CONSTRAINT BASED IMAGE RECONSTRUCTION IN DIFFRACTIVE TOMOGRAPHY

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Résumé

Cet article traite le problème de la restauration d'un objet degradé par un système passe-bande idéal. Nous examinons ici le cas dans lequel l'image a été obtenue par l'utilization d'une particulière téchnique de tomographie cohérente basée sur le traitement du champ rétrodiffusé.

Nous étudions l'application à ce problème d'une téchnique de restauration itérative basée sur l'obligation que l'objet soit positif.

En particulier nous considerons le cas dans lequel l'objet est formé par un ou deux points irradiateurs.

Summary

This paper concerns with the problem of restoring a two-dimensional object that has been distorted by an ideal band-pass imaging system. The problem is here focused to the case in which the image is obtained by using a particular tomographic technique that consists in irradiating the body to be explored by a C.W. source and processing the backscattered field. The effectiveness of constrained iterative restoration algorithms using positivity constraint is investigated with reference to the case in which the object is constituted by one and two pointform scatterers.

1. Introduction

Recently a particular tomographic technique [1] has been proposed for the reconstruction of two-dimensional images of sections of a body B using coherent radiation (ultrasound or microwaves). This technique consists in irradiating the body B to be explored (see Fig. 1) by a C.W. source with different wavelengths λ_i and suitably processing the backscattered field received from different angular positions θ_j.

It can be proved that the received field, for each angular position θ_j and each wavelength λ_i is the Fourier Transform of the scattering strength density function g(x,y) inside B evaluated at a point P of polar coordinates $(2/\lambda_i, \theta_j)$.

Supposing to perform the measurements for a suitable number of θ_j and λ_i, the straightforward way to reconstruct the image is to compute the Inverse Fourier Transform of the partially known spectrum of g(x,y).

In [1,2] it has been shown that this imaging technique provides resolution performance exceeding that predicted by the Rayleigh criterion; however, as a limit of the technique, the Point Spread Function of the system shows very high sidelobes that prevent the detection, in the image, of objects with small magnitude near objects with large magnitude.

In [3,4] the performances of this technique were examined in the case that several complete observations $(0 \leqslant \theta_j \leqslant 2\pi)$ with different wavelenghts were accomplished; the performed computer simulations showed that the resolution of the reconstruction method increases as the bandwidth and the mid-band frequency values used in the exploration increase: in particular in order to achieve satisfactory resolution performance very large exploration bandwidth is required.

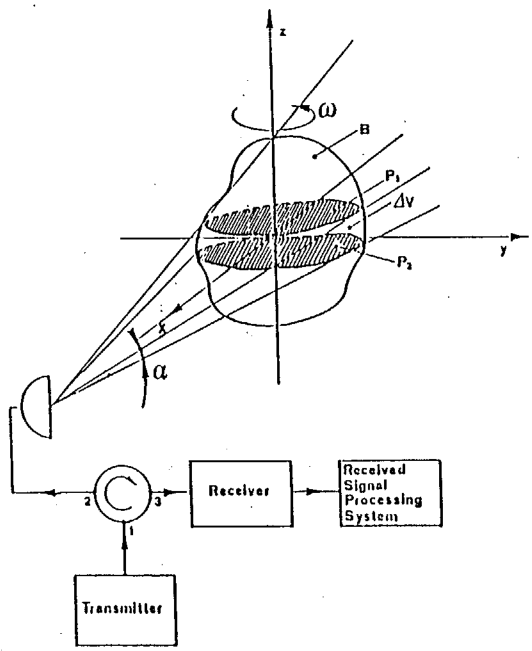

Fig. 1 - Diagram of the tomographic technique examined.

From an experimental point of view the use of large exploring bandwidth is difficult and cumbersome as it implies wide band instrumentation and expensive data acquisition systems.

The aim of this paper is to study if, by means of suitable restoration algorithms, it is possible to obtain satisfactory results for the reconstructed image also in the case where a narrow exploration bandwidth is used.

Among the several proposed restoration methods, we chose a particular class of iterative algorithms that, using some a priori information about the object, try to extend the known spectrum.

Reference was made to the case where the explored object was constituted by a single pointform scatterer or by two equal pointform scatterers at distance 2D from each other inside a homogeneous medium matched to the exploring radiation. Performance parameters were defined and their improvement due to the restoration algorithm was investigated.

