AN EFFICIENT IDENTIFICATION METHOD FOR MULTIVARIABLE SYSTEMS

M. CUNEO(') - G. TACCONI(") - A. TIANO(')

(') Istituto di Automazione Navale (IAN) - C.N.R. - GENOVA - ITALY
(") Department of Biophysical and Electronic Engineering (DIBE)- UNIVERSITY OF GENOVA - ITALY

Ce travail, après une breve introduction au problème de la identification d'un système stochastique multivariable, présente une discussion critique des plusieurs algorithmes d'identification au point de vue de leur adéquation à résoudre le problème de l'identification.

Comme résultat de tel analyse on apporté quelques modifications au méthode de minimization sans le calcul des derivées de Powell, au but d'obtenir une méthode d'identification globalement efficace spécifiquement projetée pour systèmes multivariables à grand dimension.

La méthode d'identification proposée a étè verifiée en utilisand tant de données simulées que de données réellés. Ces données ont été produites pendant expériences d'identification pour ce qui concerne différentes sorts de vehicules marins.

Ces résultats obtenuts, qui sont expliqués relativement à un navire de surface et un hydrofoil, resemblent assez encourageants et montrent un attitude plutôt satisfaisant de la méthode proposée.

This paper, after a short introduction to the identification problem of a linear multivariable stochastic system, presents a critical discussion about different minimization algorithms, from the point of view of their suitability to cope with the identification problem.

As a result of such analysis, a number of suitable modifications are introduced to the derivative-free minimization algorithm proposed by Powell, in order to obtain an efficient identification procedure specifically designed for high dimensional multivariable systems.

The proposed identification method has been checked out by using simulated as well as real data files. Such files were generated by identification experiments carried out on different types of marine vehicles.

The obtained results, which are illustrated for a surface ship and a hydrofoil craft, seem to be very encouraging and indicate quite a satisfactory behaviour of the proposed method.

1. INTRODUCTION

The problem of identification and system parameter estimation of a multivariable dynamical system is quite a complex task, even in the case when relatively low dimensions of the system input and output vectors are involved.

Such a task, in the case of a linear stochastic system, is usually carried out by utilizing a Maximum Likelihood criterion, based on maximization of the likelihood function depending on the experimental input-output data as well as on the unknown parameter vector.

In order to solve the non linear optimization problem associated with identification, it is necessary to resort to advanced numerical methods, the role of which is very important within the whole identification procedure, since the required computations are generally quite cumbersome and time-consuming. This consideration is particularly relevant in the case of multivariable systems which often give rise to the estimation of a high dimensional unknown parameter vector. In this case it is important to analyse optimization algorithms on the basis of tests with a large number of variables, since many methods which are tolerably efficient with few variables may become totally unacceptable with many.

In this paper we present an identification method based on Powell's optimization algorithm. The method has been tested for identifying the multivariable dynamics of a number of marine vehicles. As it will be shown later, the method has been proved to be quite efficient even in the presence of a large number of parameters to be estimated.

2. THE IDENTIFICATION PROBLEM

Let us consider a linear stochastic continuous time system observed at discrete time instants

$$\dot{x}(t)=A(\theta)x(t)+B(\theta)u(t)+w(t) \qquad (1)$$

$$y(t_k)=Cx(t_k)+e(t_k) \qquad (2)$$

where $x(t)\in R^n$ denotes the system state vector, $u(t)\in R^m$ is the input control vector, $y(t_k)\in R^r$ is a discrete-time observation of the output vector taken at the sampling time t_k, $w(t)\in R^n$ is a continuous-time disturbance vector and $e(t_k)\in R^r$ is a discrete-time measurement noise. Matrices $A(\theta)$, $B(\theta)$ and $C(\theta)$ have proper dimensions. It is assumed that the noise vectors $w(t)$ and $e(t_k)$ are uncorrelated zero-mean gaussian white processes having covariance matrices $R_1(\theta)$ and $R_2(\theta)$.

