SIGNAL INTERCEPTION: PERFORMANCE ADVANTAGES OF CYCLE DETECTORS

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The problem of intercepting weak modulated random signals corrupted by unknown and changing broadband noise and narrowband interference is considered. The measure of output SNR called deflection is used to compare the performances of radiometric detectors and cycle detectors which detect on the basis of spectral lines regenerated from the signal. It is shown that cycle detectors can outperform radiometers when there is non-zero variability in the noise level or the interference power and the collection time is sufficiently long. It is also explained that for sufficiently broadband signals the radiometer that uses a maximum-likelihood estimate of the noise level can be outperformed by a cycle detector.

I. INTRODUCTION

The general study of cycle detectors, or cyclic-feature detectors, presented in [1] strongly suggests that cycle detectors can outperform radiometers for signal interception in the presence of unknown and changing noise level and interference activity. As explained in [1], the single-cycle form of feature detector uses a quadratic transformation to regenerate a spectral line from the received data at some non-zero frequency, and then it detects the presence of the signal by detecting the spectral line, which is present if and only if the received signal contains a cyclostationary component. In contrast to this, the radiometer uses a quadratic transformation that generates a spectral line at zero frequency regardless of whether or not the signal is present, and it attempts to detect the presence of the signal by distinguishing between the strength of the spectral line at zero frequency due to noise and interference alone and that (at zero frequency) due to the signal plus noise and interference. The distinct disadvantage of the radiometer lies in the difficulty of discriminating between these two strengths when the spectral-line strength due to the noise and interference alone greatly exceeds that due to the signal alone and moreover the larger spectral-line strength of the noise and interference is unknown and changing. The purpose of this paper is to carry out performance comparisons in order to substantiate the proposed superiority of cycle detectors in particularly adverse noise and interference environments.

To evaluate detection performance, we adopt deflection as a measure of detection performance and doubly stochastic stationary models for the noise and interference. Deflection is a measure of output SNR that is particularly useful for weak-signal detection as explained in [2]. The doubly stochastic processes are used to model the noise level (average power spectral density) and interference strength (average power) as random variables. Then deflection measures output SNR with numerator and denominator each averaged over the ensemble of possible noise levels and interference strengths. If the time-variations of local noise level and local interference strength were modeled as slowly varying stationary ergodic processes, then the ensemble-averaged performance would indeed properly reflect the more appropriate time-averaged performance.

II. PERFORMANCE COMPARISONS FOR RANDOM NOISE LEVEL

The detection problem considered here can be interpreted as the problem of testing the hypothesis H_0: $x(u) = n(u)$ against the hypothesis H_1: $x(u) = s(u) + n(u)$, for $u \in [t-T/2, t+T/2]$, where $x(t)$ is the collected data, $s(t)$ is the random cyclostationary signal to be detected, and $n(t)$ is doubly stochastic white Gaussian noise. That is, conditioned on knowledge of the value of the random power spectral density N, $n(t)$ is a white Gaussian process. The autocorrelation for the noise is $R_n(u,v) = E\{n(u)n(v)\} = \mu_N \delta(u-v)$, where μ_N is the mean of N, and $\delta(u-v)$ is the Dirac delta. The fourth joint moment of $n(t)$ (which is needed to evaluate deflection for a quadratic detector) is

$$E\{n(s)n(t)n(u)n(v)\} = (\sigma_N^2 + \mu_N^2)[\delta(s-t)\delta(u-v) + \delta(s-u)\delta(t-v) + \delta(s-v)\delta(t-u)], \qquad (1)$$

in which Isserlis' formula has been used [3], and where σ_N^2 is the variance of N. The random signal $s(t)$ has zero mean value and its autocorrelation can be expressed in the Fourier-series form

$$E\{s(u)s(v)\} = R_s(u,v) = \sum_\alpha R_s^\alpha(u-v)e^{i\pi\alpha(u+v)} \qquad (2)$$

by virtue of the cyclostationarity (or almost cyclostationarity) of $s(t)$ as explained in [1]. The Fourier-coefficient functions in (2) are given by

$$R_s^\alpha(\tau) \triangleq \lim_{Z\to\infty} \frac{1}{Z} \int_{-Z/2}^{Z/2} R_s(t+\tau/2, t-\tau/2)e^{-i2\pi\alpha t}dt. \qquad (3)$$

The index of summation α in (2) ranges over the harmonics (integer multiples) of the fundamental frequencies (reciprocal periods) of cyclostationarity, such as carrier frequencies, chip rates, baud rates, code repetition rates, and their sums and differences.

