MULTICHANNEL RECURSIVE IN SPACE LEAST-SQUARES LATTICE ALGORITHMS

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Etant données les propriétés statistiques du second ordre, l'algorithme de Levinson, la structure du filtre en treillis et l'algorithme de la factorisation de matrice toêplitz sont bien développés dans la littérature sous forme de versions non normalisées et de versions normalisées. Les algorithmes des filtres en treillis des moindres carrés sont aussi bien développés sous forme de versions non normalisées et de versions normalisées (normalisées en variance) pour le traitement adaptatif des signaux. Au cas multivoie, ces algorithmes sont valables en prenant un vecteur comme un scalaire, mais demandent l'inversion de matrice et la racine carrée de matrice. Ces calculs rendent difficile l'implantation en VLSI.

Basé sur l'équation de Yule-Walker multivoie, ce papier donne la dérivation de la version multivoie scalaire de l'algorithme de Levinson, de la structure en treillis multivoie et de l'algorithme de la factorisation de matrice toêplitz par bloc sous forme de versions non normalisées et de versions normalisées. Aussi, ce papier donne la dérivation de la structure en treillis multivoie et de la version multivoie scalaire de l'algorithme des moindres carrés sous forme de versions non normalisées et de versions normalisées (normalisées en variance) pour le traitement adaptatif des signaux vectoriels. Cette dérivation est basée sur la notion de récursions en espace et utilise l'approche de la projection.

Nons montrons que les filtres en treillis d'une seule voie et les filtres triangulaires sont des cas particuliers des filtres en treillis multivoie. Les résultats obtenus sont à la base du traitement statistique des signaux vectoriels. Des applications sont données pour montrer la performance de ces algorithmes.

Given the second-order statistics, the Levinson algorithm, the lattice filter structure and the algorithm factorizing toeplitz matrix have been well developed in the literature in both the unnormalized and the normalized formes. The least-squares lattice algorithms have also been well developed in both unnormalized and the normalized (variance normalized) formes for adaptive signal processing. In multichannel case, they are valid by considering a vector as a scalar, thus require matrix inversion and square root. These computations make it difficult to implement in VLSI.

Based on the multichannel Yule-Walker equation, this paper gives the derivation of the multichannel scalar Levinson algorithm, the lattice structure and the algorithm factorizing block toeplitz matrix in both the unnormalized and the normalized formes. Also, this paper gives the derivation of the multichannel scalar adaptive least-squares lattice structure and algorithm in both the unnormalized and the normalized (variance normalized) formes. This derivation is based on the notion of space recursions and using the projection framework.

We show that both the conventional one channel lattice filters and the triangular filters are special cases of the multichannel lattice structures. The results obtained are at the base of statistical array signal processing. Applications are given to show the performance of these algorithms.

I. INTRODUCTION

Given the second order statistics of a random process, the Levinson algorithm, the lattice filter structure and the algorithm factorizing toeplitz matrix have been well developed in the litterature in both the unnormalized and the normalized formes [2],[4],[5],[6]. Multichannel linear prediction algorithms have also been developed for spectral estimation [7],[8]. All these algorithms require matrix inversion and matrix square-root for normalized versions.

Least-squares lattice algorithms have also been well developed in both the unnormalized and the normalized (variance normalized) formes for adaptive signal processing [3],[4],[5]. In the multichannel case, they are valid by considering a vector as a scalar, thus require again matrix inversion and matrix square-root. These computations make it difficult to implement in VLSI [9].

Recently, a family of filters called triangular lattice filters are developed [10],[12],[14]. The essential idea of these filters is the spatial channel decorrelation [13]. The algorithms based on this idea are scalar. They are applied in array signal processing [11] for spatial spectrum estimation.

This paper is extracted from the thesis [1]. Based on the multichannel Yule-Walker equation, this paper gives in the section II the derivation of the multichannel scalar Levinson algorithm, the lattice structure and the algorithm factorizing block toeplitz matrix in both the unnormalized and the normalized formes. Here the ward scalar means that all the operations are scalar.

Also, we give in the section III the derivation of the multichannel scalar adaptive least-squares lattice filter structure and algorithm in both the unnormalized and the normalized (variance normalized) formes. This derivation is based on the notion of space recursions and using the projection framework which is adopted in [5],[6].

We show that both the conventional one channel lattice filters and the triangular lattice filters are special cases of the multichannel lattice filters. The results obtained are at the base of the statistical array signal processing. Applications are given to show the performance of these algorithms.

II. Multichannel Scalar Levinson Algorithm, Lattice Structure and Factorization

Before deriving the unnormalized and the normalized versions of the multichannel scalar Levinson algorithm, the lattice structure and the algorithm factorizing block toeplitz matrix, we review the multichannel Yule-Walker equation.

II_1 Multichannel Yule-Walker Equation

Consider the linear prediction of multichannel process $\{y(i)_T, j=1..N\}$. We define by the (A1),(A2) and (A3),(A4) the p'th order predictor ,prediction error and the backward predictor,prediction error. We define the correlation sequence of matrix by (A5). See Fig.II_1. By using the orthogonal property of least-squares linear prediction errors, we obtain the multichannel Yule-Walker equation of (A6) with the block toeplitz matrix R_p given by (A7). We note that the p'th order least-squares forward and backward prediction errors covariance matrices $R_p{}^e$, $R_p{}^r$ are positive definite.

Suppose that we know the p'th order predictors $A_{p,i}$, $B_{p,i}$. By padding zeros to these two predictors in (A6), we obtain the equation (A8) with $\partial_{p+1}{}^e, \partial_{p+1}{}^r$ given by (A9),(A10). The p+1'th order solution of the predictors is obtained by a simple linear combination of the two rows of (A8) which imposes zeros at the places occupied by $\partial_{p+1}{}^e, \partial_{p+1}{}^r$. The algorithm to obtain this solution is called Levinson algorithm. In multichannel case. This algorithm involves matrix inversion.

II.2 Multichannel Scalar Levinson Algorithm, Lattice Structure and Factorization

To do not involve any matrix operations, we must work on the elements of $R_p{}^e, R_p{}^r, \partial_{p+1}{}^e$ and $\partial_{p+1}{}^r$. Now, we try to find the p+1'th order solution of the forward predictor $A_{p+1,i}$.

At the k stage,k=0,...,N-1, we pre-multiply the (A8) by $W_A(k+1)$ of (A11) whose elements are computed by

$$T(i,k+1)_p{}^r(k) = R(i,k+1)_p{}^r(k)\ R(k+1,k+1)_p{}^r(k)^{-1}, \quad i=k+1,...,N \quad (II\_1)$$

$$\begin{bmatrix} y(1)_{T/T-1} \\ \vdots \\ y(N)_{T/T-1} \end{bmatrix} = -\sum_{i=1}^{p} A_{p,i} \begin{bmatrix} y(1)_{T-i} \\ \vdots \\ y(N)_{T-i} \end{bmatrix} \quad (A1) \qquad \begin{bmatrix} y(1)_{T-p-1/T-1} \\ \vdots \\ y(N)_{T-p-1/T-1} \end{bmatrix} = -\sum_{i=1}^{p} B_{p,p+1-i} \begin{bmatrix} y(1)_{T-i} \\ \vdots \\ y(N)_{T-i} \end{bmatrix} \quad (A3)$$

$$\begin{bmatrix} e(1)_{p,T} \\ \vdots \\ e(N)_{p,T} \end{bmatrix} = \begin{bmatrix} y(1)_{T/T-1} \\ \vdots \\ y(N)_{T/T-1} \end{bmatrix} - \begin{bmatrix} y(1)_{T} \\ \vdots \\ y(N)_{T} \end{bmatrix} \quad (A2) \qquad \begin{bmatrix} r(1)_{p,T-1} \\ \vdots \\ r(N)_{p,T-1} \end{bmatrix} = \begin{bmatrix} y(1)_{T-p-1} \\ \vdots \\ y(N)_{T-p-1} \end{bmatrix} - \begin{bmatrix} y(1)_{T-p-1/T-1} \\ \vdots \\ y(N)_{T-p-1/T-1} \end{bmatrix} \quad (A4)$$

$$R(m,n)_{i-j} == E\{ y(m)_i\, y(n)_j{}^T \}, \quad m,n=1,...,N \quad (A5)$$

$$\begin{bmatrix} 1 & A_{p,1} & A_{p,2} & \cdots & A_{p,p} \\ B_{p,p} & \cdots & \cdots & B_{p,1} & 1 \end{bmatrix} R_p = \begin{bmatrix} R_p{}^e & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & R_p{}^r \end{bmatrix} \quad (A6)$$

$$R_p = \begin{bmatrix} R_0 & R_1 & R_2 & \cdots & R_p \\ R_{-1} & R_0 & R_1 & \cdots & R_{p-1} \\ R_{-2} & R_{-1} & R_0 & R_1 \cdots & R_{p-2} \\ \vdots & & & & \vdots \\ \cdots & & & R_0 & R_1 \\ R_{-p} & \cdots & & R_{-1} & R_0 \end{bmatrix} \quad (A7)$$

