ANALYSIS OF TWO CARRIER PHASE RECOVERY CIRCUITS FOR CODED AND UNCODED PSK TRANSMISSION OVER SATELLITE CHANNEL.

F.Lorenzelli, L.Testa, M.Visintin, E.Biglieri and M.Pent

Dipartimento di Elettronica - Politecnico di Torino
Corso Duca degli Abruzzi 24 -- I-10129 TORINO (Italy)

RESUME

Le contenu de cet article est une étude comparative des effets de deux systèmes pour la régénération de porteuse dans la modulation numérique de phase à quatre niveaux sans codage et à huit niveaux avec codage en treillis [2]. Le premier a été récemment proposé par T. Fujino [1], tandis que le deuxième est le circuit "classique" à multiplication de fréquence.

ABSTRACT

The aim of this paper is to compare the effects of imperfect carrier phase recovery on uncoded 4-PSK and trellis coded 8-PSK carrying two information bits per channel symbol. We first determine the amount of improvement that can be obtained using a recently proposed recovery system in both coded and uncoded environments, and how critical its use can be. Second, we assess the difference in behavior between this carrier phase recovery system and the standard frequency multiplier.

1. INTRODUCTION

Since in digital radio communications both spectrum and power are generally limited resources, to cope with the increasing demand for digital services more efficient transmission techniques are called for. Currently, quaternary phase-shift keying (4-PSK) is the prevalent modulation in use for digital satellite communications. To improve bandwidth efficiency, 8-PSK could be used instead, but at the price of a higher power needed to maintain the same error probability as for 4-PSK. Another solution maintains the same bandwidth as 4-PSK, while increasing the power efficiency: this is obtained by using a trellis-coding scheme invented by Ungerboeck [2] and based on the use of an 8-PSK signal constellation to transmit two information bits per symbol. With this scheme, we have the same bandwidth occupancy as for uncoded modulation [4], but with a reduced power requirement for the same system performance, at least on an ideal channel perturbed by additive white Gaussian noise (AWGN) only.

Although Ungerboeck's trellis codes were designed for AWGN channels, some authors have found them to be effective when used to transmit over less ideal channels: see, e.g., [3], where a nonlinear channel model perturbed by additive Gaussian noise and intersymbol interference was considered. For a realistic comparison of the relative merits of coded and uncoded transmission, one should also take into account the effects of imperfect carrier recovery: in fact, by doubling of the signal constellation size, as required by Ungerboeck's codes, the transmission becomes more sensitive to phase offsets and jitter (see, e.g., [1]).

The aim of this paper is to examine the effects of imperfect carrier phase recovery on uncoded 4-PSK and trellis-coded 8-PSK. Recently, Mitsubishi's T. Fujino proposed a new carrier recovery system based on the multiplication of the received signal by a waveform derived from the timing clock signal; by adding a relatively slight complexity it is possible to diminish the phase jitter and to eliminate partially the drawback of the coded scheme. The transmission model that we assume is a single-channel digital system, perturbed by intersymbol interference and additive white Gaussian noise on the down link. Actually, we can disregard the additive noise when considering the synchronization circuit since in a narrow-banded environment data noise is the most relevant disturbance. The presence of the additive noise will be taken into account in the evaluation of error probability, which we shall base on compuer simulation [5].

Our goal is twofold. First, to determine the amount of improvement that can be obtained using the clock-aided (CA) recovery system in both coded and uncoded environments, and how critical its use can be. Second, to assess the difference in behavior that the two carrier phase recovery systems can show with and without coding.

The study is carried out using a blend of analysis and digital simulation. The quality of the carrier phase recovery is first evaluated by introducing a parameter called tone power to background power ratio (TBR) of the recovered carrier [6]. Then, the error probability of the transmission system is evaluated and used to determine how accurate the timing clock recovery must be in Fujino's circuit to allow it to outperform the standard circuit.

2. QUALITATIVE ANALYSIS

The transmission system and the carrier recovery scheme considered in [1] are shown in Fig.2.1a and 2.1b. The input IF-signal $y(t)$, after passing through a band-pass filter, is amplitude-modulated by the recovered clock $c(t)$. It is then multiplied x8 and supplied to the "narrow-band filter/amplitude limiter". The AFC removes the slow variations of the received carrier frequency. In Fig. 2.1c we show a simplified version of Fujino's circuitry, pointing out the relevant signals. Such a clock-aided system allows a higher output CNR (Carrier to Noise Ratio) in comparison to the usual systems (see Fig. 2.1d), and the gain increases for growing values of the input CNR [1] (see Fig. 2.2).