We assume that the ignorance about the system is represented by the unknown parameter vector $\theta\in R^n$.

The identification problem consists of estimating the unknown parameter vector on the basis of N observations of the output vector $\{y(t_k)\}$ and of the input vector $\{u(t_k)\}$, k=1,N.

This estimation is usually carried out by a Maximum Likelihood approach, which attempts to maximize the likelihood function associated to the experimental data with respect to the unknown parameter vector, see /1/ and /2/.

Under the hypothesis of stationarity of the disturbances, uniformity of the sampling period $h=t_k-t_{k-1}$ and step-wise constancy of the input signal between two subsequent samplings, it can be shown that maximization of the likelihood function is equivalent to the minimization of a cost function given by:

$$J(\theta)=\det\sum_{k=1}^{N}\varepsilon(t_k)\,\varepsilon'(t_k) \qquad (3)$$

where $\varepsilon(t_k)$ denotes the innovation vector, which is obtained as a difference between the observed output vector and the one-step prediction vector $\hat{y}(t_k/t_{k-1})$:

$$\varepsilon(t_k)=y(t_k)-\hat{y}(t_k/t_{k-1}) \qquad (4)$$

The one-step prediction vector is obtained by a set of Kalman filter recursive equations:

$$\hat{y}(t_k/t_{k-1})=C\hat{x}(t_k/t_{k-1})$$

$$\hat{x}(t_{k+1}/t_k)=\Phi\hat{x}(t_k/t_{k-1})+\Gamma u(t_k)+K(t_k)\ \varepsilon(t_k)$$

$$K(t_k)=\Phi P(t_k/t_{k-1})C'\ R^{-1}(t_k)$$

$$P(t_{k+1}/t_k)=\Phi P(t_k/t_{k+1})\Phi'+R_1-K(t_k)\left[\Phi P(t_k/t_{k-1})C'\right]'$$

$$R(t_k)=CP(t_k/t_{k-1})C'+R_2 \qquad (5)$$

where

$$\Phi=e^{Ah} \quad , \quad \Gamma=\left[\int_0^h e^{As}ds\right]\cdot B \qquad (6)$$

As it can be easily recognized, the minimization of cost function (3) under constraints (4) to (6), is a rather difficult job, particularly in the case of a multivariable system with many input and output variables.

It is worth noting, furthermore, that such a minimization should be carried out with the aim of locating the global minimum of the likelihood function and not just simply a local one. In fact, only the global minimum provides an estimate with good statistical properties like unbiasedness, consistency and efficiency.

3. THE MINIMIZATION METHOD

The identification problem and the related minimization problem can be solved, at least in principle, by using standard minimization techniques, see for example /3/ and /4/.

We note, however, that a wide class of techniques which are based on the calculation of the hessian matrices of second order derivatives are not advisable for this purpose, owing to the following reasons:

(i) they are very burdensome from the computational point of view

(ii) the hessian matrix may result not positive definite

One possibility to obviate the first drawback consists in computing derivatives numerically by approximate methods. In the presence of a large number of variables, however, it may become very difficult to balance the influence of rounding errors and truncation errors when using finite differences to estimate derivatives. The latter inconvenience is very serious, since it may definitively prevent convergence of the algorithm to the global minimum, particularly when a large number of parameters have to be estimated.

It seems, therefore, that minimization methods which do not require derivatives should generally be preferable for multivariable identification.

These methods carry out minimization of the given cost function by a direct search strategy which attempts to reduce the value of the cost function by means of proper tests near a preset initial point. Such tests, which are based on function evaluations only, determine a direction of search in which the minimum is expected to be located. The procedure is iterated until convergence has been achieved.

Among the methods for minimizing a function depending on a large number of variables without the use of derivatives, Powell's method, /5/, has been demonstrated to be one of the most efficient and reliable, as reported in /7/ and /8/. The basic idea behind this method consists in searching along each of r mutually conjugate directions, which are defined by a set of r linearly independent vectors (r is the number of variables of the minimization problem).This algorithm constitutes an efficient iterative procedure for determining the r linearly independent conjugate vectors by means of a sequence of "one-at-a-time" searches in each of r sets of independent directions. The minimum of a quadratic function can thus be located exactly in r iterations. For a general function,convergence rate depends on the choice of the initial point, in the sense that an initial point "close" to the minimum gives rise to a fast convergence.