2. The imaging method

With reference to Fig. 1 the backscattered field received at the angular position for a particular wavelenght λ is [1]

$$G(\theta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)\, e^{(-4\pi j/\lambda\, (-x\sin\theta + y\cos\theta))}\, dx\, dy \qquad (2\text{-}1)$$

where $g(x,y)$ = scattering strength density function (object).

By defining

$$f_x = 2\sin\theta/\lambda \qquad f_y = -2\cos\theta/\lambda$$

eq. (2-1) becomes

$$G(f_x,f_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)\, e^{(2\pi j (f_x x + f_y y))}\, dx\, dy \qquad (2\text{-}2)$$

i.e. the received signal from direction θ may be seen as the two-dimensional Fourier Transform of the scattering strength density function $g(x,y)$ evaluated at a point P of polar coordinates $(2/\lambda, \theta)$.

In the case of a continuous exploration using a radiation with frequency values inside $\Delta F = f_2 - f_1$, the $G(f_x,f_y)$ on an annular region of inner and outer radii $R_1 = 2\cdot f_1/c$ and $R_2 = 2\cdot f_2/c$ is obtained.

In this case the PSF of the considered imaging system may be written in the form

$$h(x,y) = R_2 \frac{J_1(2\pi R_2\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} - R_1 \frac{J_1(2\pi R_1\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} \qquad (2\text{-}3)$$

where J_1 is the Bessel function of first kind, order one. Expression (2-3) becomes the Dirac impulse when $R_1 = 0$ and $R_2 = \infty$. A qualitative shape of a cross section of the PSF is shown in Fig. 2. In [3] a set of parameters were chosen to quantify the performance of the imaging system with reference to its PSF. These parameters are

$\rho = A_0/A_1$ = main lobe amplitude/first sidelobe amplitude

ω = main lobe width

Moreover a global "quality factor" ρ/ω was introduced to quantify the performance of the PSF. It was proved that for any assigned $f_m = (f_1+f_2)/2$ the ratio ρ/ω increases as $\Delta f = f_2 - f_1$ increases.

In [3,4] the case of two pointform scatterers at distance 2D was also considered, in order to investigate the resolution performance of the imaging method.

It was proved that the resolution increases as f_m increases and scarcely depends on the increase of Δf.

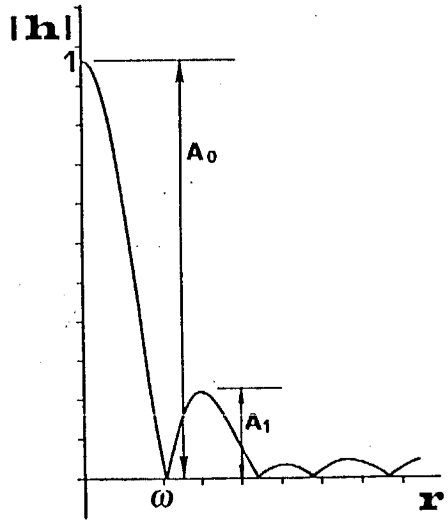

Fig. 2 - Magnitude vs. $r = \sqrt{x^2 + y^2}$ of a section of the PSF.

3. The restoration algorithm used

As shown in the previous sections the proposed tomographic technique provides a set of values of the Fourier Transform of the explored object in a finite domain of the Fourier plane given by

$$\hat{G}_0(f_x,f_y) = H(f_x,f_y)\, G(f_x,f_y) + N(f_x,f_y) \qquad (3\text{-}1)$$

where H is the frequency response of the distortion system (band-pass ideal filter), N is additive noise and G is the Fourier Transform of the object.

The problem consists in recovering the "best" approximation of the object from the knowledge of the data $\hat{G}_0(f_x,f_y)$ or, equivalently, of the image $\hat{g}_0(x,y)$.

The solution based on the direct computation of the inverse operator of H is often not practicable because of the ill-conditioned nature of the problem.

Many authors [5,6,7] evidenced the benefits of using constrained iterative restoration algorithms that consent to incorporate in the problem some constraints based upon a priori information about the properties of the desired solution such as positivity and finite extent.

