

OPTIMIZATION OF THE SIGNAL STRUCTURE  
IN FEEDBACK COMMUNICATION SYSTEMS

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Dans cette communication, on analyse l'intégration de la modulation, des signaux et des procédures de décision afin de réduire la probabilité d'erreur dans un système de communication à canal rétroactif. Les modulations de fréquence à phase continue sont considérées en particulier. Les performances des systèmes de communication sont optimisées en fonction de l'indice de modulation et du nombre moyen de transmissions de chaque symbole.

In this paper the integration of the modulation operation and of the signaling and decision structure in order to reduce the error probability in a communication system using a feedback channel is analyzed. Continuous-phase-frequency modulations are in particular considered. The performance of the communication systems determined by the Euclidean distance is optimized as a function of the modulation index and of the mean transmission number of each symbol.

1. INTRODUCTION

In many communication systems it is possible to use an essentially noiseless feedback channel to improve the communications over a noisy feedback channel. In general, the noiseless feedback channel permits a reduction in the amount of signal energy and in the complexity of coding and decoding operations required to achieve a specified performance. Typical examples are the sequential decision schemes [1],[2], and the Automatic-Repeat-Request (ARQ) techniques [3],[4], using error detecting codes. In these communication systems, when the forward channel introduces an uncorrectable error pattern, the same information is retransmitted until a correct reception of the information is detected. The error probability in all these communication systems depends on the Euclidean distance and/or the Hamming distance. The Euclidean distance is introduced during the modulation operation, while the Hamming distance depends on the channel coding scheme.

errors. Before its transmission, each symbol  $c_i$  is sent to a CPFSK modulator which associates with this symbol, in the interval  $[(i-1)T, iT]$ , a waveform  $s_{i,p}(t)$  if  $c_i = a_p$ , which is given by:

$$(1) \quad s_{i,p}(t) = \frac{\sqrt{2E}}{T} \cos[\omega_0 t + \frac{\pi h}{T} c_i + x_i]$$

In this paper, it is shown that the integration of the modulation operation in the signaling and decision structure of the feedback communication systems permits to reduce significantly the error probability. In classical feedback communication systems the signals associated with successive transmissions of the same symbol are all the same. In the scheme described in this paper the modulated waveforms corresponding to successive transmissions of the same symbol are different and they are chosen in such a way as to increase the Euclidean distance between the signals. The proposed scheme can be applied both to sequential decision feedback schemes and ARQ schemes. Continuous-Phase-Frequency-Shift-Keying (CPFSK) modulations which are very attractive for their low bandwidth occupancy with respect to classical digital modulations, are in particular analyzed.

where  $T$  is the time-signaling interval,  $E$  the signal energy,  $f_0 = \omega_0/2\pi$  the carrier frequency, and  $x_i$  is a phase term introduced in order to maintain the phase continuity at the end and at the beginning of the time-signaling intervals [5]. The phase paths in the CPFSK modulation can be represented through the phase trellis; as an example, Fig. 1 shows the phase trellis of a CPFSK modulation with  $h = 0.5$ .

Let us consider the  $j$ -th transmission of the codeword  $c$ . The received signal in the  $i$ -th time-signaling interval ( $1 \leq i \leq n$ ), corresponding to  $s_{i,p}(t)$ , is denoted with  $r_i(t)$ . The demodulator computes the  $M$  Euclidean distances  $d_{i,p}(j)$  between  $r_i(t)$  and  $s_{i,p}(t)$  for  $p = 1, 2, \dots, M$ . These distances are used to perform a cumulative report, which is updated every time a new transmission is performed. Each time a transmission of  $c$  is received, the receiver forms the matrix of the Euclidean distances  $D(j) = \{D_{i,p}(j)\}$ , for  $1 \leq p \leq M$  and  $1 \leq i \leq n$ , defined as:

$$(2) \quad D_{i,p}(j) = D_{i,p}(j-1) + d_{i,p}^2(j)$$

being  $D_{i,p}(0) = 0$ .  $D(j)$  is called the cumulative matrix of the Euclidean distances. After the matrix  $D(j)$  for  $c$  has been computed, the demodulator tries to decode  $n$  symbols. A vector  $w = (w_1, w_2, \dots, w_n)$  is constructed in the following way. The  $i$ -th component is set equal to the symbol  $a_s$  of  $A$  for which it results:

