ESTIMATION OF SOURCE LOCATION AND SPECTRAL PARAMETERS WITH ARRAYS SUBJECT TO COHERENT INTERFERENCE

H. Messer, Faculty of Engineering, Tel-Aviv Univ., Tel-Aviv 69778, Israel,
Y. Rockah, P.O. Box 2250, Haifa 31021, Israel
P.M. Schultheiss, Dept. of Electrical Eng., Yale Univ., P.O. Box 2157, New Haven, CT 06520, USA

RESUME

Cet article traite le probleme de l'utilisation d'un réseau de M senseurs. Le réseau de senseurs, dont la geometric est arbitraire, est utilisé pour localiser la source et estimer les paramètres du spectre où le champ du bruit est dominé par une interférence spatialement coherente.
Les èxpressions sont obtenues pour la matrice Fisher d'information de la source spatiale et des paramètres estimés du spectre et pour son inverse, la limite Cramèr-Rao sur les érreurs d'estimation. La source, l'interférence et le bruit de fond sont présumés être un processus de hasard gaussien dont la moyenne est nulle, et indépendants statistiquement les uns des autres.
Les entrées de la matrice Fisher d'information sont exprimées en fonction des quantites physiques suivantes: la forme du rayon conventionel (the conventional beam-pattern) et ses derivées spatialles, la forme nulle (a null-pattern), le nombre des senseurs M et les rapports de signal/bruit de la source et de l'interférence. En conséquence certains résultats sont obtenus concernant la correlation entre l'estimation des parametres de la source spatiale et l'estimation des parametres du spectre, et la manière dont la présence de l'interférence influence la précision de l'estimation de la localisation de la source, avec des paramètres de source spectrale connus ou inconnus.

SUMMARY

This paper deals with the problem of using an M sensor array of arbitrary geometry for source location and spectral parameters estimation, when the noise field is dominated by a spatially coherent interference.
Formal expressions are obtained for the Fisher information matrix (FIM) of the source spatial and spectral estimated parameters and for its inverse, the Cramer-Rao lower bound (CRLB) on the estimation errors. Source, interference and background noise are assumed to be zero mean Gaussian random process statistically independent of each other.
The FIM entries are expressed in terms of physical quantities: the conventional array beam-pattern and its spatial derivatives, a null pattern, the number of sensors M and the source and interference signal-to-noise ratios. Results are obtained concerning the correlation between source spatial and spectral parameters estimates and the effect of interference on the accuracy of the source location estimation. Both known or unknown source spectral parameters are considered.

1. INTRODUCTION

The use of sensor array to estimate source location and spectral parameters has been studied extensively during the last years [1]. If an array of M sensors in arbitrary geometry is used and the received signal at each sensor consists of a random signal plus uncorrelated noise, the data covariance matrix at each frequency, ω, is given by

$$K_x(\omega)=K_s(\omega)+K_n(\omega)T \tag{1}$$

Under the assumption that the observation time T is large compared with the correlation time of signal and noise (TW>>1), Fourier coefficients associated with different frequencies are uncorrelated. For a point source (spatially coherent), the signal covariance matrix is

$$K_s(\omega)=S(\omega)\underline{u}(\omega)\underline{u}^*(\omega) \tag{2}$$

where $S(\omega)$ is the power spectrum of the transmitted source and $\underline{u}(\omega)$ is a steering vector

$$\underline{u}(\omega)=[1,\exp(j\omega\tau_2),\ldots\ldots,\exp(j\omega\tau_M)]^T \tag{3}$$

τ_m, m=2,...,M are the differential delays between the m-th and the first (reference) sensors and they are function of the array-source geometry. Very complete results are available for the case where the source radiates a zero mean Gaussian signal and the observational noise is uncorrelated from sensor to sensor, i.e., K_n in (1) is a diagonal MxM matrix. In this case, the estimation problem has the key feature that the asymptotic estimation errors for source location parameters (bearing and range) are uncorrelated with estimation errors for parameters describing signal and noise spectra. This can be seen by studying the Fisher information matrix (FIM), which is the inverse of the Cramer-Rao lower bound (CRLB) on the estimation errors of the unknown parameters:

Assuming that $K_n(\omega)$ in (1) is known, the vector of unknown parameters, $\underline{\theta}$, can be decomposed into $\underline{\theta}=(\tau,p)^T$, where τ is the vector of differential delays, $\underline{\tau}=(\tau_2,\tau_3,\ldots\ldots,\tau_M)^T$, contains all spatial unknown parameters and p is a vector of the spectral parameters of the source (any parametization of $S(\omega)$). The FIM of $\underline{\theta}$, F_θ can be partitioned as

$$F_\theta = \begin{bmatrix} F_\tau & F_{\tau p} \\ F_{p\tau} & F_p \end{bmatrix} \tag{4}$$

For the case where $K_n(\omega)$ is a diagonal matrix, it can be shown that $F_{\tau p}=F_{p\tau}{}^T=0$ so the $CRLB(\tau)=F_\tau^{-1}$ and the estimation errors of the spatial and spectral parameters are asymptotycaly uncorrelated. Furthermore, if in addition $K_n(\omega)=N(\omega)I$, where I is an MxM unit matrix, then

$$F_\tau=\frac{T}{\pi}\int_0^\infty k(\omega)d\omega J_\Delta \tag{5}$$

where

$$k(\omega)=\frac{2\omega^2\rho^2(\omega)}{1+M\rho(\omega)}\ ;\ \text{and}\ \rho(\omega)=\frac{S(\omega)}{N(\omega)};\ J_\Delta=MI-\underline{1}\underline{1}^T \tag{6}$$

$\underline{1}$ is an (M-1) vector of all ones [2]. Notice that here F_τ is not a function of $\underline{\tau}$. The FIM (or the CRLB) for any spatial parameter vector ϕ, (bearing in the far field case or bearing and range in the near field case) can then be obtained using [3]

$$F_\phi=G^TF_\tau G\ ;\ \text{where}\ G_{ij}=\frac{\partial\tau_i}{\partial\phi_j} \tag{7}$$

Kirlin and Dewey [4] showed that for the general case of a non-diagonal noise covariance matix, F_τ becomes a function of $\underline{\tau}$. However, they did not consider $F_{\tau p}$ and F_p, i.e. - $S(\omega)$ assumed to be known.

In this paper we study the FIM for the case of a noise field dominated by a spatially coherent interference. This study provides some insight into the general problem of non-diagonal noise covariance matrix together with specific results for the important case of source parameters estimation in the presence of a spatially coherent interference.

To minimize algebric complexity we give results only for the case of far field source and interference (location specified by bearings α and β). We assume that each radiates a narrowband, zero mean Gaussian random signal in the same frequency band (around ω_0). The source and interference are uncorrelated and have spectra of levels S and I respectively, flat over the band W. We also assume that the location and power of the interference, together with the noise level N, are known so the vector of unknown source parameters is simply $\underline{\theta}=(\alpha,S)^T$. Under the narrowband assumption, the FIM is given by

$$F_\theta=\frac{TW}{\pi}J \tag{8}$$

where J is the FIM for $\underline{\theta}$ when only Fourier coefficients associated with one frequency are available as data. (The more general case of unknown β and I is discussed in [5]).

