PROBABILISTIC SEISMIC MIGRATION BASED ON BAYESIAN ESTIMATION

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Abstract

Seismic migration is a technique widely used in seismic oil exploration for wave-field reconstruction and for imaging the geometrical distribution of the reflection surfaces within the earth, from the seismic data recorded on the surface of the earth. These data are usually corrupted by noise (white noise, surface waves, multiple reflections etc.) which degrade the result of the migration. Another factor which influences the migration is the inadequate knowledge of the distribution of the acoustic wave propagation velocity in the subsurface of the earth. The objective of this work is to use estimation theory techniques to find the optimum estimate of the wave propagation velocity and of the geometrical distribution of the subsurface reflector points.

I. INTRODUCTION

The objective of seismic techniques in oil exploration is to collect information about the structure of the interior of the earth, such as interfaces between different rock layers (reflection horizons), velocity of acoustic wave propagation in the subsurfase etc. from data recorded on the surface of the earth. These seismic data are processed extensively [1,2], so that noise is suppressed and reliable information is extracted. Some of the processing techniques used are stacking and deconvolution [1,2]. When this reliable information is extracted an inverse recontruction technique called migration is used to find the distribution of the subsurface reflector points [1,2,3]. It consists of two steps:

(a) Wave-field extrapolation

(b) Imaging

Step (a) uses the wave equation to find the wave-field in depth z_o of the interior of the earth. Step (b) collects the results of step (a) around zero travel time (t=0) to find the distribution of the reflector points at depth z_o. Step (a) can be performed nonrecursively or recursively [2,3,4]. The recursive wave extrapolation has the advantage over the nonrecursive one of taking into account depth velocity variations. This is very important since the velocity c of the wave propagation changes considerably from layer to layer and generally increases with the depth. A correct choice of velocity is very important for a correct migration. Many migration techniques have been proposed [1,2], but all of them require an accurate choice of the velocity c. This velocity is usually unknown, but it can be estimated during the stacking procedure [1] or by other means, such as well-log techniques [5]. Both methods give only an approximate velocity distribution. This fact limits the success of the migration and also of the correct interpretation of the depth information in the seismic sections. Thus an accurate estimation of the velocity distribution is a very important task.

Another major problem in seismic signal processing is the reduction of the noise recorded together with the useful signal. This noise is a combination of thermal white noise, surface waves or multiple reflections. Most of the white noise is usually removed by the stacking procedure. The multiple reflections can be removed by appropriate techniques [1,2], like dereverberation. The surface waves are usually removed by velocity filtering [2,3]. Velocity filtering has several disadvantages. Sometimes noise and signal spectra are partly overlapping, and moreover the velocity filters used are not perfect and distort the signal. Thus the problem of surface wave noise reduction is only partially solved. The same conclusion is true for the other kinds of noise, since an amount of noise still remains after stacking.

Our approach is to model the seismic experiment and to use estimation theory [6] techniques to obtain parameters of the model (such as the velocity c) and to estimate the position and the amplitude of the reflector points. The same philosophy has already been used very successfully in deconvolution [7] and it will be shown to give fruitful results in migration.

II. PROBLEM STATEMENT

The wave equation:

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \qquad (1)$$

has the following solution for upward travelling plane waves [2,3] in the wavenumber frequency domain:

$$p(k_x,k_y, z=0,\omega) = P(k_x,k_y,z_o,\omega)W(k_x,k_y,z_o,\omega) \qquad (2)$$

where

$$W(k_x k_y, z_o,\omega) = \begin{cases} \exp(-i z_o\sqrt{k^2-k_x^2-k_y^2}) & k^2 \geq k_x^2+k_y^2 \\ \exp(-z_o\sqrt{k_x^2+k_y^2-k^2}) & k^2 < k_x^2+k_y^2 \end{cases} \qquad (3)$$

$$k=\frac{\omega}{c}$$

Thus the wave on the surface (z=0) can be calculated by forward extrapolation of the wave on the surface

$z=z_o$ through the operator $W(k_x,k_y,z_o,\omega)$. Equation (2) has the following formulation in the spacetime domain:

$$p(x,y,z=0,t) = p(x,y,z_o,t) * w(x,y,z_o,t) \qquad (4)$$

Usually the reflector points have a geometrical distribution which can be very well described by the distribution of the wave pressure at zero traveltime $p(x,y,z,t=0)$ $(0 \leq z \leq$ maximum depth$)$. These waves $p(x,y,z,t=0)$ propagate to the surface and the propagation is described by a corresponding operator $h(x,y,z,t)$. Thus the data recorded on the surface have the form:

$$p(x,y,0,t) = \int_{z_{max}}^{0} p(x,y,z,t=0) * h(x,y,z,t)\, dz + n(x,y,t) \qquad (5)$$

where $n(x,y,t)$ is the recorded noise. The integration along z takes into account reflections at different depths. The operator $h(x,y,z,t)$ has the form (3) when the velocity c is constant throughout the earth. This is not usually the case, since wave velocity varies with the depth and with the type of the rock. We shall assume here only a layered medium having propagation velocity c_i in the layer i, $1 \leq i \leq N$. In this case the operator $W(k_x,k_y,z_i,\omega)$ has the following form in the layer i:

$$W(k_x,k_y,z_i,\omega) = \begin{cases} \exp\left[-i(z_i-z_{i-1})\sqrt{\dfrac{\omega^2}{c_i^2}-k_x^2-k_y^2}\right] & \dfrac{\omega^2}{c_i^2} \geq k_x^2+k_y^2 \\ \exp\left[-(z_i-z_{i-1})\sqrt{k_x^2+k_y^2-\dfrac{\omega^2}{c_i^2}}\right] & \dfrac{\omega^2}{c_i^2} < k_x^2+k_y^2 \end{cases} \qquad (6)$$

where c_i is the velocity in the layer i and $z_o=0$.

