

## NOVEL METHODS FOR HARMONIC FREQUENCY TRACKING

Yu Hen Hu\* and B.C. Phan

Electrical Engineering Department  
 Southern Methodist University  
 Dallas, Texas 75275  
 U.S.A.

## RESUME

Ce document discute deux methodes, le filtre Kalman (KF) et une detection technique eventuelle pour decouvrir les signaux harmoniques avec le variable temps du developpement de bruit. Premierement, en supposant que les sinusoides additionnes du bruit modele, nous employons une methode de Pisarenko (PHD) pour la decomposition harmonique afin d'eliminer les sinusoidales frequences. Ensuite, l'evaluation du courant de frequence est alimentee dans les algorithmiques KF ou ED pour l'apres-developpement. Le but est de trouver une trajectoire adoucie des frequences harmoniques. La trajectoire evaluee facilite la prediction des valeurs des frequences. En terme, cette information pourra etre alimentee en retour pour la methode PHD adaptee comme une nouvelle condition initiale. Esperablement, en faisant ainsi, les meilleures evaluations de frequence peuvent etre obtenues.

## I. INTRODUCTION

In many adaptive estimation problems, it is often assumed that the additive observation process is a (wide sense) stationary random process with constant variances. Hence, the signal-to-noise ratio (SNR) remains to be constant even the harmonic frequency may vary with time. However, in practical applications such as sonar or radar, where harmonic frequencies tracking techniques are employed, it is not unrealistic to assume time-varying SNR which accounts for the variations of the estimated signal strengths with respect to ambient noise. This problem not yet been fully addressed in high resolution spectrum estimation researches. The major difficulty is to explore parameter changes at the presence of noise. There is always trade off between sensitivity (to change of underlying parameters) and stability (immune to noise disturbances) in adaptive signal processing. However, in many situations, there are additional information about the behavior of the harmonic frequencies which may help better predicting, estimating the actual frequency trajectories. What is needed, then is a technique to incorporating such information into the estimation procedure. In this paper, two potentially promising techniques are investigated for this purpose.

Briefly speaking, we take on a two phase approach: First use an adaptive Pisarenko Harmonic Decomposition (PHD) method to obtain a crude estimate of the frequencies. In general, the result will be quite noisy due to short data record and SNR variations. Then the results will be fed into a post-processing unit to unveil the trajectory information of the underlying frequencies. Two methods are employed for experimentation. One is the conventional Kalman Filter (KF) algorithm, the other is a new Event Detection (ED) algorithm. Both of these two methods can handle time varying data sequences

## ABSTRACT

This paper discusses two methods, the Kalman filter (KF) and an event detection (ED) technique, for tracking harmonic signals. First, by assuming a sinusoids plus noise model, we employ an adaptive Pisarenko's harmonic decomposition (PHD) method for estimating sinusoidal frequencies. Next, the current frequency estimates is fed into KF or ED algorithms for post-processing. The purpose is to find a smoothed trajectory of harmonic frequencies. The estimated trajectory facilitates prediction of next values of frequencies. In term, this information will be fed back to adaptive PHD method as new initial condition. Hopefully, in doing so, the better frequency estimates can be obtained.

and hence seem to be good candidates. However, they differ in a number of places: the Kalman filter algorithm is a statistical approach which requires a dynamic system model to describe the behavior of the frequency. Each time, an optimal estimate will be computed. Event detection method, on the other hand, employs an artificial intelligence planning strategy called *least commitment*. The idea is to hold off a decision until the last minute. As a result, at each time instant, a set of line segments will be kept to represent all possible choices (with certain confidence). As new data are observed, the set of feasible solution will be pruned out leaving a set of more restricted solutions. We intend to compare the performance of both these methods.

Below, in section II, the adaptive PHD method will be briefly surveyed. Then, the Kalman filter formulation will be presented in section III. The event detection procedure will be discussed in section IV.

## II. ADAPTIVE PISARENKO HARMONIC DECOMPOSITION FOR ESTIMATION ESTIMATION

Let us begin by defining a harmonic process model for the time-varying tracking problem. Assume that the observed time series data  $\{x(t)\}$  is composed of  $p$  complex sinusoids with additive noise  $\{w(t)\}$ :

$$x(t) = \sum_{i=1}^p A_i e^{j\omega_i t + \phi_i} + w(t) \quad (1)$$

where  $(A_i, \omega_i, \phi_i)$  denote the amplitude, the frequency, and the initial phase of the  $i$ -th sinusoidal signal.  $\{w(t)\}$  is assumed to be a white, Gaussian, zero mean process with an unknown variance  $\{\sigma^2\}$ . For the frequency track-



ing problem, often one has to assume  $\omega_i$  changes slowly with respect to time, and hence should be denoted as  $\omega_i(t)$ . With this model, the objective is to estimate the current harmonic frequency  $\hat{\omega}_i(t)$  from the observations  $\{x(t)\}$ .