$$\begin{bmatrix} 1 & A_{p,1} & \cdots & A_{p,p} & 0 \\ 0 & B_{p,p} & \cdots & B_{p,1} & 1 \end{bmatrix} R_{p+1} = \begin{bmatrix} R_p{}^e & 0 & \cdots & 0 & \partial_{p+1}{}^e \\ \partial_{p+1}{}^{rT} & 0 & \cdots & 0 & R_p{}^r \end{bmatrix} \quad (A8)$$

$$\partial_{p+1}{}^e = R_{p+1} + \sum_{i=1}^{p} A_{p,i}\, R_{p-i+1} \quad (A9) \qquad \partial_{p+1}{}^{rT} = \sum_{i=1}^{p} B_{p,i}\, R_{-p-1+i} + R_{-p-1} \quad (A10)$$

$$W_A(k+1) = \left[\begin{array}{c} \begin{matrix} & 0..0 \;\; -K(1,k+1)_{p+1}{}^r(k) \;\; 0...0 \\ I & \cdots \\ & 0...0 \;\; -K(N,k+1)_{p+1}{}^r(k) \;\; 0...0 \end{matrix} \\ \hline \begin{matrix} 1 & & & 0 \\ & 1 & & \\ 0 & & 1 & \\ & -T(k+2,k+1)_p{}^r(k) & & \\ & \vdots & 1 & \\ 0 & -T(N,k+1)_p{}^r(k) & 0 & 1 \end{matrix} \end{array}\right] \quad (A11)$$

$$W_B(k+1) = \left[\begin{array}{c} \begin{matrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ & -T(k+2,k+1)_p{}^e(k) & & 0 \\ & \vdots & 1 & \\ 0 & -T(N,k+1)_p{}^e(k) & 0 & 1 \end{matrix} \\ \hline \begin{matrix} 0..0 \;\; -K(1,k+1)_{p+1}{}^e(k)^t \;\; 0...0 & \\ \cdots & I \\ 0...0 \;\; -K(N,k+1)_{p+1}{}^e(k)^t \;\; 0...0 & \end{matrix} \end{array}\right] \quad (A12)$$

Fig.II_1 Multichannel Yule-Walker Equation and Derivation of the Multichannel Scalar Levinson Algorithm

$$K(i,k+1)_{p+1}{}^r(k) = \partial(i,k+1)_{p+1}{}^e(k)\ R(k+1,k+1)_p{}^r(k)^{-1}, \quad i=1,...,N \quad (II\_2)$$

This operation eliminates the lower elements of the column k of $R_p{}^r(k)$ and the column k of $\partial_{p+1}{}^e(k)$. The detailed multichannel scalar algorithm is given by Fig.II_2. We have $A_{p+1,i} = A_{p,i}(N)$, i=1..p+1 since $\partial_{p+1}{}^e(N) = \mathbf{0}$. Also, we have $R_{p+1}{}^e = R_p{}^e(N)$.

To find the p+1'th order solution of the backward predictor $B_{p+1,i}$, we pre-multiply,at stage k,k=0,...,N-1, the (A8) by $W_B(k+1)$ of (A12) whose elements are given by

$$T(i,k+1)_p{}^e(k) = R(i,k+1)_p{}^e(k)\ R(k+1,k+1)_p{}^e(k)^{-1}, \quad i=k+1,...,N \quad (II\_3)$$

$$K(k+1,i)_{p+1}{}^r(k)^T = \partial(k+1,i)_{p+1}{}^r(k)^T\ R(k+1,k+1)_p{}^e(k)^{-1}, \quad i=1,...,N \quad (II\_4)$$

One can show that we have $\partial_{p+1}{}^{rT}(N) = \mathbf{0}$, $R_{p+1}{}^r = R_p{}^r(N)$ and $B_{p+1,i} = B_{p,i}$, i=1,...,p+1. The Multichannel lattice (filter) predictor is given by Fig.II_3a,b.

Note that both $R_p{}^e$ and $R_p{}^r$ are diagonalized. These diagonalizations are achieved by the triangular parts of the multichannel lattice filter. Thus this filter gives the factorization of the inverse of the block toeplitz matrix of (A7). Note that the same lattice structure also factorizes the block toeplitz matrix and gives a way to compute the parameters of this filter from the correlation sequence of matrix of (A5). Detailed algorithm is not presented here.

II.3 Multichannel Scalar Normalized Levinson Algorithm, Lattice Structure and Factorization

The key for deriving the normalized forme of the multichannel scalar Levinson algorithm is the definition of the normalized variables. For obtaining the p+1'th order solution of the normalized forward predictor,this definition is given by Fig.II_5a. The normalized variables are comprised within -1 and +1. By replacing the variables used in the unnormalized algorithm by this definition, one obtains the normalized version of Fig.II_5b. Note that in doing this, one obtains

$$R(i,i)_p^e(k)^{-1/2} R(i,i)_p^e(k+1)^{1/2} = [\, I - \bar{K}(i,k+1)_{p+1}{}^r(k)\ \bar{K}(i,k+1)_{p+1}{}^r(k)^t\,]^{1/2}, \quad i=1,\ldots,N \quad \text{(II\_5)}$$

$$R(i,i)_p^r(k)^{-1/2} R(i,i)_p^r(k+1)^{1/2} = [\, I - \bar{T}(i,k+1)_p{}^r(k)\ \bar{T}(i,k+1)_p{}^r(k)^t\,]^{1/2}, \quad i=k+2,\ldots,N \quad \text{(II\_6)}$$

These two expressions mean that the variance of prediction errors along each channel of the multichannel unnormalized lattice filter is decreasing.

The structure of the multichannel Normalized lattice (filter)predictor is given by Fig.II_6. This structure gives the normalized versions of the factorization of the inverse of the block toeplitz matrix and of itself. The latter gives a way to compute the parameters of the multichannel normalized lattice filter from the correlation sequence of matrix.

III. Multichannel Recursive in Space Least-squares Lattice Algorithms

To derive the unnormalized and the normalized versions of the multichannel scalar recursive in space least-squares lattice algorithms, we introduce the notion of space recursions.

III.1 Space Recursions

The forward /backward prediction errors for the pre-windowed method can be written as (B1),(B4). The least-squares solutions of the forward/backward predictors are given by (B2),(B5) where P_Y is the projector definied by (B7) and $Y_{p,T}$ is given by (B8). Using the pinning vector π, the two last forward/backward prediction errors can be written as (B3),(B6). See Fig.III_1.

Note that the order update of $Y_{p,T}$ requires the updates of $Y_{p,T}$ by $y(j)_{0:T}{}^{p+1}$, $j=1,\ldots,N$ and the order-and-time update of $Y_{p,T}$ requires that of $Y_{p,T}$ by $y(j)_{0:T}$, $j=1,\ldots,N$. See (B9)-(B12). These updates are called here space recursions. The time update of $Y_{p,T}$ is given by (B13).

The update formulas of inner products of the unnormalizedvariables, and of the normalized (variance normalized) variables are summarized by (B14)-(B16).

$$\begin{bmatrix} e(1)_{p,0} & \cdots & e(1)_{p,T} \\ \vdots & & \vdots \\ e(N)_{p,0} & \cdots & e(N)_{p,T} \end{bmatrix} = \begin{bmatrix} y(1)_{0:T} \\ \vdots \\ y(N)_{0:T} \end{bmatrix} + [\, A_{p,1} \cdots A_{p,p} \,]\, Y_{p,T} \quad \text{(B1)}$$

$$\begin{bmatrix} 0 & r(1)_{p,0} & \cdots & r(1)_{p,T-1} \\ \vdots & \vdots & & \vdots \\ 0 & r(N)_{p,0} & \cdots & r(N)_{p,T-1} \end{bmatrix} = \begin{bmatrix} y(1)_{0:T}{}^{p+1} \\ \vdots \\ y(N)_{0:T}{}^{p+1} \end{bmatrix} + [\, B_{p,p} \cdots B_{p,1} \,]\, Y_{p,T} \quad \text{(B4)}$$

$$[\, A_{p,1} \cdots A_{p,p} \,] = \begin{bmatrix} y(1)_{0:T} \\ \vdots \\ y(N)_{0:T} \end{bmatrix} P_{Y_{p,T}} \quad \text{(B2)} \qquad [\, B_{p,p} \cdots B_{p,1} \,] = \begin{bmatrix} y(1)_{0:T}{}^{p+1} \\ \vdots \\ y(N)_{0:T}{}^{p+1} \end{bmatrix} P_{Y_{p,T}} \quad \text{(B5)}$$

$$\begin{bmatrix} e(1)_{p,T} \\ \vdots \\ e(N)_{p,T} \end{bmatrix} = \begin{bmatrix} y(1)_{0:T} \\ \vdots \\ y(N)_{0:T} \end{bmatrix} (I - P_{Y_{p,T}})\, \pi^T \quad \text{(B3)} \qquad \begin{bmatrix} r(1)_{p,T-1} \\ \vdots \\ r(N)_{p,T-1} \end{bmatrix} = \begin{bmatrix} y(1)_{0:T}{}^{p+1} \\ \vdots \\ y(N)_{0:T}{}^{p+1} \end{bmatrix} (I - P_{Y_{p,T}})\, \pi^T \quad \text{(B6)}$$