2. THE FISHER INFORMATION MATRIX

For $\underline{\theta}^T=(\alpha,S)$, J is a 2x2 matrix given by

$$J = \begin{bmatrix} J_\alpha & J_{\alpha S} \\ J_{S\alpha} & J_S \end{bmatrix} \tag{9}$$

For any zero mean Gaussian vector, J_{ij} is given by [6]:

$$J_{ij}=\mathrm{trace}\left[K_x^{-1}\left(\frac{\partial K_x}{\partial\theta_i}\right)K_x^{-1}\left(\frac{\partial K_x}{\partial\theta_j}\right)\right] \tag{10}$$

where in our case K_S in (1) is given by (2) and (3) at $\omega=\omega_0$ and K_n is given by

$$K_n(\omega_0)=I(\omega_0)\underline{v}(\omega_0)\underline{v}^*(\omega_0)+N(\omega_0)I \tag{11}$$

$\underline{v}$ is the interference steering vector given by

$$\underline{v}(\omega_0)=[1,\exp(j\omega_0 T_2),\ldots\ldots,\exp(j\omega_0 T_M)]^T \tag{12}$$

T_m, m=2,...,M are the differential delays of the interferring signal to the first (reference) sensor. Introducing

$$Z = \frac{|\underline{u}^*\underline{v}|^2}{M^2} \; ; \text{ and } \; \zeta = 1-Z \tag{13}$$

we notice that Z is the height of the normalized beam-pattern (BP) for a beam steered in the direction of the source α, when the interference is incident from direction β (or vice versa).
Also define

$$Y=\left|\frac{\partial u^*}{\partial\alpha}\left(I-\frac{uu^*}{M}\right)\underline{v}\right|^2=\omega_0^2\left|\frac{\underline{t}^T J_\Delta U^*\underline{v}}{M}\right|^2 \tag{14}$$

where U=diag($\underline{u}$); J_Δ is given in (6) and $[\underline{t}]_m=\partial\tau_m/\partial\alpha$, m=2,...,M. For a far field source, $\tau_m=(X_m\sin\alpha+Y_m\cos\alpha)/c$ where (X_m,Y_m) are the coordinates of the m-th sensor. Here we notice that Y of (14) is the output power of the system of Fig. 1 due to an input signal (interference) at bearing β . This system nulls the source (at bearing α). Its output power is simply due to the interference. (When the source and interference coincide, Y=0). The elements of J are given by:

$$J_S=\frac{M^2(1+M\rho_i\zeta)^2}{N^2R^2}=\frac{M^2}{N^2\left(1+M\rho_S+\frac{ZM\rho_i}{1+M\rho_i\zeta}\right)^2} \tag{15}$$

$$J_{\alpha S}=J_{S\alpha}=\frac{M\rho_i\rho_S}{NR^2}(1+M\rho_i\zeta)Z1 \tag{16}$$

and
$J_\alpha=J_O-\Delta J+\delta J$, where J_O is the FIM for α in the absence of the interference (the case of a diagonal noise covariance matrix) given by

$$J_O=k(\omega_0)\underline{t}^T J_\Delta\underline{t} \tag{17}$$

and $(\Delta J-\delta J)$ is a non-negative term which increases the CRLB of α and is due to the presence of the interference.

$$\Delta J=\frac{\rho_i}{R}(MZk(\omega_0)\underline{t}^T J_\Delta\underline{t}+2M\rho_S^2 Y) \; ; \; \delta J=\left(\frac{\rho_i\rho_S Z1}{R}\right)^2 \tag{18}$$

