Constrained Adaptive Estimation of Time Delay with Multipath

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RÉSUMÉ

La différence entre les temps d'arrivée d'un signal (délai temporel) en deux endroits différents peut être représentée par un filtre à réponse impulsionnelle finie dont les coefficients sont les échantillons d'une fonction sinc. Cette condition imposée sur les coefficients permet d'estimer, à partir de méthodes adaptives, précisément le délai temporel en présence de propagation avec multiple parcours. L'application de la contrainte sur les paramètres adaptatifs permet de réaliser un algorithme plus simple et plus rapide.

SUMMARY

The difference in arrival times (time delay) of a signal at two separate locations can be modeled as a finite impulse response filter whose coefficients are samples of a sinc function. By imposing this condition on the coefficients, accurate estimation of time dealy in the presence of multipath propagation can be achieved by adaptive methods. The application of constraints on the adaptive parameters results in a faster and simpler algorithm.

1. INTRODUCTION

There are many applications where it is required to determine the location of a radiating source by passive means and a well known example is in passive sonar. A standard method uses three or more receivers at separate but known locations. By measuring the differences in arrival times (time delays) of the signal at these receivers, the bearing and range of the source can be determined. Clearly, location accuracy depends directly on the time delay measurements and various time delay estimation methods have been proposed and evaluated. A special issue of the IEEE Tansactions [1] is devoted to this subject which has many other applications in the areas of bio-engineering, geophysics, seismology and more recently speed measurement [2].

While there is an abundance of literature on the topic of time delay estimation, only a few [3,4,5,6] considered the problem of multipath propagations. In its most general form, the multi-path model for two receivers is

$$x(t) = S(t) + \sum_{i=1}^{N} \alpha_1 \; S(t+M_i\,) + \phi(t) \qquad (1)$$

$$y(t) = S(t-D) + \sum_{j=1}^{L} \alpha_j \; S(t+M_j) + \psi(t) \qquad (2)$$

where S(t) is the source signal, normally taken to be stationary random process and S(t+D) its delayed version of delay D and the noise sources are $\phi(t)$ and $\psi(t)$, also random processes that are independent with respect to each other and S(t). The multipath transmissions are of amplitudes α_i, α_j and delays M_i, M_j. The receiver output x(t) contains N multipaths while y(t), L multipaths. Physically, in the case of sonar, these multipath signals come from bottom bounces or reverberations and, reflections from buildings or mountains in the case of radio transmissions. The problem is to find an estimate of D, given x(t) and y(t).

Due to the complexity of (1) and (2), there is presently no method available to determine D. Of course, ignoring the presence of multipaths and using standard procedures would result in erroneous time delay estimates. However, tractable solutions do exist when some simplifying assumptioms can be made. For example if $|M_i|, |M_j| >> |D|$, and $|M_i - M_j| >> D$, then the location of the first peak, from the origin, of the cross-correlation between x(t) and y(t) would yield an estimate of D. Another often used method [7] takes the cepstrum of x(t) and y(t) separately to first determine the existence of multipath receptions and then removes them through cepstrum filtering techniques. Again, this method will not work if M_i and M_j are small (less than 10 sampling intervals, say) or if D is close to some M_j in (2).

Compounding the above problem is the time-dependent and unstable nature of the multipath parameters α_i, M_i. Additionally, the time delay D will also vary with time if there is relative motion between the radiating source and the receivers. Estimating D in such a multipath, non-stationary environment would certainly require adaptive methods. This paper proposes a new, adaptive time delay estimation technique in the presence of multi-path reception. It is applicable to the case of a single multipath and although the time dealy can be any number, the multipath delay is assumed to be an integral multiple of the sampling interval. This is not a severe limitation if the sampling interval is sufficiently small so that the integral assumption will always be approximately valid. The time delay, multipath delay and gain can of course be time-varying.

II. THE CONSTRAINED ADAPTIVE ALGORITHM

Let

$$x(n) = S(n) + \alpha S(n-\Delta) + \phi(n) \qquad (3)$$

and

$$y(n) = S(n-D) + \psi(n) \quad , n=0,1,\ldots,N-1 \qquad (4)$$

be the sampled output of the two receivers. For convenience and without loss of generality, the sampling interval is taken to be unity. The sampled signals is $S(n)$ with its delayed version $S(n-D)$, D being any real number. The received multipath propagation is denoted $\alpha S(n-\Delta)$, with $\emptyset \leq \alpha \leq 1$ and Δ any positive integer. The bounds on the multipath parameters α and Δ arise from the knowledge that the multipath transmission is always a delayed signal and that itis always attenuated. The uncorrelated noise samplesare $\phi(n)$ and $\psi(n)$.