In the following we explain qualitatively the reasons why Fujino's carrier recovery scheme affords a performance improvement. The model we assume is a linear, noiseless, narrow-band channel. In particular, we consider raised-cosine modem filters, with a 0.5 roll-off and aperture equalization accounting for rectangular transmitted signals. The presence of intersymbol interference (ISI) due to the finite channel bandwidth causes data noise, which in turn degrades the quality of the recovered carrier. Consider in fact Fig. 2.1d: the signal present at the output of the x8-multiplier, after zonal filtering, can be written in the form

$$w(t) = -A\cos(8\omega_0 t) + a(t)\cos(8\omega_0 t) + b(t)\sin(8\omega_0 t) \quad (1)$$

where the first term is the useful one, whilst $a(t)$ and $b(t)$ are baseband, zero-mean random processes representing the in-phase and quadrature parts of the disturbance. The scattering diagram of $w(t)$, i.e., the representation of the evolution with time of the instantaneous phase and amplitude of the complex envelope $\tilde{w}(t)$, $-\infty<t<\infty$, shows clearly the effects of the data noise. The scattering diagram of an unmodulated signal would reduce to just one point, located on the negative real axis. On the contrary, for a real channel, and without multiplication by the clock signal, the vector trajectories encircle the origin of the axes (see Fig. 2.3). This means that at some instants the signal phase is opposite to the right one, which causes a certain amount of phase distortion in the band-pass filtered signal. As we have assumed a Nyquist channel, a sequence of instants $t_k=\tau+kT$ (k an integer, T the symbol period) exists in which the received signal is not perturbed by ISI, i.e., $a(t_k)=b(t_k)=0$. Let us define $c(t)$ as

$$c(t) = \begin{cases} 1, & t \in [t_k-\delta, t_k+\delta] \\ 0, & \text{elsewhere,} \end{cases} \quad (2)$$

where δ must be less than T/2 (e.g., δ=0.1 T) . If we multiply the received signal $y(t)$ by $c(t)$, we obtain the signal $w'(t)$ (see Fig. 2.1c), whose scattering diagram is shown in Fig. 2.4. It can be easily seen that the effect of amplitude modulation is to bring the vector magnitude down to zero when its phase is around zero radians. Intuitively, this implies that the phase jitter, after band-pass filtering, is reduced with respect to the previous case. This effect can be observed by inspection of the scattering diagrams of the signals $v(t)$ and $v'(t)$, shown in Fig. 2.5 : the point spread is considerably reduced when the clock-aided carrier recovery scheme is used. By reducing the parameter δ, the dispersion decreases, but at the same time the amplitude of the recovered carrier is reduced as well. These conclusions hold true even when the signal used for the multiplication is different from the one considered before. In particular,we can consider the family of waveforms

$$c_1(t) = \tfrac{1}{2} \{ 1 + \cos[2\pi(t-t_0)/T] \}$$
$$c_n(t) = [c_1(t)]^n, \quad \text{n any positive integer.} \quad (3)$$

As n increases, if $t_0=\tau$, multiplication by one of these waveforms causes the phase jitter to decrease. As a limiting case, a train of ideal impulses can be considered (for mathematical reasons, multiplication by the clock signal is to be performed after the 8th-order nonlinearity):

$$c_\delta(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT-t_0) \quad (4)$$

(In (4), $\delta(t)$ denotes the Dirac generalized function). If $t_0=\tau$ the received signal is sampled exactly when the ISI is zero. Hence, we have

$$w'(t) = w(t)\cdot c_\delta(t) = -A\cos(8\omega_0 t)\cdot c_\delta(t) \quad (5)$$

because $a(t_k)=b(t_k)=0$. The amplitude spectrum is:

$$W'(f) = -(A/2T)\sum_{k=-\infty}^{\infty} [\delta(f-8f_0-k/T)+\delta(f+8f_0-k/T)]\cdot$$
$$\cdot e^{-j2\pi k t_0/T} \quad (6)$$

which becomes, after band-pass filtering:

$$V'(f) = -(A/2T)\ [\delta(f-8f_0)+\delta(f+8f_0)] \quad (7)$$

which is exactly the amplitude spectrum of a cosinusoidal signal not affected by jitter. This last result is a further confirmation of the aforementioned conclusions. It must be observed that this clock-aided recovery system can be used not only for coded 8-PSK, but also for uncoded 4-PSK; on the other hand, we shall see in the sequel that the improvements obtainable are more appreciable for an 8-PSK than for a 4-PSK.