$$P_{Y_{p,T}} == Y_{p,T}{}^T (Y_{p,T} Y_{p,T}{}^T)^{-1} Y_{p,T} \quad \text{(B7)}$$

$$Y_{p,T} = \begin{bmatrix} y(1)_0 & \cdots & y(1)_{p-1} & \cdots & y(1)_{T-1} \\ \vdots & & \vdots & & \vdots \\ y(N)_0 & \cdots & y(N)_{p-1} & \cdots & y(N)_{T-1} \\ \vdots & & \vdots & & \vdots \\ & & y(1)_0 & \cdots & y(1)_{T-p} \\ & & \vdots & & \vdots \\ & & y(1)_0 & \cdots & y(N)_{T-p} \end{bmatrix} \quad \text{(B8)}$$

$$Y_{p,T}(k^{p+1}) = Y_{p,T} + \sum_{j=1}^{k} y(j)_{0:T}{}^{p+1} \quad \text{(B9)} \qquad Y_{p,T}(k) = Y_{p,T} + \sum_{j=1}^{k} y(j)_{0:T} \quad \text{(B11)}$$

$$Y_{p+1,T} = Y_{p,T}(N^{p+1}) \quad \text{(B10)} \qquad Y_{p+1,T+1} = Y_{p,T}(N) \quad \text{(B12)}$$

$$Y_{p,T} + \pi = Y_{p,T-1} \quad \text{(B13)}$$

$$V(I-P_{S+x})W^T = V(I-P_S)W^T - V(I-P_S)x^T [\, x(I-P_S)x^T \,]^{-1} x(I-P_S)W^T \quad \text{(B14)}$$

$$\Phi_{S+x}(V,W) = F(\, \Phi_S(V,W),\ \Phi_S(x,W),\ \Phi_S(V,x)\,) \quad \text{(B15)}$$

$$\bar{\Phi}_{S+x}(V,W) = G(\, \bar{\Phi}_S(V,W),\ \bar{\Phi}_S(x,W),\ \Phi_S(V,x)\,) \quad \text{(B16)}$$

$$F(a,b,c) == (I-cc^t)^{-1/2}\,(a-cb)(I-b^tb)^{-T/2} \quad \text{(B17)}$$

$$G(a,b,c) == (I-cc^t)^{-1/2}\,(a-cb) \quad \text{(B18)}$$

Fig.III_1 Definitions and Space Recursions for Variables Updating

III.2 Multichannel Recursive in Space Least-squares Lattice Algorithms

Define by the Fig.III_2 the variables of the stage k, $k=0,\ldots,N-1$ of the multichannel least-squares lattice filter. Note that (k^{p+1}) denotes the variables of the stage k for order update and (k) denotes that of the stage k for order-and-time update.

By applying (B14),(B9) and (B11) to these variables, one obtains the space recursions of Fig.III_3a for order update and the space recursions of Fig.III_3b for order-and-time update. The recursions for time update are summarized in Fig.III_3c. The structure of the multichannel adaptive least-squares lattice filter is given by Fig.III_4a et b.

Define the multichannel lattice predictors by Fig.III_4a. The recursions of these predictors are summarized in Fig.III_4b. The multichannel lattice predictor is given by Fig.III_5c.

III.3 Multichannel Normalized (Variance Normalized) Recursive in Space Least-squares Lattice Algorithms

Define by Fig.III_6 the variables of the multichannel normalized least squares lattice filter. By applying (B15),(B9) and (B11) to these variables, one obtains the space recursions of Fig.III_7a for order update and that of Fig.III_7b for order-and-time update. The recursions for time update are summarized in Fig.III_7c.

Similarly, define by Fig.III_8 the variables of the multichannel variance normalized least-squares lattice filter. By applying (B16), (B9) et (B11) to these variables, one obtains the space recursions of Fig.III_9a for order update and that of Fig.III_9b for order-and-time update. The recursions for time update are summarized in Fig.III_9c. The structure of the multichannel variance normalized adaptive least-squares lattice filter is given by Fig.III_11a et b.

Definie by Fig.III_10a the variance normalized predictors. They are updated by the recursions of Fig.III_10b. The multichannel variance normalized lattice predictor is given by Fig.III_11c. Note that the multichannel variance normalized adaptive least-squares lattice filter and predictor use only the parameters of the lattice filter of the normalized versions.

IV. Conclusions and Remarks

The conventional one channel lattice filters and the triangular lattice filters are special cases of the multichannel lattice filter structure. If the number of channels is reduced to one, the multichannel lattice filters become the one channel lattice filters. If the number of sections is reduced to one, the triangular parts have the same structure of the triangular lattice filters.

The multichannel scalar lattice filters and algorithms are at the base of the statistical array signal processing. We encourage the studies and tests of these scalar algorithms. At the end of this paper, we have no place to give experimental results. Note that detailed descriptions and applications can be found in [1].

References:

[1]Xing LI, chapter 4 and chapter 5 of Doctoral Thesis, 1987

[2]A.H.Gray,Jr. and J.D.Market," A normalized digital lattice structure," IEEE Trans. ASSP, vol.ASSP-23, no.3, pp.268-277, 1975.

[3]D.Lee,M.Morf and B.Friedlander, "Recursive square-root ladder estimation algorithms,"IEEE ASSP,vol.ASSP-29,no.3, pp.627-641, june 1981.

[4]J.Makhoul," Linear prediction: a tutoriel review," Proc. IEEE, vol.63, pp.501-580, 1975.

[5]B.Friedlander," Lattice filters for adaptive signal processing," Proc. IEEE, vol.70, no.8, Aug. 1982.

[6]B.Friedlabder," Lattice methods for spectral estimation," Proc. IEEE, vol.9, sept. 1982.

[7]M.Morf, A.Viera,D.Lee and T.Kailath," Recursive multichannel maximum entropy spectral estimation," IEEE Trans. Geosci. Electron., vol.GE-16, no.2, pp.85-94,Apr. 1978.

[8]S.L.Marple,Jr. and A.H.Nuttall," Experimental comparison of three multichannel linear prediction spectral estimators," IEE Proc.,vol.130, pt. F, no.3, Apr. 1983.

[9]S.Y.Kung," On supercomputing with systolic / wavefront array processors," Proc. IEEE, vol.72, no.7, july 1985.

[10]N.Ahmed and D.H.Youn," On a realization and related algorithms for adaptive prediction," IEEE Trans. ASSP, vol.ASSP-28, no.5, oct. 1980.

[11]J.W.R.Griffiths," Adaptive array processing," IEE Proc. vol.130, pts. F and H, no.1, feb. 1983.

[12]T.S.Durrani and K.C.Sharman," Eigenfilter approaches to adaptive processing," IEE Proc. vol.130, pts. F and H, no.1, feb. 1983.

[13]J.G.McWhirter and T.G.Shepherd," Adaptive algorithms in space and time domain," IEEE Proc., vol. 130, pts. F and H, no.1, feb. 1983.

[14]F.Ling, D.Manolakis and J.G.Proakis," A recursive modified Gram-Schmidt algorithm for least-squares estimation," IEEE Trans. ASSP, vol. ASSP-34, no.4, Aug. 1986.

for k = 0,..., (N-1) do

$$e(i)_p(k+1) = e(i)_p(k) - K(i,k+1)_{p+1}{}^{r}(k)\, r(k+1)_p(k) \qquad i=1,\ldots,N$$
$$r(i)_p(k+1) = r(i)_p(k) - T(i,k+1)_p{}^{r}(k)\, r(k+1)_p(k), \qquad i=k+2,\ldots,N$$

$$A(i)_p(k+1) = A(i)_p(k) - K(i,k+1)_{p+1}{}^{r}(k)\, B(k+1)_p(k), \qquad i=1,\ldots,N$$
$$B(i)_p(k+1) = B(i)_p(k) - T(i,k+1)_p{}^{r}(k)\, B(k+1)_p(k), \qquad i=k+2,\ldots,N$$

$$T(i,k+1)_p{}^{r}(k) == R(i,k+1)_p{}^{r}(k)\, R(k+1,k+1)_p{}^{r}(k)^{-1}, \qquad i=k+2,\ldots,N$$
$$K(i,k+1)_{p+1}{}^{r}(k) == \partial(i,k+1)_{p+1}{}^{e}(k)\, R(k+1,k+1)_p{}^{r}(k)^{-1}, \qquad i=1,\ldots,N$$

$$R(i,j)_p{}^{r}(k+1) = R(i,j)_p{}^{r}(k) - T(i,k+1)_p{}^{r}(k)\, R(j,k+1)_p{}^{r}(k)^{T}, \qquad i,j=k+2,\ldots,N$$
$$\partial(i,j)_{p+1}{}^{e}(k+1) = \partial(i,j)_{p+1}{}^{e}(k) - K(i,k+1)_{p+1}{}^{r}(k)\, R(j,k+1)_p{}^{r}(k)^{T}, \qquad i=1,\ldots,N;\ j=k+2,\ldots,N$$
$$\partial(i,j)_{p+1}{}^{r}(k+1)^{t} = \partial(i,j)_{p+1}{}^{r}(k)^{t} - T(i,k+1)_p{}^{r}(k)\, \partial(j,k+1)_{p+1}{}^{r}(k), \qquad i,j=k+2,\ldots,N$$
$$R(i,j)_p{}^{e}(k+1) = R(i,j)_p{}^{e}(k) - K(i,k+1)_{p+1}{}^{r}(k)\, \partial(j,k+1)_{p+1}{}^{r}(k), \qquad i,j=1,\ldots,N$$