Our approach is this: First, based on the observations  $\{x(t)\}$ , a raw frequency estimate  $y(t)$  will be computed using an adaptive PHD method. Then,  $y(t)$  will be fed into one of the two frequency tracking algorithms (namely, KF and ED) to produce an improved frequency estimate  $\hat{\omega}_i(t)$  as well as a prediction of the next frequency estimates  $\hat{\omega}_i(t+1 | t)$ . Finally, the new frequency estimates will be fed back to the adaptive PHD algorithm to produce  $y(t+1)$ .

In this section, let us summarize the adaptive PHD method [4]. To take into account the effect of time varying frequencies, we compute a time varying sample covariance matrix of  $\{x(t)\}$  using the formula:

$$R_x(t+1) = \lambda R_x(t) + (1-\lambda)x(t+1)x^*(t+1) \quad (2)$$

$\lambda$  is called the forgetting factor. Suppose  $a(t)$  is current estimate of minimum eigenvector of  $R_x(t)$ . The new minimum eigenvector estimate can be calculated as:

$$b(t+1) = a(t) - \mu R_x(t+1)a(t) \quad (3)$$

$$a(t+1) = b(t+1) / |b(t+1)| \quad (4)$$

when  $\lambda = 0$ , above equation is known as the *adaptive line enhancer* [5]. Other adaptive PHD formula can also be applied [4].  $\mu$  is an adaptation constant whose value should be bounded by the inverse of the maximum eigenvalue of the  $R_x$  ( $\triangleq \mathbf{E}\{R_x(t)\}$ ) matrix [6] to ensure the convergence. For each  $a(t)$ , an eigen-polynomial

$$a(z) = [1 \ z \ z^2 \ \dots \ z^p] a(t) \quad (5)$$

can be formed. The roots of the equation  $a(z) = 0$  should locate on or close to unit circle  $|z| = 1$  on the complex  $z$ -plane. The arguments (angles) of these roots thus will be taken to be the current estimates of the harmonic frequency estimates  $\{y_i(t)\}$ .

It has been reported that the PHD method is sensitive to statistical and numerical perturbations [7]. This effect is amplified by the use of above adaptive formula for computing  $R_x(t)$ . As a consequence, the frequency tracking performance of the adaptive PHD method will be less satisfactory. Furthermore, steady state tracking error is also a function of the signal to noise ratio of the observations. Recently, we have found that the variance of the sample frequency estimates is a function of both the SNR and the underlying frequencies [7]. Specifically, at low SNR, the estimation error is large. While at large SNR, the estimation error tends to fluctuate with respect to frequencies. Therefore, the frequencies computed using adaptive PHD method can not properly portray the trajectory of the true frequencies. This motivates us to look into other approaches which may improve the performance.

For convenience, let us assume that there is only one real sinusoidal signal ( $p=2$ ),  $\omega(t) = \omega_1(t) = -\omega_2(t)$ . Thus, the subscript "i" in frequency estimates  $\{y_i(t)\}$  can be dropped. Now, each  $y(t)$  can be regarded as an imperfect (noisy) observation (computed from the adaptive PHD method) of the true frequency  $\omega(t)$ , with time varying variances. The objective is to find the best estimates of  $\omega(t)$  based on given  $\{y(t)\}$ .

Two approaches will be discussed in the following sections. The first method assumes an internal dynamic equation for  $\omega(t)$  and apply conventional Kalman filter to smooth the noisy observations  $\{y(t)\}$  in order to produce a set of smoothed observation  $\{\hat{\omega}(t)\}$ . The second method assumes the knowledge of the variance of  $y(t)$  for each  $t$ , and maintain a set of straight lines as piecewise linear approximation of the trajectory of  $\omega(t)$ . Thus, a range of possible values of  $\omega(t)$  will be computed rather than a single value. These methods are now outlined below:

### III. KALMAN FILTER FOR FREQUENCY TRACKING

Kalman filter (KF) has been applied to many target tracking problems in the past [8]. Briefly, Kalman filter is an optimal state estimator which assumes the observation (input) is generated from a known dynamic system. In this paper, we assume that the true frequency  $\omega(t)$  is the state of a dynamic system described by the following equations:

$$\omega(t+1) = \nu(t)\omega(t) + v_1(t) \quad (6)$$

$$y(t) = \omega(t) + v_2(t) \quad (7)$$

where  $v_1(t)$  is the system model driving noise and  $v_2(t)$  is the measurement noise. The state space matrix (scaler in this case)  $\nu(t)$  is assumed to have magnitude less than unity to ensure convergence. The model parameters  $\nu(t)$ ,  $\sigma_1^2(t)$  ( $\triangleq \text{Var}\{v_1(t)\}$ ), and  $\sigma_2^2(t)$  ( $\triangleq \text{Var}\{v_2(t)\}$ ) have to be estimated indirectly from frequency estimates  $\{y(t)\}$  as described later. With above state space model, the Kalman filter equation can be summarized as follows: First, the state vector update equation:

$$\hat{\omega}(t+1 | t) = \nu(t)\hat{\omega}(t | t-1) + G(t)[y(t+1) - \nu(t)\hat{\omega}(t | t-1)] \quad (8)$$

Here  $G(t)$  is the Kalman gain which is defined as:

$$G(t) \triangleq \frac{\nu(t)K(t | t-1)}{K(t | t-1) + \sigma^2(t)}$$

where the predicted state-error correlation matrix,  $K(t, t-1)$ , is updated according to the formula:

$$K(t+1 | t) = \nu(t)\sigma_2^2(t)G(t) + \sigma_1^2(t)$$

The scaler state space matrix  $\nu(t)$  and state driving noise variance  $\sigma_1^2(t)$  can be estimated from the sample covariance lags of the state vector estimates  $\hat{\omega}(i | i)$  ( $i < t$ ). The covariance lags are computed iteratively with an forgetting factor  $\lambda_1$ :

$$r_{\omega}(m, t+1) = \lambda_1 r_{\omega}(m, t) + (1-\lambda_1)\hat{\omega}(t | t)\hat{\omega}^*(t-m | t-m) \quad (9)$$

where  $m = 0, 1$ .  $\hat{\omega}(t | t) \triangleq \hat{\omega}(t+1 | t)/\nu(t)$ . With  $r_{\omega}(0, t+1)$  and  $r_{\omega}(1, t+1)$ ,  $\nu(t+1)$  and  $\sigma_1^2(t+1)$  are computed as:

$$\nu(t+1) = r_{\omega}(1, t+1)/r_{\omega}(0, t+1) \quad (10)$$

$$\sigma_1^2(t+1) = r_{\omega}(0, t+1) - \nu(t+1)r_{\omega}(1, t+1) \quad (11)$$

These approximation equations are derived by assuming  $\{\omega(t)\}$  to be a first order Auto-Regressive (AR) process where  $\nu(t)$  is the first temporal correlation coefficient and  $\sigma_1^2$  is the temporal driving noise variance.

The variance of observation noise process,  $\sigma_2^2(t)$  is estimated iteratively from the prediction error term  $[y(t) - \hat{\omega}(t | t-1)]$ :

$$\sigma_2^2(t+1) = \lambda_2 \sigma_2^2(t) + (1 - \lambda_2)[y(t) - \hat{\omega}(t | t-1)]^2 \quad (12)$$



Using the KF prediction formula (8), each time an  $\hat{\omega}(t+1 | t)$  will be computed for given  $y(t)$  as a prediction of the next frequency  $\omega(t+1)$ . This estimation is sub-optimal since the model parameters  $\nu(t)$ ,  $\sigma_1^2(t)$ , and  $\sigma_2^2(t)$  are unknown and have to be estimated from data. To make use of the prediction, we may form a predicted eigen-polynomial

$$\hat{a}(t,z) \triangleq \prod_{i=0}^p (z - \exp(j\hat{\omega}_i(t+1 | t))) \quad (13)$$

The coefficient vector of  $\hat{a}(t,z)$ , denoted by  $\hat{a}(t)$ , then will replace the  $a(t)$  in (3) to facilitate the computation of new adaptive PHD estimate.

*Discussion:* The KF approach outlined above essentially performs two tasks: (a) To smooth the frequency estimate trajectory via the robust update formula (8). (b) To provide a new initial condition for each adaptive PHD iteration which depends directly upon the frequency estimate. Because of (b), a prior information about the system dynamics of  $\omega(t)$  can be effectively utilized to improve the frequency estimate. Finally, the time varying formulation of the KF is very suitable to practical situations where time varying SNR is encountered.