The detectors of interest each perform sliding (over time t) threshold tests

$$y(t) > \gamma(t) \Longrightarrow H_1, \qquad y(t) < \gamma(t) \Longrightarrow H_0, \quad (4)$$

where $\gamma(t)$ is a possibly time-variant threshold level, and where the detection statistics $y(t)$ can be put into the general form

$$y(t) = \int_{-T/2}^{T/2} \int_{T/2}^{T/2} k_t(u,v)x(t-u)x(t-v)dudv \; . \quad (5)$$

For the single-cycle (SC) detector, the kernel in (5) is given by [1]

$$k_t^{SC}(u,v) = k_t^{\alpha}(u,v) \triangleq \frac{1}{\mu_N^2 T} R_s^{\alpha}(u-v)e^{-i\pi\alpha(u+v)}e^{i2\pi\alpha t}, \quad (6)$$

and for the multi-cycle (MC) detector [1]

$$k_t^{MC}(u,v) = \sum_{\alpha} k_t^{\alpha}(u,v) = \frac{1}{\mu_N^2 T} R_s(t-u,t-v), \quad (7)$$

and for the radiometer (R)

$$k_t^{R}(u,v) = k_t^{0}(u,v). \quad (8)$$

We want to use the deflection measure of performance to compare these three detection statistics specified by (5)-(8). However, before proceeding with this, we note that as explained in [1] these detection statistics can be re-expressed as

$$y(t) = \frac{1}{\mu_N^2 T} \sum_{\alpha \in D} \int_{-\infty}^{\infty} S_s^{\alpha}(f)^* S_{x_T}^{\alpha}(t,f)df \; , \quad (9)$$

where $S_s^{\alpha}(f)$ is the spectral correlation function (cyclic spectral density function)

$$S_s^{\alpha}(f) = \int_{-\infty}^{\infty} R_s^{\alpha}(\tau)e^{-i2\pi\alpha\tau}d\tau \; , \quad (10)$$

and $S_{x_T}^{\alpha}(t,f)$ is the cyclic periodogram

$$S_{x_T}^{\alpha}(t,f) = \frac{1}{T} X_T(t,f+\alpha/2)X_T^*(t,f-\alpha/2) \; , \quad (11)$$

where

$$X_T(t,f) = \int_{t-T/2}^{t+T/2} x(u)e^{-i2\pi fu}du \; . \quad (12)$$

For the SC detector, the set D contains one non-zero cycle frequency $\alpha \neq 0$. For the MC detector, D contains all the cycle frequencies including $\alpha=0$. For the radiometer, D contains only $\alpha=0$, in which case $S_s^{\alpha}(f)$ reduces to the power spectral density function $S_s^0(f) \equiv S_s(f)$, corresponding to the autocorrelation function $R_s^0(\tau) \equiv R_s(\tau)$, and $S_{x_T}^{\alpha}(t,f)$ reduces to the conventional periodogram, (11) with $\alpha=0$.

The deflection is defined by

$$d(t) = \frac{|E\{y(t)|H_1\} - E\{y(t)|H_0\}|}{[VAR\{y(t)|H_0\}]^{1/2}} \quad (13)$$

in which $E\{\cdot\}$ and $VAR\{\cdot\}$ denote unconditional (on N) expectation and unconditional variance.

Substitution of (5) into (13), and use of (1) yields the formula

$$d^2(t) = \frac{|K_s(t)|^2}{\mu_N^2 L_n(t) + \sigma_N^2 [L_n(t) + |K_n(t)|^2]} \quad (14)$$

for the squared deflection, where

$$K_s(t) \triangleq \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} k_t(u,v)R_s(t-u,t-v)dudv \quad (15)$$

is the expected output due to only the signal at the input,

$$K_n(t) \triangleq \int_{-T/2}^{T/2} k_t(u,u)du \quad (16)$$

is the expected output, normalized by μ_N, due to only the noise at the input, and

$$L_n(t) \triangleq 2\int_{-T/2}^{T/2} \int_{-T/2}^{T/2} |k_t(u,v)|^2 dudv \quad (17)$$