2. CPFSK MODULATION AND EUCLIDEAN DISTANCE FOR SEQUENTIAL SIGNALING

In a classical sequential signaling scheme each block of  $k$  informative symbols is encoded in a codeword of  $n$  symbols of a code  $(n, k)$ , able to correct  $t$  errors and to detect  $s > t$  errors. The general block-diagram of the proposed scheme, denoted in the following with ARQ1, is shown in Fig. 1. The source generates symbols from a finite alphabet  $A = \{a_1, a_2, \dots, a_M\}$  with  $M$  elements. Each block of  $k$  information symbols coming out of the source is encoded in a codeword  $c = \{c_i\}$ ,  $n$  symbols long, of a code  $C$  of type  $(n, k)$ . The alphabet of the code is assumed, for simplicity, equal to that of the source, but the method can be easily generalized to the other case. Code  $C$  is assumed able to correct  $t$  errors ( $t > 0$ ) and to detect  $s > t$

$$(3) \quad D_{i,s}(j) = \min_{1 \leq p \leq M} D_{i,p}(j)$$

The decoder analyzes the vector  $w$ ; if  $w$  contains an uncorrectable error pattern, then a negative acknowledgment (NACK) is sent to the transmitter, which provides a new replica of  $c$ , while if  $c$  is a codeword of  $C$  or contains a correctable error pattern, then it is assumed as correct and a positive acknowledgment (ACK) is sent to the transmitter. The Euclidean distances among the symbols to be demodulated increase linearly with  $j$  and, therefore, the error probability at the input of the channel decoder decreases with  $j$ .



The method described herein can be applied to any classical feedback communication system. However, in some sequential signaling schemes, the same symbols or codewords are retransmitted consecutively [6],[7]. In these cases, the method can be furtherly modified in order to improve its performance.

The codeword  $c$  is transmitted  $m_j$  times consecutively and the sequence of transmitted symbols is shown in Fig. 2.a. However, different orders of the transmitted symbols can be considered; the most obvious order, which will be taken into account in this paper, is to transmit each symbol  $m_j$  consecutive times, as shown in Fig. 2.b. In this way, for the transmission of the  $i$ -th symbol, a sequence of  $m_j$  symbols equal to  $c_i$  is sent. The symbol  $c_i$  can assume  $M$  different values and, therefore,  $M$  different sequences of  $m_j$  symbols are possible.

However, this is not the only possible choice. In fact, for the transmission of  $c_i$ , we can use any group of  $M$  sequences with  $m_j$  components from the alphabet  $A$ . In this respect, if  $c_i = \alpha_p$ , the sequence used to transmit  $c_i$  is denoted with  $\alpha_p = (\alpha_{p,1}, \alpha_{p,2}, \dots, \alpha_{p,m_j})$ , where  $\alpha_{p,r} \in A$ . As an example, in the binary case  $A = \{-1, 1\}$ , two different sequences  $\alpha_{-1}$  and  $\alpha_1$  with  $m_j$  components must be chosen among the  $2^{m_j}$  possible sequences.

The choice of the  $M$  sequences  $\alpha_l$  influences significantly the Euclidean distance among the modulated signals. The Euclidean distance is also influenced by the length  $L$  of the observation interval used to perform the demodulation. In fact, the demodulation of the  $r$ -th symbol is generally performed by observing the actual symbol and the  $(L-1)$  successive symbols. The Euclidean distance between the received signals,  $LT$  sec. long, and all the possible sequences of  $L$  symbols is computed; then, the  $r$ -th symbol to be demodulated is set equal to the first symbol of the sequence having the lowest Euclidean distance from the received signal.

3. RESULTS

The most intuitive method of choosing  $\alpha_{-1}$  and  $\alpha_1$  is to set all their components equal to  $-1$  or  $1$ , respectively, i.e.

$$(4) \alpha_{-1} = (-1, -1, -1, \dots) \quad \alpha_1 = (1, 1, 1, \dots)$$

However, this choice can be in some cases non-optimum since other sequences may give a higher Euclidean distance.