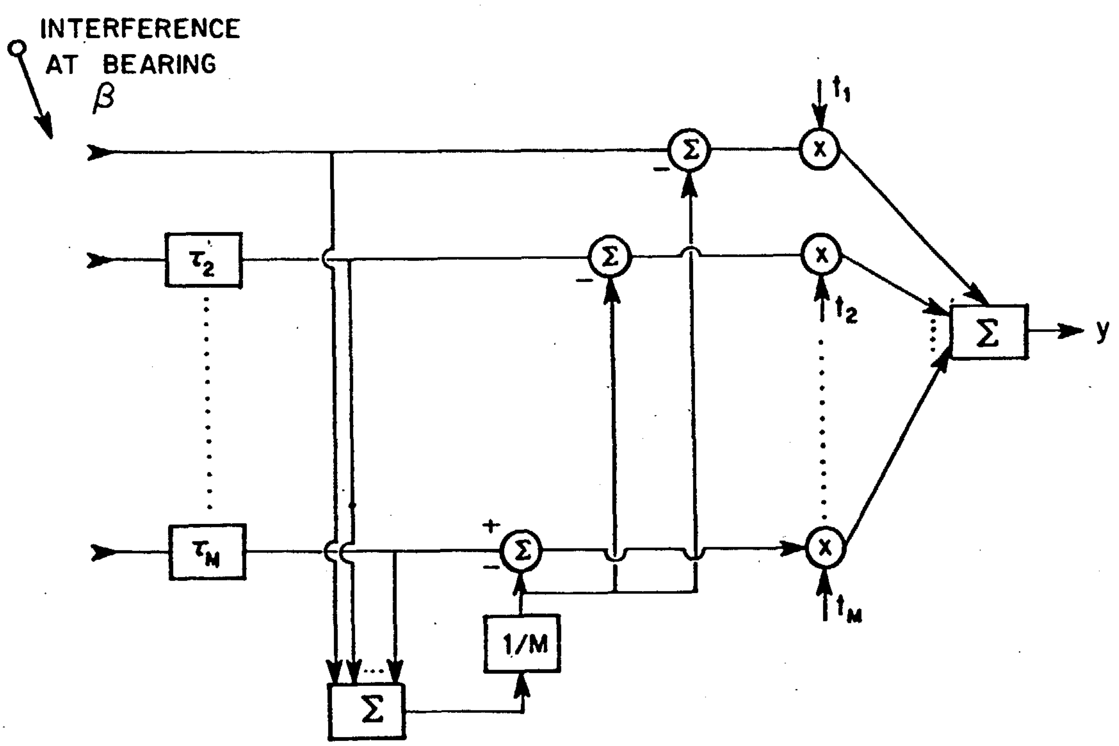

Fig. 1 : A nulling processor having output power Y due to an interference at bearing β.

In (15)-(18) $\rho_S=\frac{S(\omega_o)}{N(\omega_o)}$; $\rho_i=\frac{I(\omega_o)}{N(\omega_o)}$; $R=1+M(\rho_i+\rho_S)+M^2\rho_i\rho_S\zeta$ and $Z1=\frac{\partial Z}{\partial \alpha}$.

It is now clear that the FIM (and therefore the CRLB) for the estimation errors of the spatial and spectral parameters of a source in the presence of a coherent interference (with known parameters) can be put in terms of SNR and INR, the number of sensors M, the conventional beam-pattern Z and its first derivative , the null pattern Y and the fundamental error J_O .

All of these factors have physical meaning: hence one can describe the asymptotic performance of the optimal estimator in simple physical terms. The following observations can be made:

a. The case of a diagonal noise covariance matrix is a special case of the one presented here. By setting I=0 (ρ_i=0) the FIM J given in (15)-(18) reduces to the well known result of spatially white noise. In particular, $J_{\alpha S}=0$ for $\rho_i=0$, so the estimation errors of spatial and spectral parameters are uncorrelated. For $\rho_i\neq 0$ eq. (16) indicates that the key factor determines the geometrical coupling between spectral and spatial parameters is the BP derivative $Z1=\partial Z/\partial\alpha$. It is immediately apparent that Z=0 implies Z1=0. Thus, at nulls of the BP $J_{\alpha S}=0$. However, it does not follow that small values of Z insure weak coupling. Hence one cannot argue that source and interference separated by more then a beamwidth insures weak coupling, even if there are no high sidelobes. What is required is that a beam steered on the interference exhibits a pattern which is essentially flat in the direction of the source. Whether its height at that point is large or small is of secondary importance.

b. The derivatives of Z do not appear in Js (Eq. (15)). For spectral estimation with known source bearing the height of the BP for a given source interference separation summarizes all relevant geometrical information. In particular, at nulls of the BP the CRLB on S is the same as in the absence of the interference.