is the output variance, conditional on N=1, due to only the noise at the input. Equation (14) can be re-expressed as

$$d^2(t) = \frac{d_0^2(t)}{1 + \rho[1 + |K_n(t)|^2/L_n(t)]} \; , \quad (18)$$

where $\rho \triangleq \sigma_N^2/\mu_N^2$, which is the coefficient of variation for the random parameter N, and

$$d_0^2(t) \triangleq |K_s(t)|^2/\mu_N^2 L_n(t) \cong \frac{T}{2}|K_s(t)|, \quad (19)$$

which is the maximum deflection (for each of the cases SC, MC, R) when $\rho=0$, that is, when N is nonrandom. In obtaining the close approximation (19), it is assumed that the length T of the data collection interval greatly exceeds the longest period of cyclostationarity, $\alpha_{min}T \gg 1$, and is greater than the largest width τ_α of the cyclic autocorrelation functions $R_s^{\alpha}(\tau)$, $T > \max\{\tau_\alpha\}$.

It can be shown that

$$d_0^{SC}(t) \leq d_0^{R}(t) \leq d_0^{MC}, \quad (20)$$

where the superscript SC indicates deflection for any $(\alpha \neq 0)$ single-cycle detector. Thus, when the noise level N is known exactly, the radiometer has the largest possible deflection compared with all SC detectors. However, this result (which has been verified for various ad hoc feature detectors [4] and forms the basis for a popular bias in favor of the radiometer) is misleading since as we shall see the inequality (20) is reversed when N is random and T is large. Nevertheless, the deflection (19) for $\rho=0$ is a useful benchmark, and it has therefore been evaluated for various types of modulated signals and various optimum SC detectors, optimum MC detectors, and the optimum radiometer (using the formulas for $S_s^{\alpha}(f)$ given in [3]). The results are summarized in Table I for BPSK and QPSK spread spectrum signals in which the spreading code is modeled as a white binary sequence, f_c is the carrier frequency, T_0 is the chip interval, and the chip envelope is a full-duty-cycle rectangle. In this table, the values of deflection have been normalized by the product of the detector processing gain factor T/T_0 and the squared input SNR, $SNR_{in}^2 \triangleq [T_0 P_s/2N]^2$ where $2/T_0$ is the approximate signal bandwidth, and P_s is the signal power. With the preceding assumption that $T \gg 1/\alpha_{min}$ and $T \gg \max\{\tau_\alpha\}$ (and also $\alpha T \gg 1/2\pi^2\alpha\tau_\alpha$ for (d^{SC})) (16)-(18) yield the close approximations

$$d^{SC} \cong d_0^{SC}\sqrt{D_{SC}}, \quad d^{MC} \cong d_0^{MC}\sqrt{D_{MC}}, \quad d^{R} \cong d_0^{R}\sqrt{D_R}\; , \quad (21)$$

where the degradation factors (due to the randomness of N) are given by

$$D_{SC} = [1 + \rho]^{-1} \quad (22)$$

$$D_{MC} = \left[1 + \rho[1 + \frac{T[R_s(0)]^2}{2\sum_{\alpha} \int_{-\infty}^{\infty} |S_s^{\alpha}(f)|^2 df}]\right]^{-1} \quad (23)$$

$$D_R = \left[1 + \rho[1 + \frac{T[R_s(0)]^2}{2\int_{-\infty}^{\infty} [S_s(f)]^2 df}]\right]^{-1} . \quad (24)$$

It follows that $D_{SC} > D_{MC} > D_R$ and as either ρ or T gets larger the strengths of these inequalities increase. In fact, for any $\rho > 0$, as T increases, the inequalities in (20) will eventually be reversed to $d^{SC} > d^{MC} > d^{R}$. It follows from (21)-(24) that

$$\frac{d^{SC}}{d^{R}} \cong a\,\frac{d_o^{SC}}{d_o^{R}}\left[\frac{\rho}{1+\rho}\right]^{1/2}\sqrt{T/T_o}\ , \qquad (25)$$

where

$$a \triangleq \sqrt{T_o}\,[R_s(0)]\left[2\int_{-\infty}^{\infty}[S_s(f)]^2 df\right]^{-1/2}. \qquad (26)$$