The algorithm which we actually use for multivariable identification incorporates two modification of Powell's method suggested by Zangwill, /6/ and Brent /8/.

Both modifications should prevent that some conjugate direction may become linearly dependent on the other ones. The former consists in changing in a proper way that conjugate direction which is responsible of linear dependence. The latter modification is based on a "restarting" procedure, which is applied every r iterations.

It has already been observed that convergence and convergence rate of the minimization algorithm may be critically affected by the choice of the initial point, which has to be as close as possible to the global minimum.

In order to obtain an initial point estimate, an opproximate identification routine is used, which basically is a deterministic approximation of the dynamical system represented by eq.(1) when the disturbance w(t) is neglected. We also assume, without loss of generality, that the initial state vector x(0)is zero.

In this case we are induced to minimize the least squares criterion:

$$J_1(\vartheta)=\sum_{k=1}^{N}\left[y_k-Cx_k(\vartheta)\right]'\left[y_k-Cx_k(\vartheta)\right] \qquad (7)$$

where $y_k=y(t_k)$ and $x_k(\vartheta)=x(t_k;\vartheta)$ denotes the state vector at k-th sampling interval:

$$x_k(\vartheta)=\sum_{i=0}^{k-1}\Phi^{i}\Gamma u_{k-i-1} \qquad (8)$$

Minimization of cost function (7) taking into account (8) is now a much simpler task in comparison with the original one and standard minimization techniques can be used for this purpose.

4. IDENTIFICATION RESULTS

The above described identification method has been used for determining the multivariable behaviour of a number of marine vehicles, including a surface ship, a hydrofoil and a submarine.

As concerns the surface ship, a set of simulated data obtained by trial manoeuvres of a 60.000 bulk-carrier have been analysed in order to test the validity of the method. During the simulation experiments,the rudder and the number of revolution of the propeller acted as control inputs and all the variables describing the ship dynamics in the horizontal plane, i.e. longitudinal speed, sway velocity, yaw rate and heading, were assumed as output variables. There were 11 parameters to be estimated in a state space model of the type represented by eqs.(1) and (2).

Since the values of the coefficients assumed in the simulation were known, the identified values of the coefficients could be compared directly with the known values. Quite a good agreement was found, as reported in /9/. Some plots of the measured and predicted output variables are shown in fig.1.

A number of identification experiments were carried out in the last years on board an RHS-160 class hydrofoil of Rodriquez Shipyard, which displaces 90 tons and has an overall length of 31 meters. During the sea trials an onboard computer applied suitable excitation sequences to the input variables constituted by four ailerons, while the rudder was changed manually. The same computer provided to record also the complete set of variables describing the hydrofoil dynamics, i.e. roll, pitch, yaw,heave, etc.
As described in /10/, the mathematical model contains 48 parameters to be estimated. If some couplings between state variables are neglected, the number of parameters to be estimated is 21.

The identification method was successfully applied also in this case and a good agreement was found between mathematical model and experimental data. In fig. 2 there are illustrated some plots of measured and predicted output variables.