Fig.II_2 Recursions of Variables of the Upper Half of Section p+1 of the Multichannel Lattice Filter

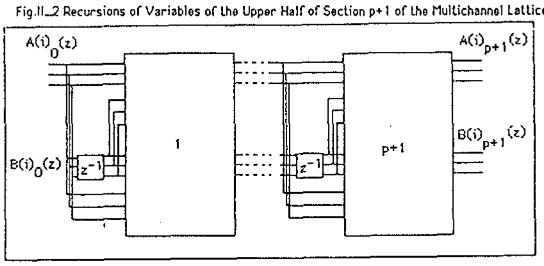

Fig.II_3a Structure of the Multichannel Lattice Predictor

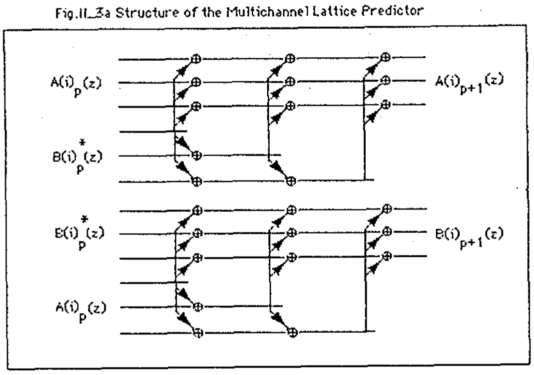

Fig.II_3b Section p+1 of the Multichannel Lattice Predictor

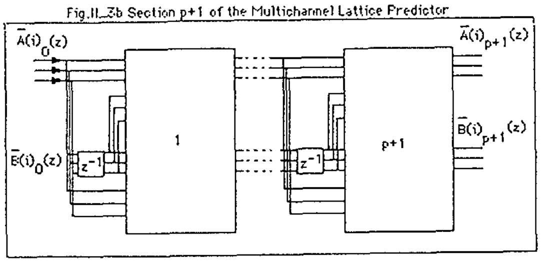

Fig.II_6a Structure of the Multichannel Normalized Lattice Filter

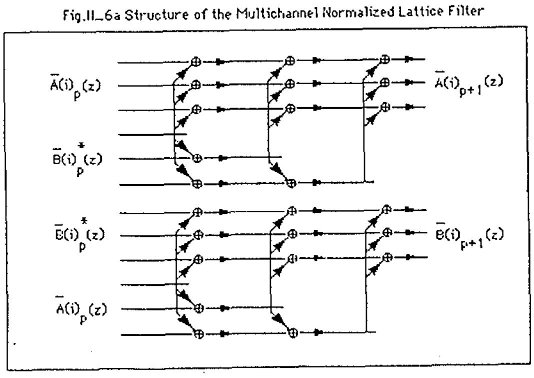

Fig.II_6b Section p+1 of the Multichannel Normalized Lattice Filter

$$\bar{e}(i)_p(k) == R(i,i)_p{}^{e}(k)^{-1/2}\, e(i)_p(k), \qquad i=1,\ldots,N;$$
$$\bar{r}(i)_p(k) == R(i,i)_p{}^{r}(k)^{-1/2}\, r(i)_p(k), \qquad i=k+1,\ldots,N$$
$$\bar{A}(i)_p(k) == R(i,i)_p{}^{e}(k)^{-1/2}\, A(i)_p(k), \qquad i=1,\ldots,N$$
$$\bar{B}(i)_p(k) == R(i,i)_p{}^{r}(k)^{-1/2}\, B(i)_p(k), \qquad i=k+1,\ldots,N$$
$$\bar{R}(i,j)_p{}^{e}(k) == R(i,i)_p{}^{e}(k)^{-1/2}\, R(i,j)_p{}^{e}(k)\, R(j,j)_p{}^{e}(k)^{-1/2}, \qquad i,j=1,\ldots,N$$
$$\bar{R}(i,j)_p{}^{r}(k) == R(i,i)_p{}^{r}(k)^{-1/2}\, R(i,j)_p{}^{r}(k)\, R(j,j)_p{}^{r}(k)^{-1/2}, \qquad i,j=k+1,\ldots,N$$
$$\bar{K}(i,j)_{p+1}{}^{r}(k) == R(i,i)_p{}^{e}(k)^{-1/2}\, \partial(i,j)_{p+1}{}^{e}(k)\, R(j,j)_p{}^{r}(k)^{-1/2}$$
$$== R(i,i)_p{}^{e}(k)^{-1/2}\, K(i,j)_{p+1}{}^{r}(k)\, R(j,j)_p{}^{r}(k)^{1/2}, \qquad i=1,\ldots,N;\ j=k+1,\ldots,N$$
$$\bar{K}(i,j)_{p+1}{}^{r}(k)^{t} == R(i,i)_p{}^{r}(k)^{-1/2}\, \partial(i,j)_{p+1}{}^{r}(k)^{t}\, R(j,j)_p{}^{e}(k)^{-1/2}, \qquad i,j=k+1,\ldots,N$$
$$\bar{T}(i,j)_p{}^{r}(k) == R(i,i)_p{}^{r}(k)^{-1/2}\, R(i,j)_p{}^{r}(k)\, R(j,j)_p{}^{r}(k)^{1/2}$$
$$== \bar{R}(i,j)_p{}^{e}(k), \qquad i,j=k+1,\ldots,N$$

a) Definition of Variables of the Multichannel Normalized Lattice Filter

for k = 0,...,(N-1) do

$$\bar{e}(i)_p(k+1) = G(\bar{e}(i)_p(k),\ \bar{r}(k+1)_p(k),\ \bar{K}(i,k+1)_{p+1}{}^{r}(k)), \qquad i=1,\ldots,N$$
$$\bar{r}(i)_p(k+1) = G(\bar{r}(i)_p(k),\ \bar{r}(k+1)_p(k),\ \bar{T}(i,k+1)_p{}^{r}(k)), \qquad i=k+2,\ldots,N$$
$$\bar{A}(i)_p(k+1) = G(\bar{A}(i)_p(k),\ \bar{B}(k+1)_p(k),\ \bar{K}(i,k+1)_{p+1}{}^{r}(k)), \qquad i=1,\ldots,N$$
$$\bar{B}(i)_p(k+1) = G(\bar{B}(i)_p(k),\ \bar{B}(k+1)_p(k),\ \bar{T}(i,k+1)_p{}^{r}(k)), \qquad i=k+2,\ldots,N$$

$$\bar{R}(i,j)_p{}^{r}(k+1) = G2(\bar{R}(i,j)_p{}^{r}(k),\ \bar{R}(k+1,j)_p{}^{r}(k),\ \bar{T}(i,k+1)_p{}^{r}(k),\ \bar{T}(j,k+1)_p{}^{r}(k)), \qquad i,j=k+2,\ldots,N$$
$$\bar{K}(i,j)_{p+1}{}^{r}(k+1) = G2(\bar{K}(i,j)_{p+1}{}^{r}(k),\ \bar{R}(k+1,j)_p{}^{r}(k),\ \bar{K}(i,k+1)_{p+1}{}^{r}(k),\ \bar{T}(j,k+1)_p{}^{r}(k)), \qquad i=1,\ldots,N;\ j=k+2,\ldots,N$$
$$\bar{K}(i,j)_{p+1}{}^{r}(k+1)^{t} = G2(\bar{K}(i,j)_{p+1}{}^{r}(k)^{t},\ \bar{K}(k+1,j)_{p+1}{}^{r}(k)^{t},\ \bar{T}(i,k+1)_p{}^{r}(k),\ \bar{K}(j,k+1)_{p+1}{}^{r}(k)), \qquad i,j=k+2,\ldots,N$$
$$\bar{R}(i,j)_p{}^{e}(k+1) = G2(\bar{R}(i,j)_p{}^{e}(k),\ \bar{K}(k+1,j)_{p+1}{}^{r}(k)^{t},\ \bar{K}(i,k+1)_{p+1}{}^{r}(k),\ \bar{K}(j,k+1)_{p+1}{}^{r}(k)), \qquad i,j=1,\ldots,N$$

$$G(a,b,c) == (I-cc^{T})^{-1/2}(a-cb)$$
$$G2(a,b,c,d) == (I-cc^{T})^{-1/2}(a-cb)(I-d^{T}d)^{-T/2}$$

b) Recursions of the Normalized Variables of the Multichannel Lattice Filter

Fig.II_5 Definition and Recursions of the Variables of the Upper Half of Normalized Filter