Example: For both BPSK and QPSK, we have $a = \sqrt{3/2}$. For $T/T_0 = 10^3$ (a typical value for weak-signal detection) and $\rho = 1/10$ (a modest value), (25) yields

$$\frac{d^{SC}}{d^{R}} \cong \begin{cases} 15/\sqrt{3}\ , & \alpha = 2f_c \text{ (BPSK only)} \\ 15/\pi\ , & \alpha = 1/T_o \text{ (BPSK and QPSK).} \end{cases}$$

III. PERFORMANCE COMPARISONS FOR RANDOM INTERFERENCE STRENGTH

The detection problem considered here can again be interpreted as the hypothesis testing problem described in Section II except that now the process $n(t)$ models the sum of white Gaussian noise $w(t)$ (not doubly stochastic) and doubly stochastic narrowband interference $i(t)$, $n(t) = w(t) + i(t)$. It is assumed that the bandwidth $2/T_0$ of the signal of interest $s(t)$ is much broader than that of the interference $i(t)$, in which case $i(t)$ can be modeled as a sine wave with time-invariant but random amplitude and phase, $i(t) = a\cos(2\pi f_i t+\theta)$. The phase θ is assumed to be uniformly distributed on $[-\pi, \pi)$, and the amplitude a is modeled as a doubly stochastic Rayleigh variable with random conditional mean square value $A \triangleq E\{a^2\}$. Thus, the power $I = A/2$ of $i(t)$ is random, analogous to the random power spectral density N in Section II. Conditional on I, $i(t)$ is a zero-mean stationary Gaussian process, and therefore $n(t)$ is a zero-mean stationary Gaussian process. Consequently Isserlis' formula [3] can be used (as in Section II) to determine the fourth joint moment of $n(t)$ in terms of only the autocorrelation functions of $w(t)$ and $i(t)$, $R_w(u,v) = N\delta(u-v)$ and $R_i(u,v) = \mu_I\cos(2\pi f_i[u-v])$.

The model for the signal $s(t)$ is the same as in Section II, (2)-(3), and so is the class of detectors to be analyzed, (4)-(12), as well as the measure of performance (13).

Substitution of (5) into (13), and use of Isserlis' formula yields the result

$$d^2(t) = \frac{d_*^2(t)}{1+\rho\left[\dfrac{L_i(t)+|K_i(t)|^2}{(T_o/r)^2L_w(t)+L_i(t)+(T_o/r)L_{wi}(t)}\right]} \qquad (27)$$

where $\rho \triangleq \sigma_I^2/\mu_I^2$, $r \triangleq 2T_o\mu_I/N$, which is an interference-to-noise power ratio (INR), and

$$d_*^2(t) = \frac{T|K_s(t)|}{2}\left[1+\left(\frac{r}{T_o}\right)^2\frac{L_i(t)}{L_w(t)}+\frac{r}{T_o}\frac{L_{wi}(t)}{L_w(t)}\right]^{-1}, \qquad (28)$$

which is the deflection when $\rho = 0$, that is, when I is nonrandom. In (27) and (28), $K_s(t)$ is given by (6)-(8) and (15), $K_i(t)$ is given by (15) (with R_s replaced by $\frac{1}{2\mu_I}R_i$) and (6)-(8), $L_w(t)$ is given by (17) (with n replaced by w) and (6)-(8), and

$$L_i(t) \triangleq \frac{1}{2\mu_I^2}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} k_t(u,v)k_t^*(u',v')\,R_i(u,u')R_i(v,v')\,du\,dv\,du'\,dv' \qquad (29)$$

is the variance of the output due to only the interference at the input, conditioned on $A = 1$, and

$$L_{wi}(t) \triangleq \frac{2}{\mu_I}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} k_t(u,v)k_t^*(u,v')\,R_i(v,v')\,du\,dv\,dv' \qquad (30)$$

reflects the contributions to output variance due to the cross-product interaction of the noise and interference in the quadratic detector.

The formula (27) can be put into the forms

$$d^R = d_o^R\sqrt{D_R^I\,D_R(\rho)} \ \text{ and } \ d^{SC} = d_o^{SC}\sqrt{D_{SC}^I\,D_{SC}(\rho)}\ , \qquad (31)$$

where d_o^R and d_o^{SC} are the optimum deflections due only to white noise (and are given in Table I), D_R^I and D_{SC}^I are the degradations in deflection due to the presence of narrowband interference in addition to broadband noise, and $D_R(\rho)$ and $D_{SC}(\rho)$ are the degradations due to the randomness of the interference power level I.