c. Denote the CRLB on α for unknown S by CRLB(α) and the bound on α for known S by CRLB(α/s) we have that

$$CRLB(\alpha/s)=\frac{\pi}{TW}J_\alpha^{-1}=\frac{\pi}{TW}(J_O-\Delta J+\delta J)^{-1} \quad (19)$$

$$CRLB(\alpha)=\frac{\pi}{TW}(J_O-\Delta J)^{-1} \quad (20)$$

It follows that unknown spectral parameters cause an incremental error in source bearing (related to ΔJ) which is not a function of Z1. For ΔJ to be small, a beam steered on the source must carry little interference power. If the source spectral parameters are known, the improvement in the bearing estimator performance is related to δJ, which is a function of Z1. This suggests that the knowledge of S can be used via the sensitivity of the beamformer output to variations in α .

d. For large M and Z≠1, R varies as M^2 . It follows that J_α and J_S are not a function of M while $J_{\alpha S}$ varies as 1/M. That is, when the source and interference are separated by more than a beamwidth, one can find M large enough so as the spectral and spatial parameters estimation errors are practically uncoupled. Furthermore, also ΔJ and δJ vary as 1/M and $1/M^4$, respectively so that the bounds on α in the presence and absence of interfernce do not differ markedly for separations in excess of a beamwidth.

3. NUMERICAL EXAMPLES

Consider the regular hexagonal array shown in Fig. 2. If the array is steered in the direction α=0, Fig. 3a shows the beam pattern Z and its first derivative Z1 (Fig. 3b) as a function of β . In Fig. 3c the null pattern Y (eq. (14)) is depicted. Fig. 4 shows the CRLB(α) and the CRLB(α/s) for α=0 as a function of β . The source SNR is ρ_S=1 and the interference INR is ρ_i=20. Also shown in Fig. 4 the CRLB for α when the interference is absent, $CR_O=(\pi/TW)Jo^{-1}$. This is the

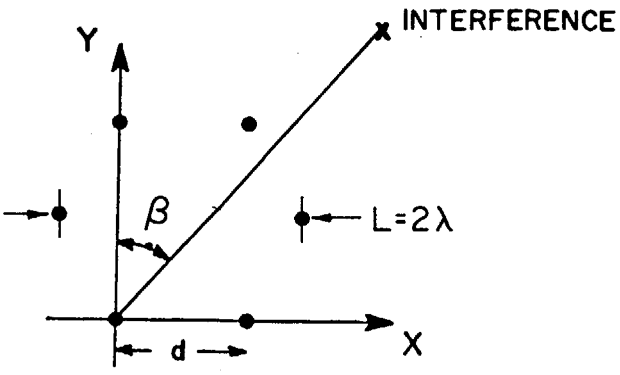

Fig. 2 : The array used for the numerical examples.

dashed horizontal line. Fig. 5 gives equivalent results for an array of the same shape and dimension with M=13 instead of M=6 . (One sensor was added in the center of the array of Fig. 2 and 6 more were added symmetrically on the hexagon edges). By looking at Figs. 3 to 5 the following observations can be made:

a. As expected, CRLB(α)≥CRLB(α/s). Also, since at β=0 Z1=0 and Y=0, the bounds are equal at α=0.

b. CRLB(α) is similar to CRLB(α/s) in regions where Z1 is small (20°≤β≤50° , 75°≤β≤120° and β≥140°). This confirms the result that with small Z1 the estimation errors of spectral and spatial parameters are uncoupled. Notice, however, that with large Z1 the difference between the CRLB(α) and the CRLB(α/s) is as high as 13dB.

c. In the presence of interference the bound is as much as 12-13dB higher then when the interference is absent. The CRLB(α) is similar to CR_O only in regions of small Z, small Y and small Z1 (β≈36° , 80°≤β≤110°).

d. Figs. 4 and 5 show similar behavior of the bounds within the mainlobe of the BP. However, for β>30° , CRLB(α)≈CRLB(α/s)≈CR_O with M=13 (Fig. 5) only. This confirms our contention that for large M and separations larger then a beamwidth the presence of interference has little influance on the bound.