In the optical field, it is often necessary to restore diffraction limited images distorted by a low-pass ideal filter [8]. In that case the above mentioned class of iterative constraints based algorithms has shown good performance in increasing the image resolution or, equivalently, in extending the band of the image beyond the cutoff frequency of the low-pass filter. However the performance of the algorithms when H is a band-pass filter still needs more insight. As a first approach, in this paper reference will be made to noise free simulated data [6].

The implemented algorithm is given by:

$$F[\hat{g}_k(x,y)] = \gamma \hat{G}_0(f_x,f_y) + [1 - \gamma H(f_x,f_y)] F[C[\hat{g}_{k-1}(x,y)]], \quad (3\text{-}2)$$

where F indicates the Fourier Transform operator, C is the operator of the imposed constraints such that g = C [g], γ is a real number chosen on the basis of convergence rate considerations. If the constraint operator C is the support limiting operator and $\gamma = 1$ the proposed algorithm coincides with that first proposed by Gerchberg [5] for low-pass filtered images and by Mensa [2] for band-pass filtered images.

4. Simulation results

In order to investigate the performance of the algorithm (3-2) in restoring images two experimental cases were investigated: in the first one the object was supposed to be a single pointform scatterer inside a homogeneous medium; in the second the object was supposed to be constituted by two equal pointform scatterers at distance 2D from each other. In both cases the Fourier Transforms were sampled using a frequency interval $\Delta f_x = \Delta f_y = 1/(\Delta s.N)$ with Δs spatial sampling step equal to 10^{-3}m and N number of samples equal to 512; 2D was chosen equal to 2 cm.

The iterative algorithm (3-2) was implemented taking into account the positivity and finite extent of the object; with regard to the finite support constraint it was assumed an overstimated circular object bound with radius equal to $N \cdot \Delta S/2$; γ value was assumed equal to 1.

With reference to microwave tomography several experimental simulations were accomplished for different values of f_1 and f_2 in the range 1-10 GHz.

a. single scatterer case

In Tab. 1 some results are summarized with reference to the "quality factor" $r = \rho/\omega$. This ratio was evaluated both for the image $\hat{g}_0(x,y)$ and the restored image after 150 iterations $\hat{g}_{150}(x,y)$ and called respectively r_0 and r_{150}. Some general observations can be made:

1. in all the examined cases a significant improvement is obtained for r_{150} with respect to r_0;
2. The use of restoration method with 150 iterations is equivalent to use an experimental exploration bandwidth about four times wider.
3. Looking at the shape of the reconstructed images before and after the restoration process, it is always possible to appreciate the noticeable reduction of the spurious sidelobes produced by the restoration algorithm (see Fig. 3).

b. two scatterers case

Also for this case several experimental conditions in the range 1-10 GHz were simulated. On the basis of the obtained results the following considerations can be drawn:

1. In all cases a noticeable improvement in resolution and spurious side lobes amplitude reduction was achieved (see fig. 4);
2. The comparison between the shape of restored and unrestored images evidenced again that the use of the restoration algorithm is equivalent to the use of a wider experimental exploration bandwidth; however it is to note that to obtain now the performance associated to an equivalent band four times wider a number of iterations larger than that relative to the single scatterer case is required (about 600).

Tab. 1 : Simulation results obtained for a single scatterer.

f_m	Δf	r_0	r_{150}
3.6	0.45	131	493
3.6	0.9	240	1250
3.6	1.8	413	1862
3.6	3.6	1261	2050
5.25	0.45	163	396
5.25	0.9	349	813
5.25	1.8	409	1486
5.25	3.6	915	2492

5. Conclusions

The problem of restoring two-dimensional objects distorted by ideal band-pass imaging systems has been investigated with reference to a backscattering based microwave tomographic technique.

The use of constrained iterative algorithms has been considered and the cases in which the object is constituted by one or two pointform scatterers have been analyzed.

The results obtained, although relative to noiseless simulated data, evidence the good performance of the iterative algorithm when the positivity and limited extent constraints are imposed. In particular, the analysis showed that the use of the restoration algorithm is equivalent to broaden the bandwidth of the imaging system.

This result seems to be very interesting because the experimental implementation of the considered tomographic technique is possible without expensive and cumbersome procedure only in the case of maximum relative bandwidth values of about 30-40 %, for which the performance of the imaging system are normally too poor.