3. ANALYSIS AND RESULTS

We want now to determine the amount of gain obtainable by inserting the clock multiplication in both cases: 4-PSK and coded 8-PSK. The two analyses follow the same path so we will focus our attention on 4-PSK.

In Fig.3.1 we describe the model of the transmission system:

- the source emits an i.i.d. sequence (α_k) $-\infty<k<\infty$ of quaternary symbols;
- the 4-PSK modulator sends out the signal $x(t)$

$$x(t) = \Sigma_k\, u_T(t-kT)\cos(\omega_0 t+\varphi_k) \quad (8)$$

  where φ_k is chosen from the set $\{\pm\pi/4,\pm3\pi/4\}$ according to the value taken by α_k, and $u_T(t)=1$ for $0\le t<T$, $u_T(t) = 0$ elsewhere;
- the channel is modelled as a raised cosine filter, whose impulse response is $h(t)$;
- the instant within the symbol interval when there is no ISI is $\tau=T/2$.

The power of the recovered carrier is not a sufficient measure of the system efficiency : the quadrature part $b(t)$ of the signal $w(t)$ (see Fig.3.1) gives rise to background noise, which in turn produces phase jitter [6]. Our aim is to reduce the power of $b(t)$ at the frequencies around $4f_0$ as compared to the power of the carrier. It is therefore justifiable to use as a merit figure the tone power to background power ratio TBR :

$$TBR_4 = \hat{G}^d_w(4f_0)/G^c_{qw}(4f_0) \quad (9)$$

where the numerator is the amplitude of the discrete power spectrum of $w(t)$ at the frequency $4f_0$ and the denominator is the value of the continuous power spectrum of the quadrature component of $w(t)$. Note that TBR is measured in hertz.

In a similar way it is possible to evaluate TBR_8 for a coded 8-PSK, even if it is more cumbersome because of the correlation among symbols. The code we dealt with is the Ungerboeck code with 4 states and constraint length 2 (see [2]). The transmission system model is described in Fig.3.2 .
The TBR can be defined as for the 4-PSK :

$$TBR_8 = \hat{G}^d_w(8f_0)/G^c_{qw}(8f_0) \quad (10)$$

We calculated TBR_4 and TBR_8 in case of clock aided recovery scheme using the waveforms $c_1(t)$, $c_2(t)$ and $c_\delta(t)$ with different values of t_0, and we compared these values with those obtained without clock multiplication. At the same time we simulated these different transmission systems and estimated the power spectra of the signals after the nonlinearity. Analysis and simulation have shown a good agreement.

There are two aspects of the problem to take into consideration: the first one is how coding changes the power spectrum of the signal, and the second one is how the multiplication by the different clock signals influences the values of TBR_4 and TBR_8. The results obtained are the following:

- Before the nonlinearity the power spectra of the signals deriving from 4-PSK, 8-PSK and coded 8-PSK are equal [4].
- At the output of the nonlinearity the power spectra are all different from each other.
- The discrete part of the spectrum does not change if coding is inserted.
- TBR_8 evaluated for the coded system is slightly lower as compared to that of the uncoded system, but the difference never rises above 1 dB so that it is practically negligible (see Fig. 4.1).
- If the multiplying signal is centered so that $t_0=T/2$, then it is possible to observe a notch in the continuous quadrature power spectrum of $w'(t)$ and a consequent increase in TBR (see Fig.4.2a and 4.2b for the case of coded 8-PSK).
- If we compare the value of TBR_8 obtained in absence

of multiplication with that obtained with multiplication by $c_1(t)=\frac{1}{2}[1+\cos(2\pi(t-T/2)/T]$, then we can see that in the second case TBR_8 is larger, and the increase in TBR_8 reaches the value of 23 dB. In the same situation, the increase for TBR_4 is of about 17 dB. However, the absolute values are such that TBR_4 always exceeds TBR_8 (see Fig.4.3).

