log \frac{1+\dfrac{1-\varepsilon}{\varepsilon}\,\gamma\,\exp\left[-\dfrac{(r_n - s_n(\mathbf{b}))^2}{2\sigma_n^2}\left(1-\dfrac{1}{\gamma^2}\right)\right]}{1+\dfrac{1-\varepsilon}{\varepsilon}\,\gamma\,\exp\left[-\dfrac{r_n^2}{2\sigma_n^2}\left(1-\dfrac{1}{\gamma^2}\right)\right]}, \quad (10)$$

where

$$s_n(\mathbf{b}) = \frac{T_c}{T}\sum_{k=1}^{K} A_k b_k a_n^k,$$

with b_k the kth component of the *bit vector* **b**, i.e., the bit sent, in the considered interval, by the kth user. Of course in such a case, maximization of (10) over the *bit vector* **b** requires exhaustation over 2^K choices.

The detector based on the approximation of the log-likelihood function (6) is obtained by maximizing

$$\ell^{lod-m} = \sum_{n=0}^{N-1} g_{\ell o}(r_n) s_n(\mathbf{b}) - \frac{1}{2}\sum_{n=0}^{N-1} s_n^2(\mathbf{b})\overline{g}(r_n).$$

Moreover, if the data-independent term given in (8) is utilized, the previous detector becomes

$$\ell^{lod-ng} = \sum_{n=0}^{N-1} g_{\ell o}(r_n) s_n(\mathbf{b}) - \frac{1}{2} I(f) \sum_{n=0}^{N-1} s_n^2(\mathbf{b}).$$

Note that the nonlinearities $g_{\ell o}$ and $\overline{g}$ involved in the implementation of the detection structures can be easily obtained taking into account the adopted noise model.

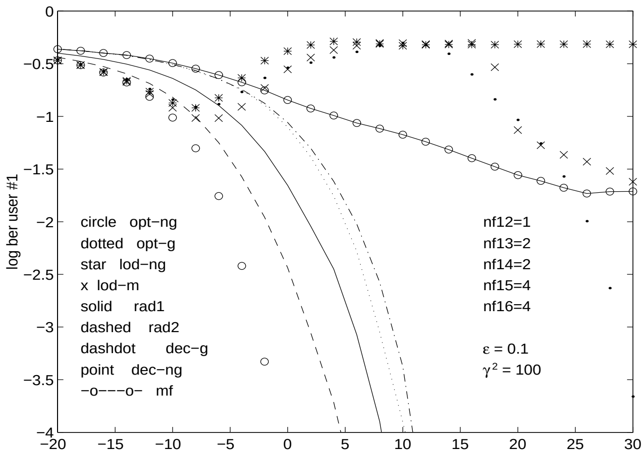

Fig.1

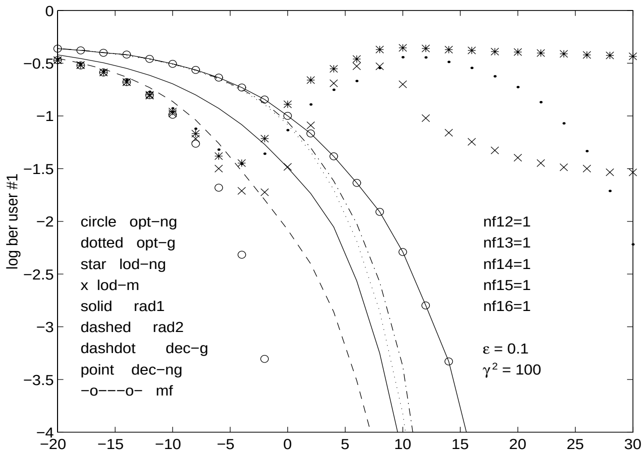

Fig.2

The optimum multiuser detector for the Gaussian noise environment is immediately obtained by putting $\varepsilon = 1$ in (10) and thus maximizes

$$\ell^{opt-g} = \sum_{n=0}^{N-1} r_n s_n(\mathbf{b}) - \frac{1}{2}\sum_{n=0}^{N-1} s_n^2(\mathbf{b}).$$

Finally, in the figures, the matched filter detector is denoted by the acronym mf, the linear decorrelating detector is denoted by the acronym dec-g and, moreover, the detector based on (4) where the nonlinearity g is chosen to be coincident with $g_{\ell o}$ and the correlator weights are chosen to be coincident with those of the decorrelating detector, is denoted by the acronym dec-ng.

Figure 1 shows the performance of these detectors, in a 6-user synchronous channel, as a function of the signal-to-noise ratio (snr) converted to dB, in a very impulsive noise environment ($\varepsilon = 0.1$, $\gamma^2 = 100$). Specifically, the bit error rate (ber) for user 1 as function of the snr of user 1 is reported, for the case where the received power of the ith user is given by E_i = nf1i E_1 ($i = 2, 3, ..., 6$), with nf1i given in the figure. The results show that significant performance gain, with respect to the optimum detector for Gaussian noise (and with respect to the decorrelator), is achieved by using the optimum non-Gaussian based multiuser detector. On the other hand, the proposed suboptimum detection structures that attempt to account for the non-Gaussian nature of the noise (lod-ng, lod-m, dec-ng) achieve nearly optimum performance only if the users' powers are weak. The range of good performance for these detectors is extended in a power-controlled situation (see Fig. 2), but the nonlinearities eventually defeat the multiple-access capability when the power level increases.

From these simulations, we see that there is potential for significant performance gain (more than 10dB in some cases) over Gaussian-optimal techniques in multiple-access channels when the ambient noise is impulsive. However, this gain is achieved with a significant penalty on complexity, in that the standard low-complexity multiuser detectors are not easily modified to account for non-Gaussian ambient noise. A compromise in complexity can be achieved by applying a two-stage detector, in which tentative bit decisions are made by the linear decorrelator, and then these decisions are refined by maximizing likelihood within a fixed Hamming radius of the decorrelator outputs. Performance of this type detector for Hamming radius 1 and Hamming radius 2 are also shown on Figs. 1 and 2. It is seen from these results that this compromise detector does in fact show promise of closing the performance gap between Gaussian-based optimal detection and non-Gaussian-based optimal detection.

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