The adaptive system configuration is shown in Figure 1, where the coefficients a_i and b_i are adjusted to minimize the sum of the squares of the error sequence

$$e(n) = z_1(n) - z_2(n) \qquad (5)$$

It is seen from Figure 1 that ideally, if the upper filter has transfer function

$$H_1(Z) = \frac{1}{1 + \alpha Z^{-\Delta}} \qquad (6)$$

and the lower filter can produce a pure time delay equal to D, then $z_1(n) = z_2(n)$ and the unknown parameters can be computed from the values of the coefficients a_i, b_i.

It is shown in [8] that

$$S(n-D) = \sum_{k=-\infty}^{\infty} \xi_k S(n-k) \qquad (7)$$

if

$$\xi_k = \frac{\sin \pi(D+k)}{\pi(D+k)} \triangleq \text{sinc } (D+k) \qquad (8)$$

That is, the delayed signal $S(n-D)$ can be obtained by passing $S(n)$ through a filter (7) whose coefficients are given by (8). It is necessary to truncate the filter order in (7) to a manageable number and [8] contains an analysis of the errors due to truncation. The requisite filter order is rather modest. For example, summing k from -6 to +6 in (7) would ensure an error in D of less than 10%.

The adaptive system's input-output is related by

$$z_1(n) = x(n) - \sum_{i=1}^{M} a_i z_1(n-i) \qquad (9)$$

$$z_2(n) = \sum_{i=-N}^{N} b_i\, y(n-i) \qquad (10)$$

From (9), it follows that the multipath gain α equals a_h, where a_h is the maximum of all a_i and the multipath delay $\Delta = h$. The delay estimate D is computed from (10), by noting the index m and the value of b_m, the maximum of the b_i. The details are given below.

The filter coefficients are adapted by the LMS algorithm [9]

$$a_i^{(n+1)} = a_i^{(n)} + \mu_a \nabla_{a_i} \qquad (11)$$

$$b_i^{(n+1)} = b_i^{(n)} + \mu_b \nabla_{b_i} \qquad (12)$$

where $a_i^{(n+1)}$ denotes the value of the coefficient a_i at time n+1, μ_a, μ_b are the convergence factors and the gradients

$$\nabla_{a_i} = \frac{\partial e^2(n)}{\partial a_i} \quad , \quad \nabla_{b_i} = \frac{\partial e^2(n)}{\partial b_i} \qquad (13)$$

Using the approximate gradients given in [10], (11) and (12) become

$$a_i^{(n+1)} = a_i^{(n)} + \mu_a\, e(n)\, x(n-i) \quad , \quad 1 \leq i \leq M \qquad (14)$$

$$b_i^{(n+1)} = b_i^{(n)} + \mu_b\, e(n)\, y(n-i) \quad , \quad -N \leq i \leq N \qquad (15)$$

In implementing the algorithm, it is beneficial and indeed necessary to utilize the a priori constraints, i.e., that Δ is an integer, $0 \leq \alpha \leq 1$, and the b_i coefficients equal the sinc function in (8). It was found in the simulation studies that without applying the constraints, convergence was difficult and sometimes impossible to obtain.

To apply the constraints of (8) on the b_i coefficients, a look-up table is first constructed. It is a matrix Φ of dimension K x (2r + 1) with elements

$$\xi_{ij} = \frac{\sin \pi(D_i + j)}{\pi\,(D_i + j)}, \quad j = -r, \ldots, -1,0,1,\ldots r \qquad (16)$$

where

$$D_i = \frac{0.5}{K}(1 + i), \quad i = 0,1,\ldots,K-1 \qquad (17)$$

Note that even though the maximum value of D_i in the table is 0.5, values from 0.5 to 1 are also covered because sinc (D_i+j), for $0.5 \leq D_i \leq 1$, equals sinc $[(1 - D_i) - (j + 1)]$ for $0 \leq 1 - D_i \leq 0.5$. The purpose of this look-up table is to assign values to the coefficients b_i according to (8). The maximum of the b_i, b_m is first mappped (by finding the closest ξ_{io} to b_m) to the centre of the sinc function in the table. Then the coefficients preceding and following b_m are assigned the corresponding values from the left and right of the centre, respectively. In this manner, only b_m needs to be adapted and all other coefficients are simply assigned values from the table.