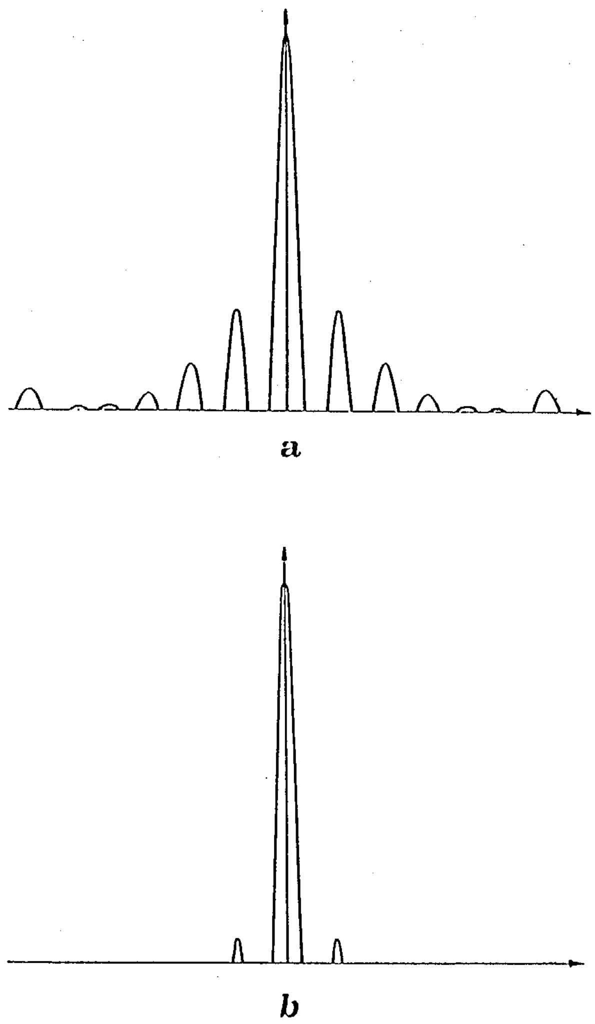

Fig. 3 - Sections of reconstructed images for a single scatterer where f_m = 3.6GHz and f = 0.9GHZ
a : before the restoration
b : after the restoration

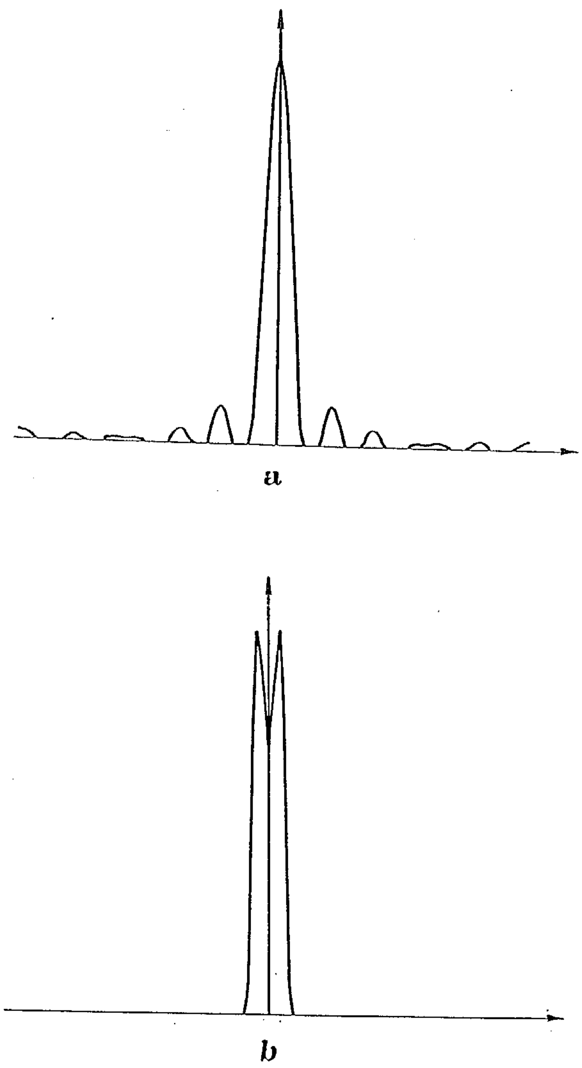

Fig. 4 - Sections of reconstructed images for two scatterers with f_m = 3.6GHz and f = 1.44 Hz
a : before the restoration
b : after the restoration

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