$s = Y_{p,T}(k^{p+1})$				$s = Y_{p,T}(k)$			
V	W	$V(I-P_s)W^T$	range	V	W	$V(I-P_s)W^T$	range
$y(i)_{0:T}$	π	$e(i)_{p,T}(k^{p+1})$	i=1,...,N	$y(i)_{0:T}{}^{p+1}$	π	$r(i)_{p,T-1}(k)$	i=1,..,N
$y(i)_{0:T}$	$y(j)_{0:T}$	$R(i,j)_{p,T}{}^{e}(k^{p+1})$	i,j=1..N	$y(i)_{0:T}{}^{p+1}$	$y(j)_{0:T}{}^{p+1}$	$R(i,j)_{p,T-1}{}^{r}(k)$	i,j=1..N
$y(i)_{0:T}{}^{p+1}$	π	$r(i)_{p,T-1}(k^{p+1})$	i=k+1..N	$y(i)_{0:T}$	π	$e(i)_{p,T}(k)$	i=k+1,...,N
$y(i)_{0:T}{}^{p+1}$	$y(j)_{0:T}{}^{p+1}$	$R(i,j)_{p,T-1}{}^{r}(k^{p+1})$	i,j=k+1..N	$y(i)_{0:T}$	$y(j)_{0:T}$	$R(i,j)_{p,T}{}^{e}(k)$	i,j=k+1..N
$y(i)_{0:T}$	$y(j)_{0:T}{}^{p+1}$	$\partial(i,j)_{p+1,T}(k^{p+1})$	i=1,...,N j=k+1..N	$y(i)_{0:T}$	$y(j)_{0:T}{}^{p+1}$	$\partial(i,j)_{p+1,T}(k)$	i=1,..,N j=k+1,...,N
π	π	$G_{p-1,T-1}{}^{c}(k^{p+1})$		π	π	$G_{p-1,T-1}{}^{c}(k)$	

Fig.III_2. Definition of Variables of the Multichannel Least-squares Lattice Filter

$s = Y_{p,T}(k^{p+1})$, $x = y(k+1)_{0:T}{}^{p+1}$, $x(I-P_s)x^T = R(k+1,k+1)_{p,T-1}{}^{r}(k^{p+1})$, k=0,...,N-1

V	W	$V(I-P_{s+x})W^T$	$V(I-P_s)W^T$	$V(I-P_s)x^T$	$x(I-P_s)W^T$	range
$y(i)_{0:T}$	π	$e(i)_{p,T}(k+1^{p+1})$	$e(i)_{p,T}(k^{p+1})$	$\partial(i,k+1)_{p+1,T}(k^{p+1})$	$r(k+1)_{p,T-1}(k^{p+1})$	i=1,...,N
$y(i)_{0:T}$	$y(j)_{0:T}$	$R(i,j)_{p,T}{}^{e}(k+1^{p+1})$	$R(i,j)_{p,T}{}^{e}(k^{p+1})$	..	$\partial(j,k+1)_{p+1,T}(k^{p+1})$	i,j=1..N
$y(i)_{0:T}{}^{p+1}$	π	$r(i)_{p,T-1}(k+1^{p+1})$	$r(i)_{p,T-1}(k^{p+1})$	$R(i,k+1)_{p,T-1}{}^{r}(k^{p+1})$	$r(k+1)_{p,T-1}(k^{p+1})$	i=k+2..N
$y(i)_{0:T}{}^{p+1}$	$y(j)_{0:T}{}^{p+1}$	$R(i,j)_{p,T-1}{}^{r}(k+1^{p+1})$	$R(i,j)_{p,T-1}{}^{r}(k^{p+1})$	$R(i,k+1)_{p,T-1}{}^{r}(k^{p+1})$	$R(k+1,j)_{p,T-1}(k^{p+1})$	i,j=k+2,...,N
π	π	$G_{p-1,T-1}{}^{c}(k+1^{p+1})$	$G_{p-1,T-1}{}^{c}(k^{p+1})$	$r(k+1)_{p,T-1}(k^{p+1})^T$	$r(k+1)_{p,T-1}(k^{p+1})$	
$y(i)_{0:T}$	$y(j)_{0:T}{}^{p+1}$	$\partial(i,j)_{p+1,T}(k+1^{p+1})$	$\partial(i,j)_{p+1,T}(k^{p+1})$	$\partial(i,k+1)_{p+1,T}(k^{p+1})$	$R(k+1,j)_{p,T-1}{}^{r}(k^{p+1})$	i=1..N;j=k+2..N

definition: $K(i,k+1)_{p+1,T}(k^{p+1}) == \partial(i,k+1)_{p+1,T}(k^{p+1})\, R(k+1,k+1)_{p,T-1}{}^{r}(k^{p+1})^{-1}$, i=1,...,N

$T(i,k+1)_{p,T-1}(k^{p+1}) == R(i,k+1)_{p,T-1}{}^{r}(k^{p+1})\, R(k+1,k+1)_{p,T-1}{}^{r}(k^{p+1})^{-1}$, i=k+1,...,N

a) recursions in space for order update

$s = Y_{p,T}(k)$, $x = y(k+1)_{0:T}$, $x(I-P_s)x^T = R(k+1,k+1)_{p,T}{}^{e}(k)$, $k=0,\dots,N-1$

V	W	$V(I-P_{s+x})W^T$	$V(I-P_s)W^T$	$V(I-P_s)x^T$	$x(I-P_s)W^T$	range
$y(i)_{0:T}{}^{p+1}$	π	$r(i)_{p,T-1}(k+1)$	$r(i)_{p,T-1}(k)$	$\partial(k+1,i)_{p+1,T}(k)^T$	$e(k+1)_{p,T}(k)$	$i=1,\dots,N$
$y(i)_{0:T}{}^{p+1}$	$y(j)_{0:T}{}^{p+1}$	$R(i,j)_{p,T-1}{}^{r}(k+1)$	$R(i,j)_{p,T-1}{}^{r}(k)$	$\partial(k+1,i)_{p+1,T}(k)^T$	$\partial(k+1,j)_{p+1,T}(k)$	$i,j=1,\dots,N$
$y(i)_{0:T}$	π	$e(i)_{p,T}(k+1)$	$e(i)_{p,T}(k)$	$R(i,k+1)_{p,T}{}^{e}(k)$	$e(k+1)_{p,T}(k)$	$i=k+2..N$
$y(i)_{0:T}$	$y(j)_{0:T}$	$R(i,j)_{p,T}{}^{e}(k+1)$	$R(i,j)_{p,T}{}^{e}(k)$	$R(i,k+1)_{p,T}{}^{e}(k)$	$R(k+1,j)_{p,T}{}^{e}(k)$	$i,j=k+2..N$
π	π	$G_{p-1,T-1}{}^{c}(k+1)$	$G_{p-1,T-1}{}^{c}(k)$	$e(k+1)_{p,T}(k)^T$	$e(k+1)_{p,T}(k)$	
$y(i)_{0:T}$	$y(j)_{0:T}{}^{p+1}$	$\partial(i,j)_{p+1,T}(k+1)$	$\partial(i,j)_{p+1,T}(k)$	$R(i,k+1)_{p,T}{}^{e}(k)$	$\partial(k+1,j)_{p+1,T}(k)$	$j=1..N;i=k+2..N$

definition: $K(k+1,i)_{p+1,T}(k)^T == \partial(k+1,i)_{p+1,T}(k)^T\, R(k+1,k+1)_{p,T}{}^{e}(k)^{-1}$, $i=1\dots N$

$T(i,k+1)_{p,T}(k) == R(i,k+1)_{p,T}{}^{e}(k)\, R(k+1,k+1)_{p,T}{}^{e}(k)^{-1}$, $i=k+2,\dots,N$

b) recursions in space for order and time update

$s = Y_{p,T}$, $x = \pi$, $x(I-P_s)x^T = G_{p-1,T-1}{}^{c}$.

V	W	$V(I-P_{s+x})W^T$	$V(I-P_s)W^T$	$V(I-P_s)x^T$	$x(I-P_s)W^T$	range
$y(i)_{0:T}$	$y(j)_{0:T}{}^{p+1}$	$\partial(i,j)_{p+1,T-1}$	$\partial(i,j)_{p+1,T}$	$e(i)_{p,T}$	$r(j)_{p,T-1}$	$i,j=1,\dots,N$
$y(i)_{0:T}{}^{p+1}$	π	$r(i)_{p,T-2}$	$r(i)_{p,T-1}$	$r(i)_{p,T-1}$	$G_{p-1,T-1}{}^{c}$	$i=1,\dots,N$

c) time update

Fig.III_3 Recursions of Variables of the Multichannel Least-squares Lattice Filter