Figs. 3.a and 3.b show the Euclidean distance  $d_e^2$  for a CPFSK modulation versus the modulation index  $h$  for  $m = 2$  and  $m = 4$ , respectively. When no memory is used at the receiver side (classical ARQ scheme), and the length of the observation interval is  $L = 2$ , the Euclidean distance is represented by curve a. For the ARQ1 strategy, the Euclidean distance is represented by curve b. The Euclidean distance for the ARQ2 strategy using sequences (4) is given by curves c and d for  $q = 1$  and  $q = 2$ , respectively. As can be seen from these results, the ARQ2 strategy using sequences (4) permits the achievement of an increase in the Euclidean distance for low values of  $h$  and the gain increases with  $m$ . For mean values of the modulation index ( $h = 0.5$ ), the ARQ1 strategy gives higher Euclidean distances. However, these results depend on the choice of the sequences  $\alpha_{-1}$  and  $\alpha_1$ .

The Euclidean distance in the ARQ2 strategy is improved by increasing  $q$  and, therefore,  $L$ . Figs. 4.a, 4.b and 4.c show the results obtained for  $m = 2$ ,  $m = 3$  and  $m = 4$ , respectively. In these figures curve a gives the results for the ARQ1 strategy, and curves b and c give the Euclidean distance when sequences (4) are used for  $q = 2$  and  $q = 3$ , respectively.

The optimum choice of  $\alpha_{-1}$  and  $\alpha_1$ , which give the highest Euclidean distance for each modulation index, has been determined through a computer search, as depicted by Figs. 4 (curves d and e).

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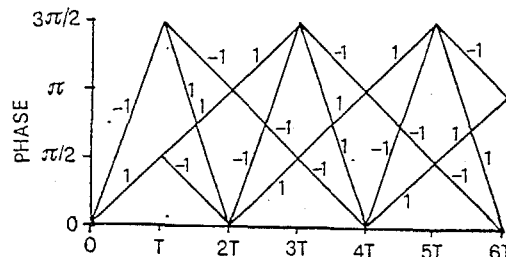


Fig. 1 - Phase trellis of a CPFSK modulation with  $h = \frac{1}{2}$ .

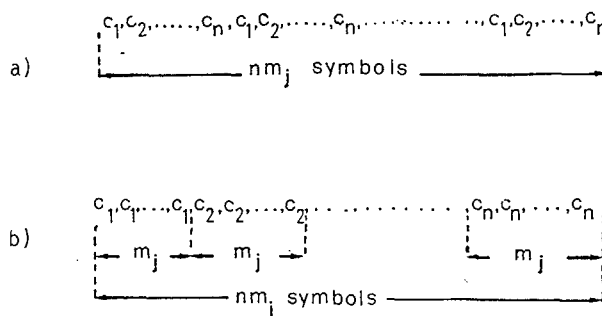


Fig. 2 - Retransmission protocol of an ARQ scheme in which a codeword is transmitted some times consecutively: a) classical scheme; b) modified protocol.

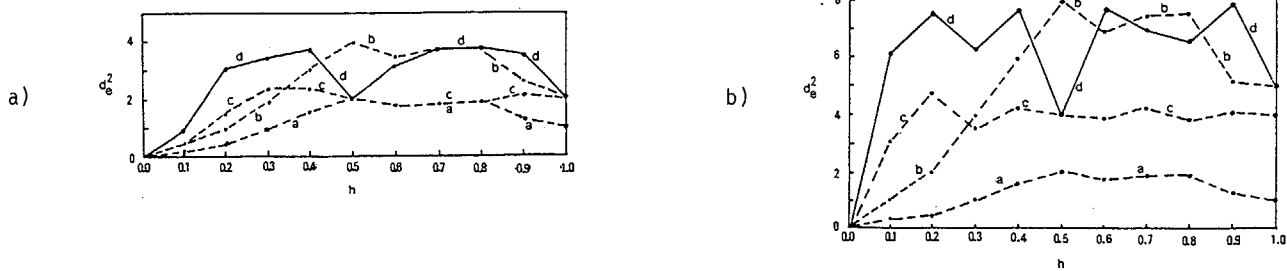


Fig. 3 - Minimum Euclidean distance  $d_e^2$  versus  $h$ : a)  $m = 2$ ; b)  $m = 4$ .