The constrained adaptive algorithm is as follows:

1. The coefficients are adapted according to (14) and (15) for the first 200 iterations. The initial conditions are $a_i = 0$, $b_i = 0$ for all i except $b_o = 1$.

2. At the 201st iteration,
   a) find the maximum of the a_i. Let this be a_h. If $a_h < 0.5$, set $a_h = 0.5$. If $a_n > 0.5$, leave it unchanged. Set all other $a_i = 0$.

   b) find the maximum of the b_i. Let this be b_m. If $b_m > 0.9$, leave it unchanged. If $b_m < 0.9$ set $b_m = 0.9$. This corresponds to a fractional delay D_i of 0.75 or -0.25. The proper value is determined from the signs of the two adjacent coefficients of b_m. Once the value of D_i is determined, the values for the other b_i are taken directly from the look-up table. The delay estimate is $N+1-m + D_i$.

3. At each iteration after the 201st,

   a) if $a_h < 0.1$, then all a_i are adjusted according to (14). If $a_h > 0.1$, only a_h is adjusted according to (14). Set all other $a_i = 0$.

b. Compute

$$b_{m-1}^{(n+1)} - b_{m-1}^{(n)} = \delta_{m-1} = \mu_b e(n)\ y(n-m+1) \quad (18)$$

$$b_{m}^{(n+1)} - b_{m}^{(n)} = \delta_{m} = \mu_b e(n)\ y(n-m) \quad (19)$$

$$b_{m+1}^{(n+1)} - b_{m+1}^{(n)} = \delta_{m+1} = \mu_b e(n)\ y(n-m-1) \quad (20)$$

If $D_i > 0$, and the sign conditions

$$\delta_{m-1} < 0,\ \delta_m > 0,\ \delta_{m+1} > 0 \quad (21)$$

are satisfied, then update b_m according to (15). From this new b_m, get an updated a_i from the table together with updated values for all other b_i. If condition (21) is violated, leave b_m unchanged. Similar procedures follow if $D_i < 0$.

A detailed explanation of the particular steps taken in the algorithm is next given. First, the constraints are not applied for the initial 200 iterations because the coefficients may be peaking at the wrong positions during the initial transients, especially when noise is present. Secondly, the minimum value for the ξ_{io} column in the matrix Φ is 0.6366, corresponding to $D_i = 0.5$. Using the bounds that $0.6366 \leq \xi_{io} \leq 1$, the adaptation can be speeded up by setting $b_m = 0.9$, if it is below 0.9, on the first application of the constraint at the 201st iteration. The choice of 0.9 is ad hoc, any value between 0.6366 and 1 will work. The checking of the sign conditions in step 3(b) is to ensure that an update on b_m will be carried out only if the changes in the three most significant coefficients, b_{m-1}, b_m, b_{m+1}, are moving in the correct directions. Finally, the filter order N is chosen, as discussed in [8], to be sufficiently large to ensure an acceptable modeling error. The analysis in [8] shows that for N=5, the worst case delay error is 8.2%. If this is acceptable, and the largest delay expected is $|R|$, then $N=5+|R|$ is adequate. A higher order is needed if smaller modeling error is desired.

III. SIMULATION RESULTS

To verify the algorithm and to study its performance, two simulation experiments were performed. In both cases, the filters' orders are M=12, N=10. The look-up table has dimension 512 x 31, i.e. K = 512 and r = 15. This value of K is sufficient to give delay estimates of approximately 0.0001 resolution. With r = 15, maximum delays of ∓ 10 are adequately covered. The sampling interval is unity.

The output of a gaussian random number generator served as the sequence S(n) which is then filtered according to (7) to give S(n-D). The plots in Figures 2 and 3 are the averages of 10 independent runs. In test 1 (Figure 2), at the 6000th iteration, all the three unknowns of multipath gain and delay, and time delay, were given step off-sets. It is seen that the adaptive system responded to these changes and was able to give accurate estimates after the transients although the multipath delay transient was much shorter. This can be attributed to the simple constraint used which sets all a_i to zero except the largest one. The convergence factors $\mu_a = \mu_b = 0.005$.

In Test 2, all the unknown parameters were given a series of off-sets designed to cover the different possible combinations of parameter changes. Again, the results in Figure 3 show that the adaptive system can track these changes effectively. The convergence factors $\mu_a = \mu_b = 0.003$.

The unconstrained algorithm, i.e., adaptation according to (14) and (15), was tested without much success. Convergence was either not achieved or excessively high. Noise at signal to noise ratio of 20 db were also added and the results were only slightly different and were therefore not shown in the figures.