Filter Variables	Predictor	range	Filter Variables	Predictor	range
$e(i)_{p,T}(k^{p+1})$	$A(i)_{p,T}(k^{p+1})(z)$	$i=1,\dots,N$	$r(i)_{p,T-1}(k)$	$B(i)_{p,T-1}(k)(z)$	$i=1,\dots,N$
$r(i)_{p,T-1}(k^{p+1})$	$B(i)_{p,T-1}(k^{p+1})(z)$	$i=k+1..N$	$e(i)_{p,T}(k)$	$A(i)_{p,T}(k)(z)$	$i=k+1..N$
$G_{p-1,T-1}{}^{c}(k^{p+1})$	$C_{p,T-1}(k^{p+1})(z)$		$G_{p-1,T-1}{}^{c}(k)$	$C_{p,T-1}(k)(z)$	

a) Definition of the Multichannel Lattice Predictors

$B(i)_{p,T-1}(z) = B(i)_{p,T}(z) - r(i)_{p,T}\, G_{p-1,T}{}^{-c}\, C_{p,T}(z)$; $i=1,\dots,N$

for $k=0,\dots,(N-1)$ do

$A(i)_{p,T}(k+1^{p+1})(z) = A(i)_{p,T}(k^{p+1})(z) - K(i,k+1)_{p+1,T}(k^{p+1})\, B(k+1)_{p,T-1}(k^{p+1})$; $i=1,\dots,N$

$B(i)_{p,T-1}(k+1^{p+1})(z) = B(i)_{p,T-1}(k^{p+1})(z) - T(i,k+1)_{p,T-1}{}^{r}(k^{p+1})\, B(k+1)_{p,T-1}(k^{p+1})(z)$; $i=k+2,\dots,N$

$C_{p,T}(k+1^{p+1})(z) = C_{p,T}(k^{p+1})(z) - r(k+1)_{p,T-1}(k^{p+1})\, R(k+1,k+1)_{p,T-1}{}^{r}(k^{p+1})^{-1}\, B(k+1)_{p,T-1}(k^{p+1})(z)$

$B(i)_{p,T-1}(k+1)(z) = B(i)_{p,T-1}(k)(z) - K(k+1,i)_{p+1,T}(k)^T\, A(k+1)_{p,T}(k)(z)$; $i=1,\dots,N$

$A(i)_{p,T}(k+1)(z) = A(i)_{p,T}(k)(z) - T(i,k+1)_{p,T}{}^{e}(k)\, A(k+1)_{p,T}(k)(z)$; $i=k+2,\dots,N$

$C_{p,T}(k+1)(z) = C_{p,T}(k)(z) - e(k+1)_{p,T}(k)\, R(k+1,k+1)_{p,T}{}^{e}(k)^{-1}\, A(k+1)_{p,T}(k)(z)$

b) Recursions of the Multichannel Lattice Predictors

Fig.III_4 Definition and Recursions of the Multichannel Lattice Predictors

$s = Y_{p,T}(k)$, $x = y(k+1)_{0:T}$, $k=0,\dots,N-1$

V	W	$\Phi_{s+x}(V,W)$	$\Phi_s(V,W)$	$\Phi_s(V,x)$	$\Phi_s(x,W)$	range
$y(i)_{0:T}{}^{p+1}$	π	$\bar{r}(i)_{p,T-1}(k+1)$	$\bar{r}(i)_{p,T-1}(k)$	$\bar{K}(k+1,i)_{p+1,T}(k)^T$	$\bar{e}(k+1)_{p,T}(k)$	$i=1,\dots,N$
$y(i)_{0:T}{}^{p+1}$	$y(j)_{0:T}{}^{p+1}$	$\bar{R}(i,j)_{p,T-1}{}^{r}(k+1)$	$\bar{R}(i,j)_{p,T-1}{}^{r}(k)$	$\bar{K}(k+1,i)_{p+1,T}(k)^T$	$\bar{K}(k+1,j)_{p+1,T}(k)$	$i,j=1..N$
$y(i)_{0:T}$	π	$\bar{e}(i)_{p,T}(k+1)$	$\bar{e}(i)_{p,T}(k)$	$\bar{R}(i,k+1)_{p,T}{}^{e}(k)$	$\bar{e}(k+1)_{p,T}(k)$	$i=k+2,\dots,N$
$y(i)_{0:T}$	$y(j)_{0:T}$	$\bar{R}(i,j)_{p,T}{}^{e}(k+1)$	$\bar{R}(i,j)_{p,T}{}^{e}(k)$	$\bar{R}(i,k+1)_{p,T}{}^{e}(k)$	$\bar{R}(k+1,j)_{p,T}{}^{e}(k)$	$i,j=k+2..N$
$y(i)_{0:T}$	$y(j)_{0:T}{}^{p+1}$	$\bar{K}(i,j)_{p+1,T}(k+1)$	$\bar{K}(i,j)_{p+1,T}(k)$	$\bar{R}(i,k+1)_{p,T}{}^{e}(k)$	$\bar{K}(k+1,j)_{p+1,T}(k)$	$i=k+2..N$ $j=1,\dots,N$
π	π	$\bar{G}_{p-1,T-1}{}^{c}(k+1)$	$\bar{G}_{p-1,T-1}{}^{c}(k)$	$\bar{e}(k+1)_{p,T}(k)^T$	$\bar{e}(k+1)_{p,T}(k)$	

b) Recursions in space for order and time update

$s = Y_{p,T}$, $x = \pi$.

V	W	$\Phi_{s+x}(V,W)$	$\Phi_s(V,W)$	$\Phi_s(V,x)$	$\Phi_s(x,W)$	range
$y(i)_{0:T}$	$y(j)_{0:T}{}^{p+1}$	$\bar{K}(i,j)_{p+1,T-1}$	$\bar{K}(i,j)_{p+1,T}$	$\bar{e}(i)_{p,T}$	$\bar{r}(j)_{p,T-1}$	$i,j=1,\dots,N$
$y(i)_{0:T}{}^{p+1}$	π	$\bar{r}(i)_{p,T-2}$	$\bar{r}(i)_{p,T-1}$	$\bar{r}(i)_{p,T-1}$	$\bar{G}_{p-1,T-1}{}^{c}$	$i,j=1,\dots,N$

c) time update

Fig.III_7 Recursions of Variables of the Multichannel Normalized least-squares Lattice Filter

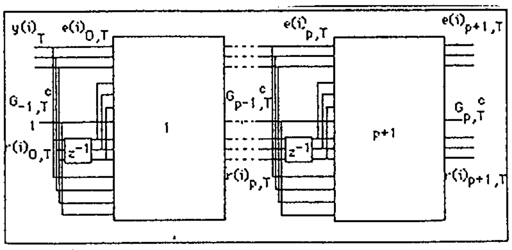

Fig.III_5a Multichannel Least-squares Lattice Filter Structure

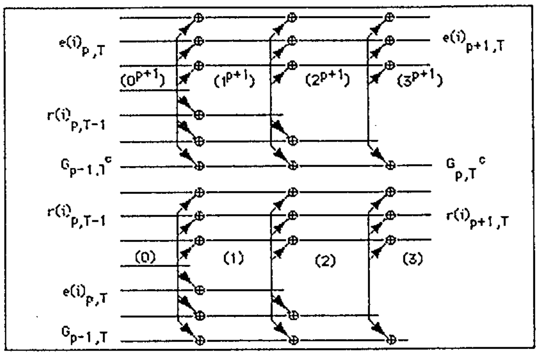

Fig.III_5b Section p+1 of the Fig.III_5a

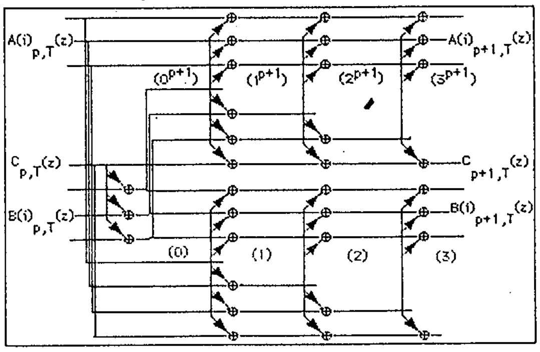

Fig.III_5c Section p+1 of the Multichannel Lattice Predictor

$s = Y_{p,T}(k^{p+1})$				$s = Y_{p,T}(k)$			
V	W	$\Phi_s(V,W)$	range	V	W	$\Phi_s(V,W)$	range
$y(i)_{0:T}$	π	$\bar{e}(i)_{p,T}(k^{p+1})$	$i=1,\dots,N$	$y(i)_{0:T}{}^{p+1}$	π	$\bar{r}(i)_{p,T-1}(k)$	$i=1,\dots,N$
$y(i)_{0:T}$	$y(j)_{0,T}$	$\bar{R}(i,j)_{p,T}{}^{e}(k^{p+1})$	$i,j=1,\dots,N$	$y(i)_{0:T}{}^{p+1}$	$y(j)_{p,T}{}^{p+1}$	$\bar{R}(i,j)_{p,T-1}{}^{r}(k)$	$i,j=k+1..N$
$y(i)_{0:T}{}^{p+1}$	π	$\bar{r}(i)_{p,T-1}(k^{p+1})$	$i=k+1,\dots,N$	$y(i)_{0:T}$	π	$\bar{e}(i)_{p,T}(k)$	$i=k+1,\dots,N$
$y(i)_{0:T}{}^{p+1}$	$y(j)_{0,T}{}^{p+1}$	$\bar{R}(i,j)_{p,T-1}(k^{p+1})$	$i,j=k+1..N$	$y(i)_{0:T}$	$y(i)_{0:T}$	$\bar{R}(i,j)_{p,T}{}^{e}(k)$	$i,j=k+1,\dots,N$
$y(i)_{0:T}$	$y(j)_{0,T}{}^{p+1}$	$\bar{K}(i,j)_{p+1,T}(k^{p+1})$	$i=1,\dots,N$ $j=k+1,\dots,N$	$y(i)_{0:T}$	$y(j)_{0:T}{}^{p+1}$	$\bar{K}(i,j)_{p+1,T}(k)$	$i=k+1,\dots,N$ $j=1,\dots,N$
π	π	$\bar{G}_{p-1,T-1}{}^{c}(k^{p+1})$		π	π	$\bar{G}_{p-1,T-1}{}^{c}(k)$	