#### IV. EVENT DETECTION METHOD FOR HARMONIC FREQUENCY TRACKING

Event detection (ED) method [3] is a method derived to track trajectories of multiple objects. In this method, it is assumed that each observation  $y(t)$  will experience some measurement error with known upper and lower bounds. That is, there exist  $\alpha_t$  and  $\beta_t$  such that

$$\alpha_t \leq y(t) \leq \beta_t$$

The objective of the ED method is to produce a piecewise linear approximation of the trajectory  $\{y(t)\}$  as a function of time  $t$ . It is required that the line segment at time  $t$  should also be bounded by  $[\alpha_t, \beta_t]$ . It is also desired to minimize the number of line segments used. The unique feature of the ED method is that at each time, the set of all feasible solutions (line segments) will be maintained. Each additional data observation  $y(t)$  will impose additional constraints on the set of current feasible line segment and therefore reduce the number of feasible line segments. If the feasible solution set reduces to an empty set, it produces an event. In general, an event signals that the new data deviates significantly from its previous course and a new line segment should be assigned to fit the following data. The ED method can easily be generalized to handle other pre-determined trajectories.

The key of the ED method is to find the error interval  $\{\alpha_t, \beta_t\}$ . In this paper, we assume:

$$y(t) - \alpha_t = \beta_t - y(t) = \text{Std.}\{y(t)\}$$

where  $\text{Std.}\{y(t)\}$  stands for the standard deviation of  $y(t)$ . Hence, the interval  $[\alpha_t, \beta_t]$  can be regarded as a confidence interval of the estimate  $y(t)$ . In adaptive PHD method, the standard deviation of  $y(t)$  is not available. Two methods can be used to estimate this quantity: First method is to compute the sample mean and hence sample variance of  $y(t)$  using, perhaps, an exponential window:

$$\text{Var}\{y(t)\} = \lambda_3 \text{Var}\{y(t-1)\} + (1 - \lambda_3)[y(t) - \bar{y}(t)]^2$$

$$\bar{y}(t) = \lambda_3 \bar{y}(t-1) + (1 - \lambda_3)y(t)$$

The second method relies on a recent result reported by the authors regarding the empirical variance of PHD frequency estimates as a function of frequencies and SNR. Thus, by table look-up and interpolation, it is possible to estimate roughly what the variance of  $y(t)$  will be. In practical application, the first method seems more favorable as it relates directly to sample data.

With the knowledge of  $\text{Var}\{y(t)\}$ ,  $\alpha_t$  and  $\beta_t$  can be estimated for each  $y(t)$ . This facilitates the application of the ED method: Let  $y(t) = mt + b$  be the family of all the straight lines passing through  $y(t)$  with slope  $m$ . Clearly,

$$\alpha_t \leq mt + b \leq \beta_t$$

Equivalently, we have two inequalities:

$$b \leq -tm + \beta_t,$$

$$b \geq -tm + \alpha_t$$

In the  $(m,b)$  plane, above equations represent the interior region between two parallel lines both having the same slope  $-t$ . Each point with this region represents a feasible solution. Now, given the next observation  $y(t+1)$  and associated  $\{\alpha_{t+1}, \beta_{t+1}\}$ , a similar interior region can also be identified in the  $(m,b)$  parameter plane. However, these new parallel lines will have slope  $-t-1$  instead of  $-t$ . Thus, these two pairs of parallel lines form a quadrangle of which each interior point represents a feasible solution. As the next data  $y(t+2)$  arrives, a new set of parallel lines (with slope  $-t-2$ ) will be generated. The intersection between the two parallel lines and the interior of the quadrangle will be the set of feasible solution. As more and more data points are collected, the intersection will assume a polygon shape and reduce in size. To accomplish this, a polygon-update algorithm is developed. Due to space limitation, the detail of this algorithm has to be omitted in this paper. When, at last, some arrives such that the interior of corresponding parallel lines does not have any intersection with the polygon, an event will be flagged. An event is an indication that the new data (frequency) is about to change its value so that previous line segment can no longer fit this data point within the given bounds. At this moment, we will initiate a new line segment for the current  $y(t)$  and repeat previous procedure. The union of all the line segments thus will be a piecewise linear approximation of the frequency trajectory.

#### V. CONCLUSION

In this paper, two methods (KF and ED) are proposed to enhance the performance of adaptive Pisarenko harmonic retrieval techniques. Simulation results will be presented in the conference.

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