- If the clock used is $c_n(t)$, $n>1$, then the maximum values of TBR_8 and TBR_4 increase with the value of n. If $c_\delta(t)$ is used, then it is theoretically possible to bring TBR to infinity by maintaining t_0 equal to T/2 (see Figs.4.4 and 4.5).
- Whichever clock is used, there is a restricted range of values of t_0 such that TBR is greater than without multiplication. This range slightly diminishes as n increases.
- By simulation, we measured the phase variance at the output of a Butterworth filter in cascade to the nonlinearity (BPF in Fig.2.1b) for all the different carrier recovery systems. The results confirmed what we deduced on the basis of TBR.

In the sequel we give the explanation of this behavior. In T seconds the 4-PSK signal can change its phase by $\pm\pi/2$ or π. This means that, after frequency multiplication, the phase of $\tilde{y}^4(t)$ changes by 2π or 4π, so that the vector representing the complex envelope crosses the positive real axis once or twice during T. Clearly, in the second case the vector must move more quickly than in the first case because it encircles the origin twice instead of once; therefore, if in the first case, the vector phase takes Δt to pass from π to $\pi-\varphi$, for a given $|\varphi|<\pi$, necessarily in the second case it takes $\Delta t'<\Delta t$. The aim of clock multiplication should be to bring to zero the vector when its phase does not belong to $[\pi-\varphi,\pi+\varphi]$. Therefore in the second case it would be necessary to use a clock signal with a window width equal to $2\Delta t'$ instead of $2\Delta t$. As a matter of fact, just $\frac{1}{4}$ of all the possible signal transitions give rise to a π phase change. In the case of an 8-PSK signal, the possible phase changes and their probabilities are:
$\pm\pi/4$ ($p=\frac{1}{4}$); $\pm\pi/2$ ($p=\frac{1}{4}$); $\pm3\pi/4$ ($p=\frac{1}{4}$); π ($p=\frac{1}{8}$).
In terms of $\tilde{y}^8(t)$, these phase changes correspond to 1,2,3 and 4 crossings through the positive real axis, respectively. The probability that the vector crosses the positive real axis more than once in T seconds is 5/8, so it is possible to assess that, on the average, the vector $\tilde{y}^8(t)$ moves more quickly than the vector $\tilde{y}^4(t)$. This means that, in order to keep the phase of the signal v'(t) in the same interval around π, it is necessary to use a narrower window for an 8-PSK than for a 4-PSK modulation.

From figures 4.4 and 4.5 it can be seen that, if the multiplying signal is not correctly centered, then the clock-aided carrier recovery system can give a performance worse than the usual system. Moreover, it is impossible to maintain the clock delay t_0 fixed and equal to T/2 during the whole transmission. Under the assumption that t_0 is a Gaussian random variable with mean value T/2 and variance $\sigma_{t_0}^2$ ($\Pr(|t_0-T/2|>T/2)\cong0$), we evaluated a lower bound for the bit error probability in both cases of 4-PSK and uncoded 8-PSK. Let φ be the carrier instantaneous phase at the output of the frequency divider. Then, under the assumptions that φ has zero mean and small variance, we have that the bit error probability conditioned on a given value of t_0 can be approximated by :

$$P(e|t_0) \cong P(e|\varphi=0,t_0) + \frac{d^2}{d\varphi^2} P(e|\varphi,t_0)\Big|_{\varphi=0} \sigma_\varphi^2 . \quad (11)$$

For a fixed value of t_0, it is possible to estimate σ_φ^2 by simulation and therefore evaluate $P(e|t_0)$: from the plots of $P(e|t_0)$ as function of t_0, we have seen that the interval in which the clock-aided carrier recovery system outperforms the usual one is of about $0.25\cdot T$ for 4-PSK and $0.3\cdot T$ for 8-PSK. Moreover, the different clock signals used give rise to small differences. In order to compute P(e), it is necessary to perform an average on t_0, which can be obtained using the Gauss-Hermite quadrature formulas. We evaluated P(e) with different values of $\sigma_{t_0}^2$, for 4-PSK and 8-PSK carrier recovery systems with and without clock multiplication. The results are the following:

- The interval of t_0 in which CA-σ_φ^2 remains smaller than non-CA-σ_φ^2 is wider for 8-PSK than for 4-PSK modulation. This means that the improvements that can be obtained with the clock aided carrier recovery system are more relevant for 8-PSK than for 4-PSK.
- By changing the value of $\sigma_{t_0}^2$, only a slight difference can be observed in P(e) by passing from $c_1(t)$ to $c_\delta(t)$: in fact, apart from a narrow interval around $t_0=T/2$, the values of $\sigma_\varphi^2(t_0)$ (and TBR) are almost the same independently of the clock used.