Fig.III_6. Definition of Variables of the Multichannel Normalized Least-squares Lattice Filter

$s = Y_{p,T}(k^{p+1})$, $x = y(k+1)_{0:T}{}^{p+1}$, $k=0,\dots,N-1$

V	W	$\Phi_{s+x}(V,W)$	$\Phi_s(V,W)$	$\Phi_s(V,x)$	$\Phi_s(x,W)$	range
$y(i)_{0:T}$	π	$\bar{e}(i)_{p,T}(k+1^{p+1})$	$\bar{e}(i)_{p,T}(k^{p+1})$	$\bar{K}(i,k+1)_{p+1,T}(k^{p+1})$	$\bar{r}(k+1)_{p,T-1}(k^{p+1})$	$i=1..N$
$y(i)_{0:T}$	$y(j)_{0:T}$	$\bar{R}(i,j)_{p,T}(k+1^{p+1})$	$\bar{R}(i,j)_{p,T}(k^{p+1})$	$\bar{K}(i,k+1)_{p+1,T}(k^{p+1})$	$\bar{K}(j,k+1)_{p+1,T}(k^{p+1})^T$	$i,j=1..N$
$y(i)_{0:T}{}^{p+1}$	π	$\bar{r}(i)_{p,T-1}(k+1^{p+1})$	$\bar{r}(i)_{p,T-1}(k^{p+1})$	$\bar{R}(i,k+1)_{p,T-1}{}^{r}(k^{p+1})$	$\bar{r}(k+1)_{p,T-1}(k^{p+1})$	$i=k+2..N$
$y(i)_{0:T}{}^{p+1}$	$y(j)_{0:T}{}^{p+1}$	$\bar{R}(i,j)_{p,T-1}{}^{r}(k+1^{p+1})$	$\bar{R}(i,j)_{p,T-1}{}^{r}(k^{p+1})$	$\bar{R}(i,k+1)_{p,T-1}{}^{r}(k^{p+1})$	$\bar{R}(k+1,j)_{p,T-1}(k^{p+1})$	$i,j=k+2,\dots,N$
$y(i)_{0:T}$	$y(j)_{0:T}{}^{p+1}$	$\bar{K}(i,j)_{p+1,T}(k+1^{p+1})$	$\bar{K}(i,j)_{p+1,T}(k^{p+1})$	$\bar{K}(i,k+1)_{p+1,T}(k^{p+1})^T$	$\bar{R}(k+1,j)_{p,T-1}{}^{r}(k^{p+1})$	$i=1,\dots,N;j=k+2..N$
π	π	$\bar{G}_{p-1,T-1}{}^{c}(k+1^{p+1})$	$\bar{G}_{p-1,T-1}{}^{c}(k^{p+1})$	$\bar{r}(k+1)_{p,T-1}(k^{p+1})^T$	$\bar{r}(k+1)_{p,T-1}(k^{p+1})$	

b ←

a) Recursions in space for order update

$s = Y_{p,T}(k^{p+1})$				$s = Y_{p,T}(k)$			
V	W	$\bar{\Phi}_s(V,W)$	range	V	W	$\bar{\Phi}_s(V,W)$	range
$y(i)_{0:T}$	π	$\bar{e}(i)_{p,T}(k^{p+1})$	i=1..N	$y(i)_{0:T}^{p+1}$	π	$\bar{r}(i)_{p,T-1}(k)$	i=1..N
$y(i)_{0:T}$	$y(j)_{0:T}$	$\bar{R}(i,j)_{p,T}^{e}(k^{p+1})$	i,j=1..N	$y(i)_{0:T}^{p+1}$	$y(j)_{0:T}^{p+1}$	$\bar{R}(i,j)_{p,T-1}^{r}(k)$	i,j=1..N
$y(i)_{0:T}^{p+1}$	π	$\bar{r}(i)_{p,T-1}(k^{p+1})$	i=k+1..N	$y(i)_{0:T}$	π	$\bar{e}(i)_{p,T}(k)$	i=k+1,...,N
$y(i)_{0:T}^{p+1}$	$y(j)_{0:T}^{p+1}$	$\bar{R}(i,j)_{p,T-1}^{r}(k^{p+1})$	i,j=k+1..N	$y(i)_{0:T}$	$y(j)_{0:T}$	$\bar{R}(i,j)_{p,T}^{e}(k)$	i,j=k+1..N
$y(i)_{0:T}$	$y(j)_{0:T}^{p+1}$	$\bar{K}(i,j)_{p+1,T}(k^{p+1})$	i=1,...,N j=k+1..N	$y(i)_{0:T}$	$y(j)_{0:T}^{p+1}$	$\bar{K}(i,j)_{p+1,T}(k)$	j=1,...,N i=k+1..N
π	π	$\bar{G}_{p-1,T-1}^{c}(k^{p+1})$		π	π	$\bar{G}_{p-1,T-1}^{c}(k)$	

Fig.III_8 Definition of Variables of the Multichannel Variance Normalized Least-squares Lattice Filter

$s = Y_{p,T}(k^{p+1})$, $x = y(k+1)_{0:T}^{p+1}$, $k=0,\dots,N-1$

V	W	$\bar{\Phi}_{s+x}(V,W)$	$\bar{\Phi}_s(V,W)$	$\Phi_s(V,x)$	$\bar{\Phi}_s(x,W)$	range
$y(i)_{0:T}$	π	$\bar{e}(i)_{p,T}(k+1^{p+1})$	$\bar{e}(i)_{p,T}(k^{p+1})$	$\bar{K}(i,k+1)_{p+1,T}(k^{p+1})$	$\bar{r}(k+1)_{p,T-1}(k^{p+1})$	i=1..N
$y(i)_{0:T}$	$y(j)_{0:T}$	$\bar{R}(i,j)_{p,T}^{e}(k+1^{p+1})$	$\bar{R}(i,j)_{p,T}^{e}(k^{p+1})$	$\bar{K}(i,k+1)_{p+1,T}(k^{p+1})$	$\bar{K}(j,k+1)_{p+1,T}(k^{p+1})^{T}$	i,j=1,...,N
$y(i)_{0:T}^{p+1}$	π	$\bar{r}(i)_{p,T-1}(k+1^{p+1})$	$\bar{r}(i)_{p,T-1}(k^{p+1})$	$\bar{R}(i,k+1)_{p,T-1}^{r}(k^{p+1})$	$\bar{r}(k+1)_{p,T-1}(k^{p+1})$	i=k+2,...,N
$y(i)_{0:T}^{p+1}$	$y(j)_{0:T}^{p+1}$	$\bar{R}(i,j)_{p,T-1}^{r}(k+1^{p+1})$	$\bar{R}(i,j)_{p,T-1}^{r}(k^{p+1})$	$\bar{R}(i,k+1)_{p,T-1}^{r}(k^{p+1})$	$\bar{R}(k+1,j)_{p,T-1}^{r}(k^{p+1})$	i,j=k+2..N
$y(i)_{0:T}$	$y(j)_{0:T}^{p+1}$	$\bar{K}(i,j)_{p+1,T}(k+1^{p+1})$	$\bar{K}(i,j)_{p+1,T}(k^{p+1})$	$\bar{K}(i,k+1)_{p+1,T}(k^{p+1})$	$\bar{R}(k+1,j)_{p,T-1}^{r}(k^{p+1})$	i=1,...,N; j=k+2..N
π	π	$\bar{G}_{p-1,T-1}^{c}(k+1^{p+1})$	$\bar{G}_{p-1,T-1}^{c}(k^{p+1})$	$\bar{r}(k+1)_{p,T-1}(k^{p+1})^{T}$	$\bar{r}(k+1)_{p,T-1}(k^{p+1})$	