IV. CONCLUSIONS

A constrained adaptive algorithm for time delay estimation in the presence of multipath reception has been presented. The scheme is based on the LMS algorithm but through constraints, the number of parameters to be adapted is greatly reduced and the convergence is much faster. Results from two experiments have demonstrated the algorithm's effectiveness and its ability to track time-varying parameters.

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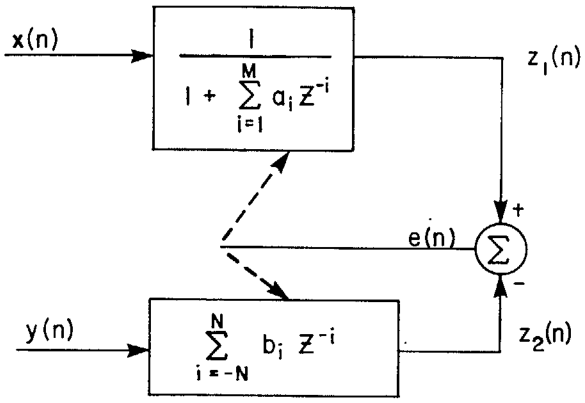

FIGURE I SYSTEM CONFIRMATION

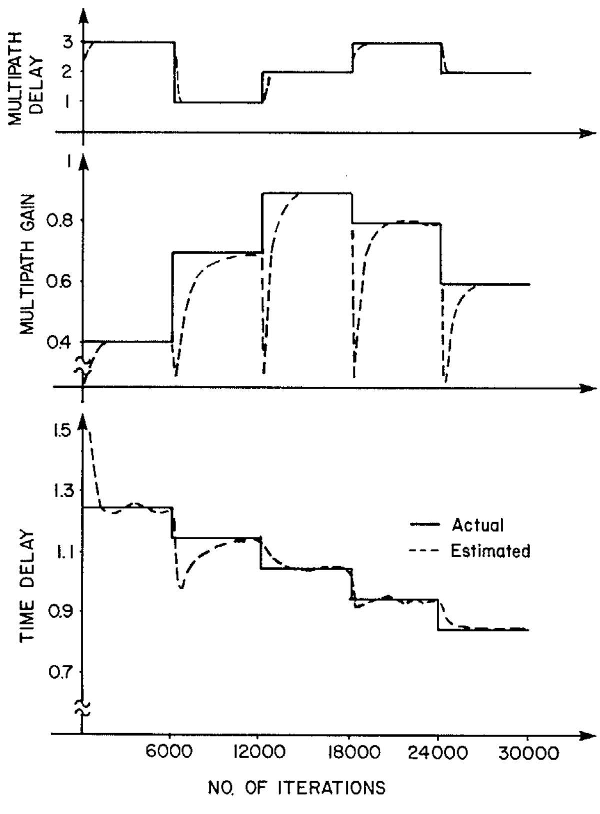

FIGURE 3 TEST 2

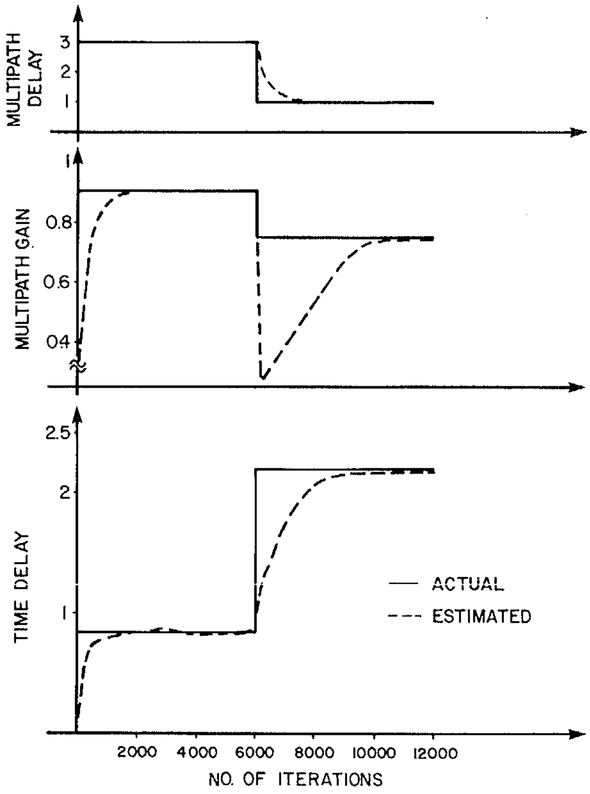

FIGURE 2 TEST I