For BPSK ($\alpha = 2f_c$), it can be shown (assuming $T \gg \tau_\alpha$, $T \gg T_0 \gg 1/f_c$, and $|f_i - f_c| \ll f_c$) that

$$[D_R^I]^{-1} \cong 1 + \frac{3r^2T}{32T_o} + 24r\,\frac{\mathrm{sinc}(2f_iT)}{(2\pi f_cT_o)^2} \qquad (32)$$

$$[D_R(\rho)]^{-1} \cong 1 + \frac{2\rho}{1+\dfrac{32T_o}{3r^2T}+\dfrac{256T_o}{rT}\dfrac{\mathrm{sinc}(2f_iT)}{(2\pi f_cT_o)^2}} \qquad (33)$$

$$[D_{SC}^I]^{-1} \cong 1 + \frac{3r}{2}\mathrm{sinc}(2\Delta fT)\left[1+\frac{rT}{16T_o}\mathrm{sinc}(2\Delta fT)\right] \qquad (34)$$

$$[D_{SC}(\rho)]^{-1} \cong 1 + \frac{\rho\,\mathrm{sinc}^2(2\Delta fT)}{\dfrac{32T_o}{3r^2T}+\mathrm{sinc}(2\Delta fT)\left[\dfrac{16T_o}{rT}+\mathrm{sinc}(2\Delta fT)\right]} \qquad (35)$$

where $\Delta f \triangleq f_c-f_i$ and $\mathrm{sinc}(x) \triangleq \sin(\pi x)/\pi x$.

For QPSK ($\alpha = 1/T_0$), it can be shown (with the preceeding assumptions plus $|f_c - f_i| \ll 1/T_0$) that

$$[D_R^I]^{-1} = 1 + \frac{3r^2T}{32T_o} + 24r\,\frac{\mathrm{sinc}(2f_iT)}{(2\pi f_cT_o)^2} \qquad (36)$$

$$[D_R(\rho)]^{-1} = 1 + \frac{2\rho}{1+\dfrac{32T_o}{3r^2T}+\dfrac{256T_o}{rT}\dfrac{\mathrm{sinc}(2f_iT)}{(2\pi f_cT_o)^2}} \qquad (37)$$

$$[D_{SC}^I]^{-1} = 1 + \frac{2r^2T}{\pi^2T_o}\mathrm{sinc}^2(T/T_o) + \frac{8r/\pi^2}{(2T_of_c)^4}\sum_{p\in\{-1,1\}}\mathrm{sinc}(T/T_o+2pf_iT)[1+\cos(2\pi f_cT_o)]^2 \qquad (38)$$

$$[D_{SC}(\rho)]^{-1} = 1 + \rho\,\mathrm{sinc}^2(T/T_o)\left[\frac{\pi^2T_o}{2r^2T}+\frac{1}{2}\mathrm{sinc}^2(T/T_o) + \frac{4T_o/rT}{(2f_cT_o)^4}\sum_{p\in\{-1,1\}}\mathrm{sinc}(T/T_o+2pf_iT)[1+\cos(2\pi f_cT_o)]^2\right]^{-1} \qquad (39)$$

It follows from (32)-(39) that the limiting values of the degradation factors, as $T\to\infty$, are

$$D_R^I \to 32T_o/3r^2T \quad \text{and} \quad D_R(\rho) \to (1+2\rho)^{-1} \qquad (40)$$

$$D_{SC}^I \to 1 \quad \text{and} \quad D_{SC}(\rho) \to 1. \qquad (41)$$

For both BPSK and QPSK, therefore, the ratio of deflections approaches the asymptote

$$d^{SC}/d^R \to r[d_o^{SC}/d_o^R]\sqrt{\frac{3}{32}(1+2\rho)T/T_o}\ , \qquad (42)$$

as $T\to\infty$, which indicates that the inequality $d_o^{SC} < d_o^R$ will be reversed to $d^{SC} > d^R$ for sufficiently long collection times T.