a

$s = Y_{p,T}(k)$, $x = y(k+1)_{0:T}^{p+1}$, $k=0,\dots,N-1$

V	W	$\bar{\Phi}_{s+x}(V,W)$	$\bar{\Phi}_s(V,W)$	$\Phi_s(V,x)$	$\bar{\Phi}_s(x,W)$	range
$y(i)_{0:T}^{p+1}$	π	$\bar{r}(i)_{p,T-1}(k+1)$	$\bar{r}(i)_{p,T-1}(k)$	$\bar{K}(k+1,i)_{p+1,T}(k)^{T}$	$\bar{e}(k+1)_{p,T}(k)$	i=1,...,N
$y(i)_{0:T}^{p+1}$	$y(j)_{0:T}^{p+1}$	$\bar{R}(i,j)_{p,T-1}^{e}(k+1)$	$\bar{R}(i,j)_{p,T-1}^{e}(k)$	$\bar{K}(k+1,i)_{p+1,T}(k)^{T}$	$\bar{K}(k+1,j)_{p+1,T}(k)$	i,j=1..N
$y(i)_{0:T}$	π	$\bar{e}(i)_{p,T}(k+1)$	$\bar{e}(i)_{p,T}(k)$	$\bar{R}(i,k+1)_{p,T}^{e}(k)$	$\bar{e}(k+1)_{p,T}(k)$	i=k+2,...,N
$y(i)_{0:T}$	$y(j)_{0:T}$	$\bar{R}(i,j)_{p,T}^{e}(k+1)$	$\bar{R}(i,j)_{p,T}^{e}(k)$	$\bar{R}(i,k+1)_{p,T}^{e}(k)$	$\bar{R}(k+1,j)_{p,T}^{e}(k)$	i,j=k+2..N
$y(i)_{0:T}$	$y(j)_{0:T}^{p+1}$	$\bar{K}(i,j)_{p+1,T}(k+1)$	$\bar{K}(i,j)_{p+1,T}(k)$	$\bar{R}(i,k+1)_{p,T}^{e}(k)$	$\bar{K}(k+1,j)_{p+1,T}(k)$	i=k+2,...,N j=1,...,N
π	π	$\bar{G}_{p-1,T-1}^{c}(k+1)$	$\bar{G}_{p-1,T-1}^{c}(k)$	$\bar{e}(k+1)_{p,T}(k)^{T}$	$\bar{e}(k+1)_{p,T}(k)$	

b) Recursions in space for order and time update

$s = Y_{p,T}$, $x = \pi$

V	W	$\bar{\Phi}_{s+x}(V,W)$	$\bar{\Phi}_s(V,W)$	$\Phi_s(V,x)$	$\bar{\Phi}_s(x,W)$	range
$y(i)_{0:T}$	$y(j)_{0:T}^{p+1}$	$\bar{K}(i,j)_{p+1,T-1}$	$\bar{K}(i,j)_{p+1,T}$	$\bar{e}(i)_{p,T}$	$\bar{r}(j)_{p,T-1}$	i,j=1..N
$y(i)_{0:T}^{p+1}$	π	$\bar{r}(i)_{p,T-2}$	$\bar{r}(i)_{p,T-1}$	$\bar{r}(i)_{p,T-1}$	$\bar{G}_{p-1,T-1}^{c}$	i=1,...,N

c) time update

Fig.III_8 Recursions of the Variables of the Multichannel Variance Normalized Lattice Filter

Lattice Filter Variables	Predictors	range	Lattice Filter Variables	Predictors	range
$\bar{e}(i)_{p,T}(k^{p+1})$	$\bar{A}(i)_{p,T}(k^{p+1})(z)$	i=1..N	$\bar{r}(i)_{p,T-1}(k)$	$\bar{B}(i)_{p,T-1}(k)(z)$	i=1..N
$\bar{r}(i)_{p,T-1}(k^{p+1})$	$\bar{B}(i)_{p,T-1}(k^{p+1})(z)$	i=k+1..N	$\bar{e}(i)_{p,T}(k)$	$\bar{A}(i)_{p,T}(k)(z)$	i=k+1,...,N
$\bar{G}_{p-1,T-1}^{c}(k^{p+1})$	$\bar{C}_{p,T-1}(k^{p+1})(z)$		$\bar{G}_{p-1,T-1}^{c}(k)$	$\bar{C}_{p,T-1}(k)(z)$	

a) Definition of the Multichannel Variance Normalized Predictors

$\bar{B}(i)_{p,T-1}(z) = G(\bar{B}(i)_{p,T}(z), \bar{C}_{p,T}(z), \bar{r}(i)_{p,T})$, $\quad i=1,\dots,N$

For k=0,....,(N-1) do

$\bar{A}(i)_{p,T}(k+1^{p+1})(z) = G(\bar{A}(i)_{p,T}(k^{p+1})(z), \bar{B}(k+1)_{p,T-1}(k^{p+1})(z), \bar{K}(i,k+1)_{p+1,T}(k^{p+1}))$, $\quad i=1,\dots,N$

$\bar{B}(i)_{p,T-1}(k+1^{p+1})(z) = G(\bar{B}(i)_{p,T-1}(k^{p+1})(z), \bar{B}(k+1)_{p,T-1}(k^{p+1})(z), \bar{R}(i,k+1)_{p,T-1}^{r}(k^{p+1}))$, $\quad i=k+2..N$

$\bar{C}_{p,T}(k+1^{p+1})(z) = G(\bar{C}_{p,T}(k^{p+1})(z), \bar{B}(k+1)_{p,T-1}(k^{p+1})(z), \bar{r}(k+1)_{p,T-1}(k^{p+1}))$,

$\bar{B}(i)_{p,T-1}(k+1)(z) = G(\bar{B}(i)_{p,T-1}(k)(z), \bar{A}(k+1)_{p,T}(k^{p+1})(z), \bar{K}(k+1,i)_{p+1,T}(k)^{T})$, $\quad i=1,\dots,N$

$\bar{A}(i)_{p,T}(k+1)(z) = G(\bar{A}(i)_{p,T}(k)(z), \bar{A}(k+1)_{p,T}(k)(z), \bar{R}(i,k+1)_{p,T}^{e}(k))$ $\quad i=k+2,\dots,N$

$\bar{C}_{p,T}(k+1)(z) = G(\bar{C}_{p,T}(k)(z), \bar{A}(k+1)_{p,T}(k)(z), \bar{e}(k+1)_{p,T}(k)^{T})$,

b) Recursions of the Multichannel Variance Normalized Predictors

Fig.III_10 Definitions and Recursions of the Multichannel Variance Normalized Lattice Predictor

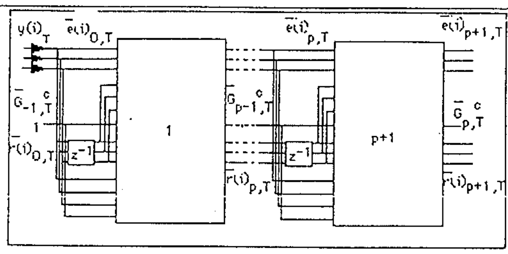

Fig.III_11a Structure of the Multichannel Variance Normalized Lattice Filter

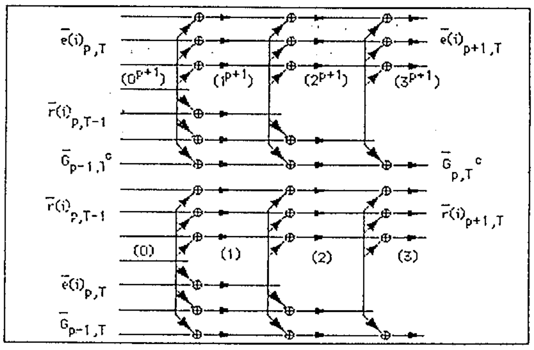

Fig.III_11b Section p+1 of the Multichannel Variance Normalized Lattice Filter

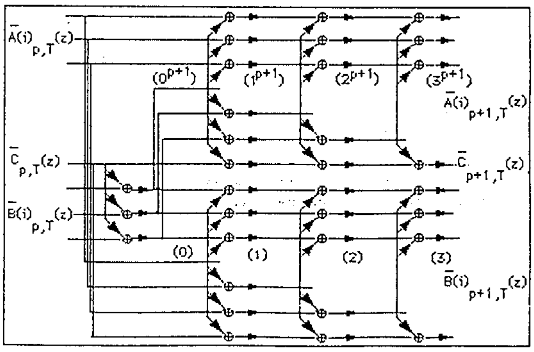

Fig.III_11c Section p+1 of the Multichannel Variance Normalized Predictor

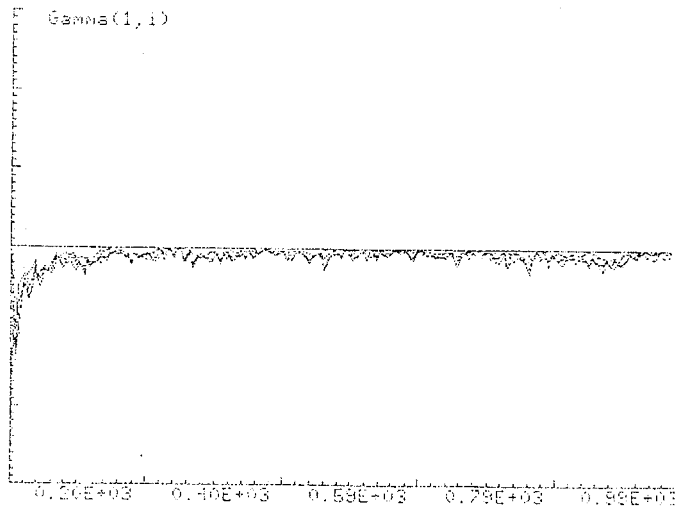

Fig.IV-6 Trace of the Likelihood Variable of a Channel of Different Sections