e. Within the mainlobe of the BP (β≤20°) the bound on α has certain features regardless of array geometry:
With unknown spectral parameters it has a single peak at $0<\beta_m<(\lambda/L)$, where λ is the signals wavelength and L is the array diameter , λ/L in radians is approximately the beamwidth of the array beam pattern. That is, at spatial separation less then a beamwidth, the estimation accuracy is poor, with smaller estimation errors at β=0 then at β_m. With known spectral parameters the bound shows a main lobe near β=0. This main lobe is much narrower than the BP width, suggesting that one can achieve much better resolution than with the conventional beamformer. This is not the case with unknown spectral parameters. Notice also the existence of a sidelobe of the bound within the BP main lobe (having a peak at about the first peak of the null pattern). This sidelobe (which can be higher than the mainlobe for smaller interference to source ratios) was also observed by Heilweil [7].

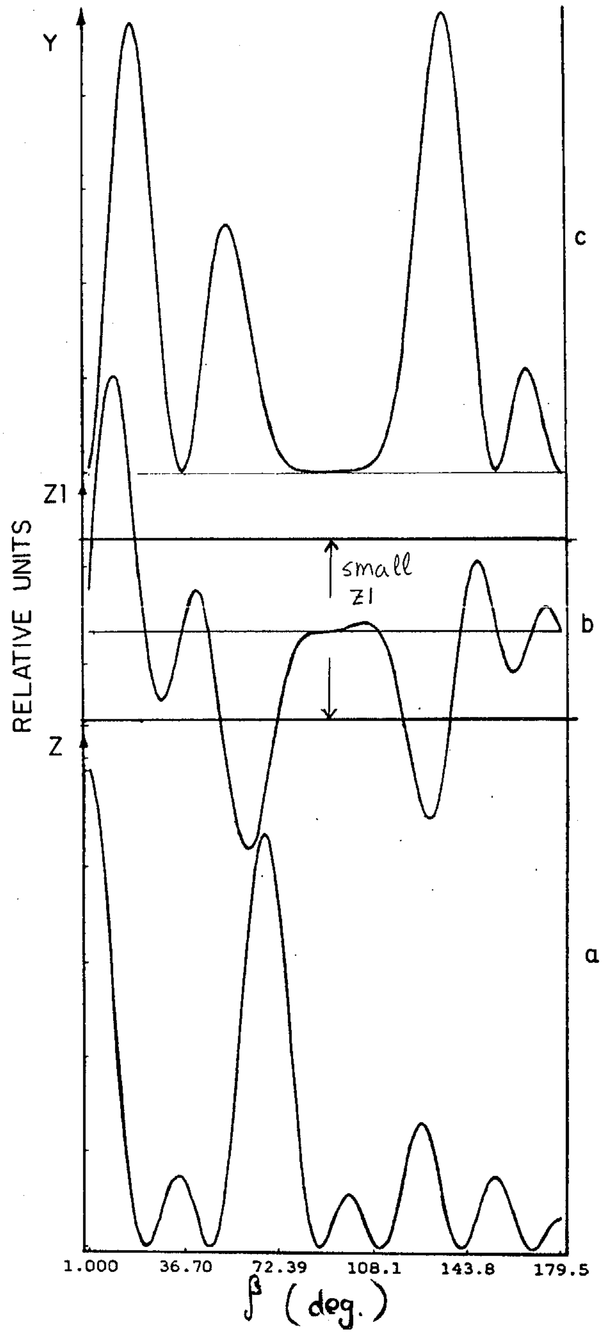

Fig. 3 : a. The conventional array beam-pattern, Z.
b. The derivative $Z1=\partial Z/\partial\alpha$.
c. The null-pattern Y.

4. REFERENCES

[1] G.C. Carter, "Time delay estimation for passive sonar signal processing", IEEE Trans. ASSP-29, June 1981.

[2] W.R. Hahn, S.A. Tretter, "Optimum processing for delay-vector estimation in passive signal arrays", IEEE Trans. IT-19, pp. 608-614, Sept. 1973.