Using (31)-(39), the ratio of deflections has been calculated and graphed as a function of the collection time T for BPSK and QPSK signals with a wide range of values of interference-to-noise ratio (r = 1,10,100) and with coefficient of variation ρ = 1/10 and interference frequency $f_i = f_c - 1/4T_0$. The results are shown in Figure 1.

IV. CONCLUSIONS

The suggestion in [1] that cycle detectors can outperform radiometers for signal interception in unknown and changing noise and interference environments has been investigated by evaluation and comparison of deflection for these two types of detectors. It has been shown that for broadband noise with random power spectral density level N, the single-cycle detector can outperform the radiometer when the coefficient of variation of N is non-zero and the collection time T is sufficiently long. It has also been shown that the single-cycle detector can even outperform the multi-cycle detector, which is optimum (locally maximum-likelihood) for known N, because of the sensitivity to the variability of N of the radiometer component of the latter detector.

For narrowband interference with random power level I, it has been shown that again the single-cycle detector can outperform the radiometer when the coefficient of variation of I is non-zero and the collection time T is sufficiently long.

It has also been established (but is not presented herein due to lack of space) that the optimum (locally maximum-likelihood) detector that jointly estimates the unknown noise level N and detects the signal, modeled as stationary (rather than cyclostationary), is a radiometric detector, and that it can be outperformed by the single-cycle detector when the signal is sufficiently broadband.

In conclusion, in interception environments where broadband noise level and/or interference activity are variable, single-cycle detectors, in comparison with radiometric detectors, hold the promise of providing more effective interception of weak signals.

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TABLE I

Normalized Deflection for Known N (ρ=0)

Detector	BPSK	QPSK
MC d_0^{MC} Multi-cycle (all α)	$\sqrt{2}$ (3 dB)	1 (0 dB)
d_0^{SC} Single-Cycle ($\alpha = 2f_c$)	$1/\sqrt{3}$ (-4.8 dB)	0
d_0^{SC} Single-Cycle ($\alpha = 1/T_0$)	$1/\pi$ (-9.9 dB)	$1/\pi$ (-9.9 dB)
d_0^R Radiometer (α=0)	$\sqrt{2/3}$ (-1.8 dB)	$\sqrt{2/3}$ (-1.8 dB)

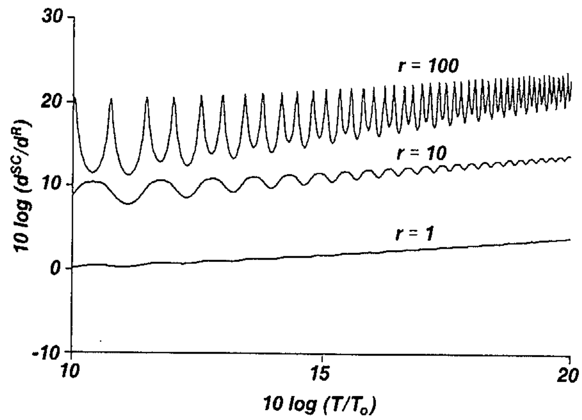

Figure 1(a). Graph of the log ratio of deflections 10 log(d^{SC}/d^R) versus the log of normalized collect time 10 log(T/T_0) for the single-cycle detector and radiometer for BPSK ($\alpha = 2f_c$) with interference-to-noise ratio of r = 1, 10, 100, and coefficient of variation for interference power of ρ = 0.1, interference frequency of $f_i = f_c - 1/4T_0$, and carrier frequency of $f_c = 3/T_0$.

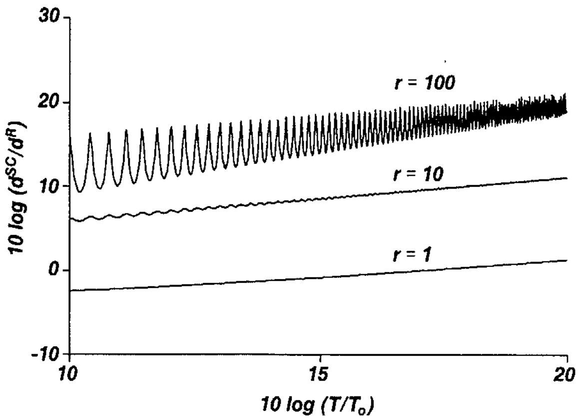

Figure 1(b). Same as (a) except for QPSK ($\alpha = 1/T_0$).