One conclusion that we can draw is that, instead of using a 4-PSK modulation, it is convenient to adopt Ungerboeck codes with Fujino's carrier recovery system using $c_1(t)$ as multiplying signal, provided that the clock recovery system is accurate enough. The analysis techniques we developed should allow one to evaluate the required accuracy for the clock.

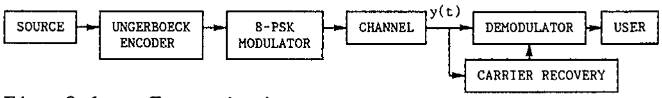

Fig. 2.1a - Transmission system

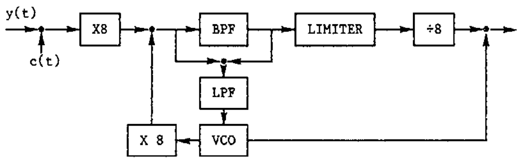

Fig. 2.1b - Fujino's carrier recovery scheme

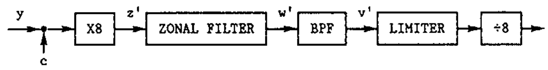

Fig. 2.1c - CA recovery system

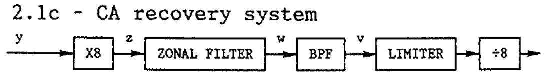

Fig. 2.1d - Non-CA recovery system

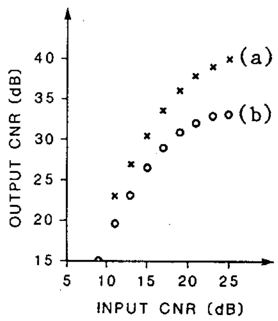

Fig. 2.2 - CNR characteristic of X8 multiplier : a) CA system, b) Non-CA system (from [1])

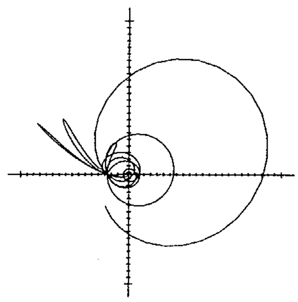

Fig. 2.3 - Scattering diagram of signal w(t)

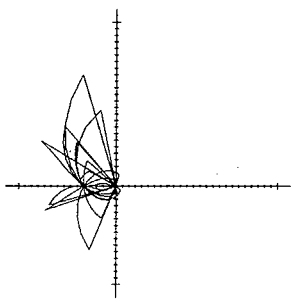

Fig. 2.4 - Scattering diagram of signal w'(t), $\delta=0.2T$

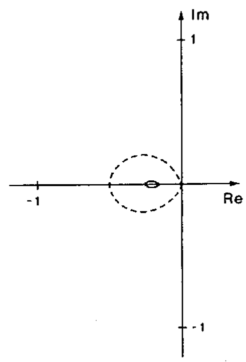

Fig. 2.5 - Scattering diagrams of signals v(t) (dotted line) and v'(t) (continuous line)

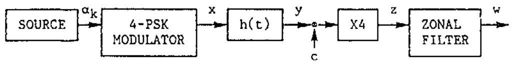

Fig. 3.1 - 4-PSK transmission and carrier recovery systems

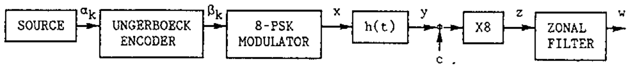

Fig. 3.2 - Coded 8-PSK transmission and carrier recovery systems

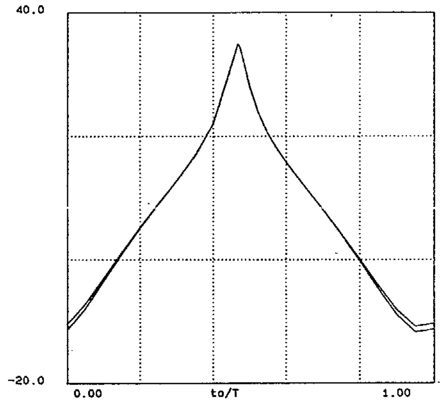

Fig. 4.1 - TBR_8 for coded (lower curve) and uncoded systems (upper curve); $c(t)=c_1(t)$