[3] P.M. Schultheiss, "Locating a passive source with array measurments, a summary of results", IEEE Proc. ICASSP, p. 967, 1979.

[4] R.L. Kirlin, L.A. Dewey, "Optimal delay estimation in a multiple sensor array having spatially correlated noise", IEEE Trans. ASSP-33, No. 6, p. 1387, Dec. 1985.

[5] H. Messer, P.M. Schultheiss, "Parameter estimation for two sources with overlapping spectra using an arbitrary M sensor array", IEEE 3rd Workshop on Spectral Estimation and Modeling, Boston, Nov. 17-18, 1986.

[6] W.J. Bangs, P.M. Schultheiss, "Space-time processing for optimal parameter estimation" in Signal Processing Proc. NATO Adv. Study Inst., Academic Press, 1973.

[7] B. Heilweil, "Direction estimation for multiple remote sources", Ph.D. Dissertation, Yale University, May 1983

5. ACKNOWLEDGMENT

The work reported in this paper was supported by the office of Naval Research under contract N00014-80-C-0092.

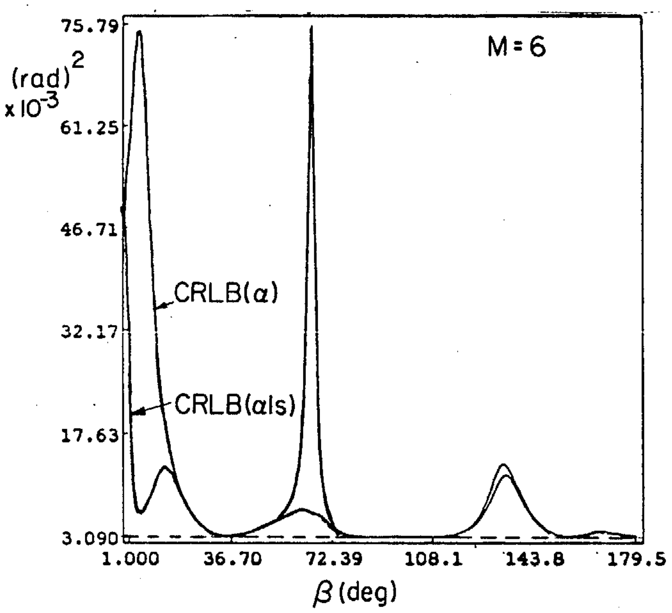

Fig. 4 : The Cramer-Rao bounds, M=6.

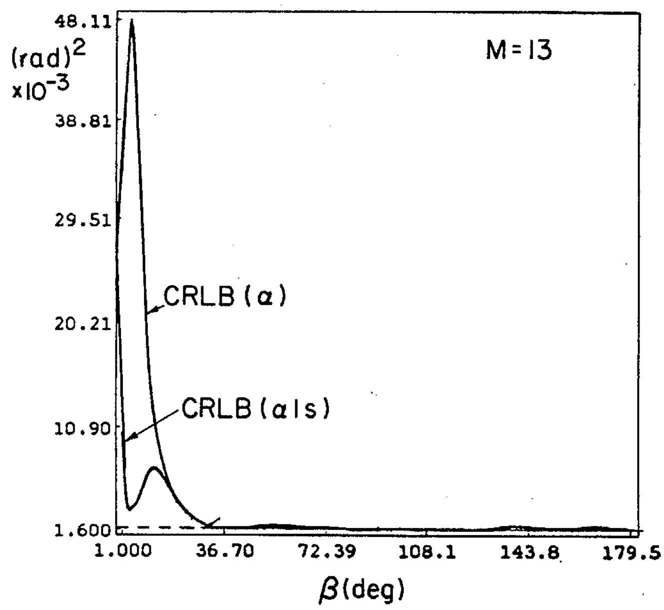

Fig. 5 : The Cramer-Rao bounds, M=13.