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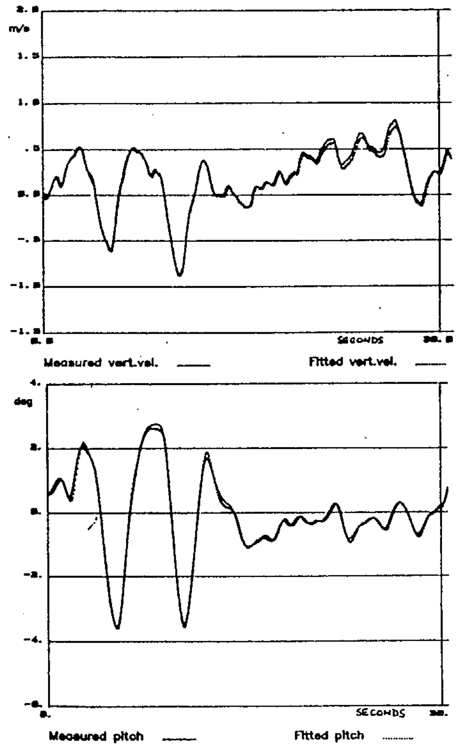
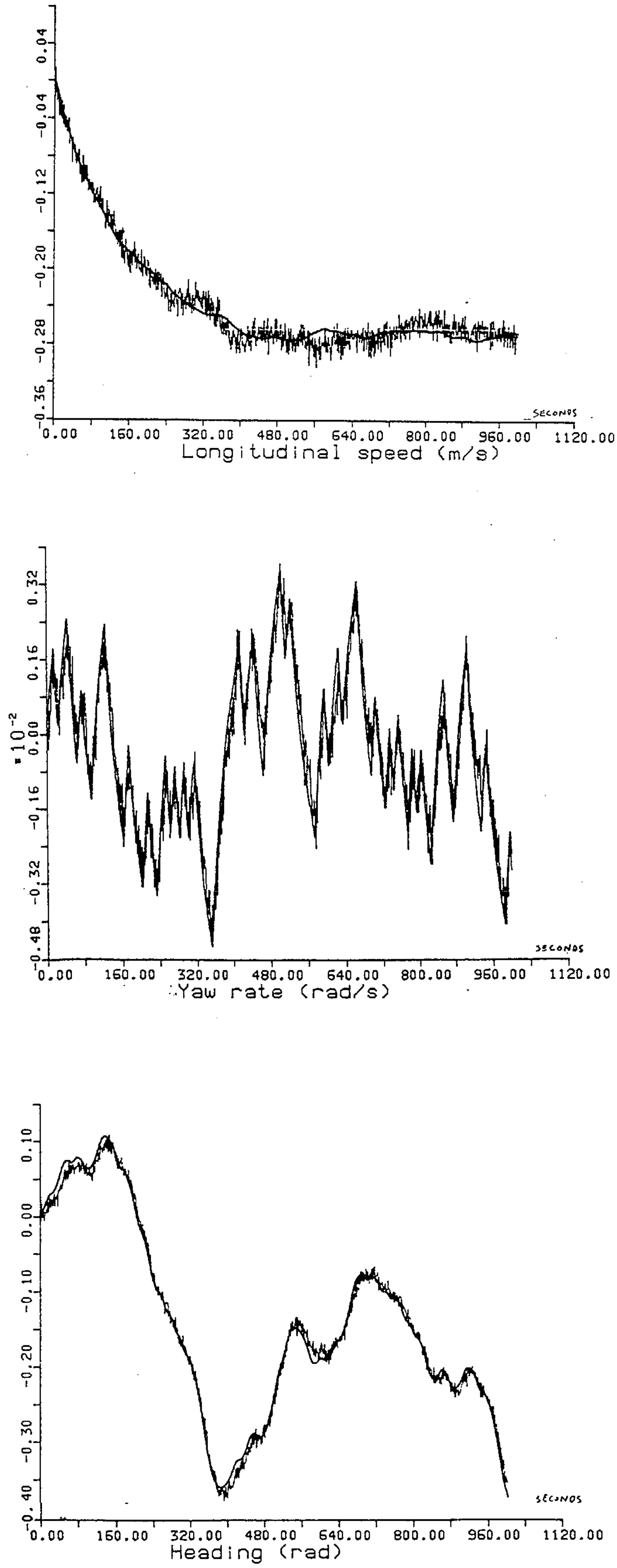


Plotting of one-step predicted and measured: vertical velocity/pitch

fig.2



Plotting of one-step predicted and measured: longitudinal speed/yaw rate/heading

fig.1