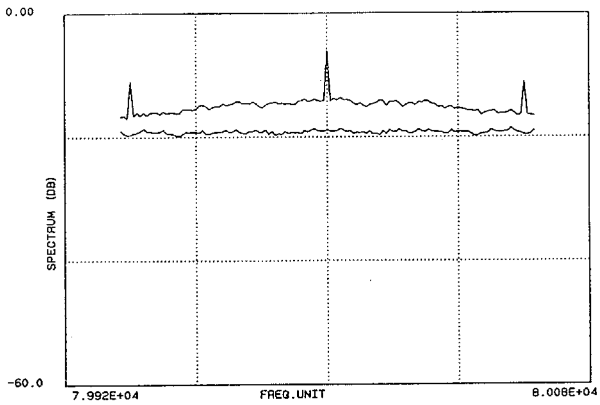

Fig. 4.2a - Power spectrum of w(t) (upper curve) and of the quadrature component of w(t) (lower curve).

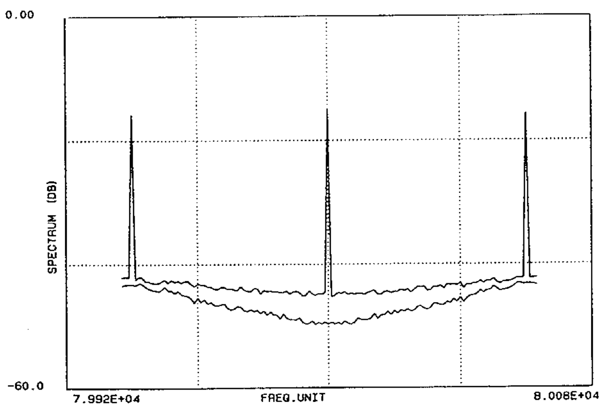

Fig. 4.2b - Power spectrum of w'(t) (upper curve) and of the quadrature component of w'(t) (lower curve); $c(t)=c_1(t)$ with $t_0=T/2$

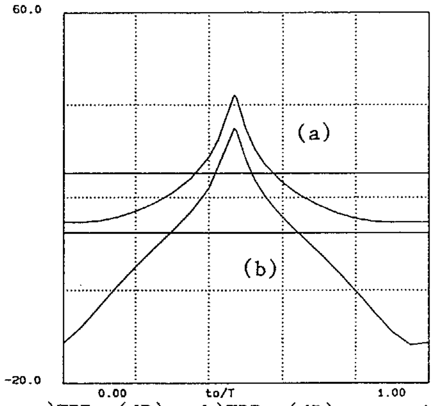

Fig. 4.3 - a)TBR_4 (dB), b)TBR_8 (dB); straight lines refer to Non-CA systems, the others to CA systems with $c(t)=c_1(t)$

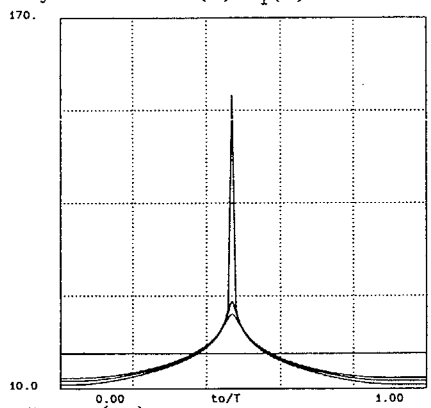

Fig. 4.4 - TBR_4 (dB) with different c(t) : $c_1(t)$ (lower curve), $c_2(t)$ (middle curve), $c_\delta(t)$ (upper curve)

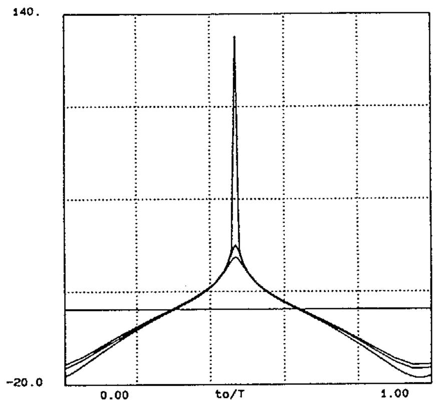

Fig. 4.5 - TBR_8 (dB) with different c(t) : $c_1(t)$ (lower curve), $c_2(t)$ (middle curve), $c_\delta(t)$ (upper curve)

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