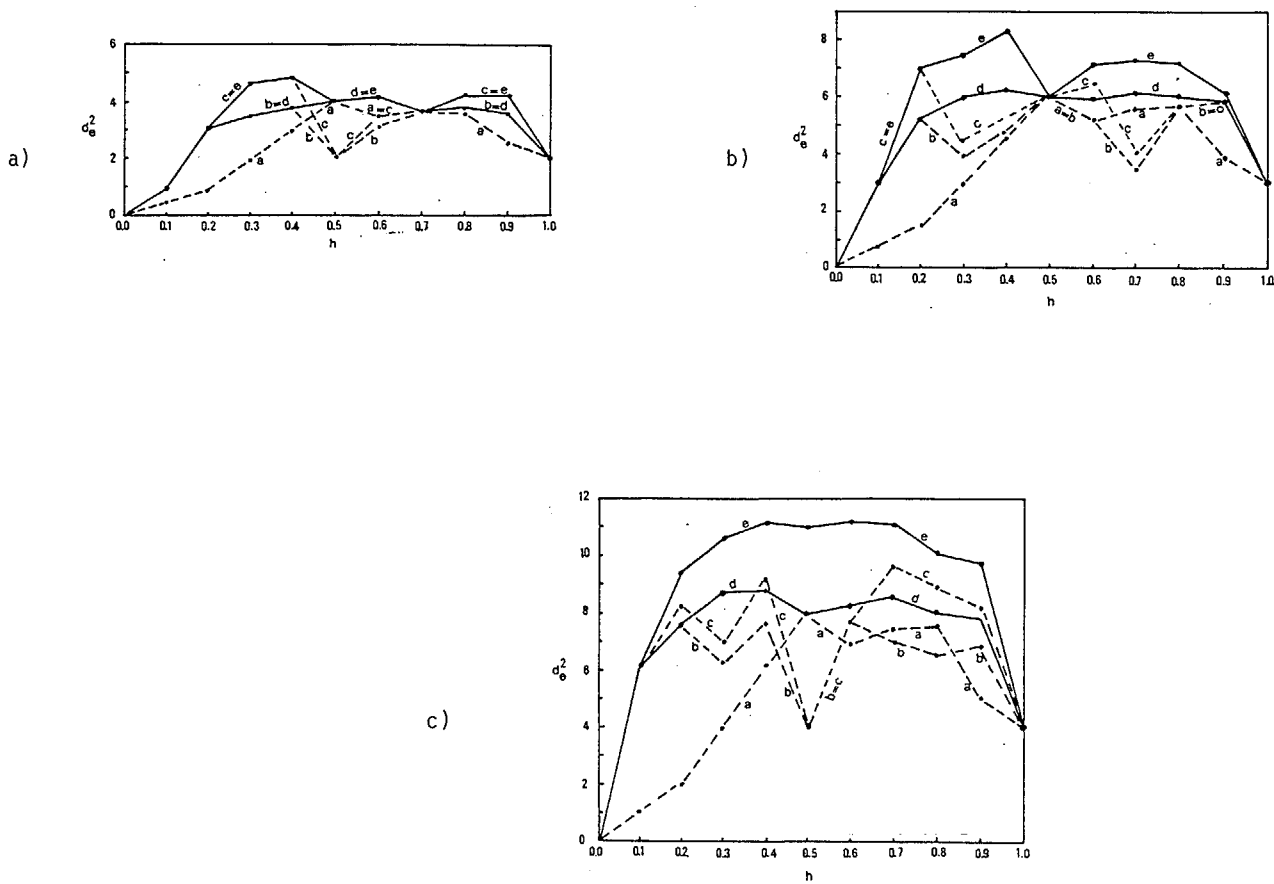


Fig. 4 - Minimum Euclidean distance  $d_e^2$  versus  $h$  for the ARQ1 and ARQ2 schemes:  
 a)  $m = 2$ ; b)  $m = 3$ ; c)  $m = 4$ .