We shall use vector y for the measurements $p(x,y,z=0,t)$ and vectors X_i, $1 \leq i \leq N$, of dimension M, for $p(x,y,z=z_i,t)$. If the convolution operator $h(x,y,z_i,t)$ is described by the matrix H_i (MxM) and the propagation operator $w(x,y,z_i,t)$ by the matrix W_i (MxM), the following relation holds:

$$H_i = \prod_{j=1}^{i} W_j \qquad (7)$$

Thus (5) becomes:

$$y = \sum_{i=1}^{N} H_i X_i + n \qquad (8)$$

We shall formulate the migration problem in the following way:

Given the measurements y find the optimum estimate of X_i and c_i

Estimation theory is very rich in methods for solving problems similar to (8). We shall describe in the next sections the maximum a posteriori (MAP) estimation [6], which has already been sucessfully used in such areas as image restoration [7] and image reconstruction from projections [8].

III. MAXIMUM APOSTERIORI (MAP) ESTIMATION

The MAP estimation [6] maximizes the logarithm of the aposteriori pdf $p(X_1,\ldots,X_N,c_1,\ldots,c_N|y)$. It is known from basic probability theory that:

$$p(X_1,\ldots,X_N,c_1,\ldots,c_N|y) = \frac{p(y|X_1,\ldots,X_N,c_1,\ldots,c_N)\; p(X_1,\ldots,X_N,c_1,\ldots,c_N)}{p(y)} \qquad (9)$$

where $p(X_1,\ldots,X_N,c_1,\ldots,c_N)$ is the joint pdf of X_i,c_i $i=1,N$.

We can assume that $X_1,\ldots,X_N,c_1,\ldots,c_N$ are statistically independent of each other. This is a reasonable assumption for $c_1,\ldots,c_N$, since there is little, if any, relation between the velocities in differnt rock layers. Such an assumption does not take into account that velocity generally increases with the depth $c_1 \leq c_2 \ldots \leq c_N$. At the same time it does not exclude such a possiblility. The assumption of the statistical independence of $X_1,\ldots,X_N$ is justified for horizontal or almost horizontal reflector horizons.

Based on the assumption of the statistical independence (9) becomes:

$$p(X_1,\ldots,X_N,c_1,\ldots,c_N|y) = \frac{p(y|X_1,\ldots,X_N,c_1,\ldots,c_N)\prod_{i=1}^{N} p(X_i)p(c_i)}{p(y)} \qquad (10)$$

The signals X_i $i=1,N$ are assumed to have Gaussian pdf with $\bar{X}_i$ apriori means and R_{X_i} covariance matrix:

$$p(X_i) = \frac{1}{(2\pi)^{M/2}|R_{X_i}|^{1/2}} \exp\{-\tfrac{1}{2}(X_i-\bar{X}_i)^T R_{X_i}^{-1}(X_i-\bar{X}_i)\} \qquad (11)$$

where M is the dimension of vector X_i. The velocities c_i $i=1,N$ are also assumed to have Gaussian pdf with $\bar{c}_i$ apriori means and $\sigma_{c_i}^2$ variances.

$$p(c_i) = \frac{1}{2\pi\sigma_{c_i}} \exp\left\{-\frac{(c_i-\bar{c}_i)^2}{2\sigma_{c_i}^2}\right\} \qquad (12)$$

By combining (10), (11), (12) and by dropping some constant terms we find that the function to be maximized is:

$$L(X_1,\ldots,X_N,c_1,\ldots,c_N) = \ln p(X_1,\ldots,X_N,c_1,\ldots,c_N) =$$
$$= -\frac{1}{2}(y-\sum_{i=1}^{N} H_i X_i)^T R_n^{-1}(y-\sum_{i=1}^{N} H_i X_i) -$$
$$-\frac{1}{2}\sum_{i=1}^{N}\left[(X_i-\bar{X}_i)^T R_{X_i}^{-1}(X_i-\bar{X}_i) + \frac{(c_i-\bar{c}_i)^2}{\sigma_{c_i}^2}\right] \qquad (13)$$

Taking the derivatives of L with respect to X_k equal to 0 we find:

$$\nabla_{X_k} L = H_k^T R_n^{-1}(y-\sum_{i=1}^{N} H_i X_i) - R_{X_k}^{-1}(X_k-\bar{X}_k) = 0 \qquad k=1,N \qquad (14)$$

By equating the derivatives $\frac{\partial L}{\partial c_k}$ $k=1,N$ to zero we find:

$$\frac{\partial L}{\partial c_k} = -\left(\sum_{i=1}^{N}\frac{\partial H_i}{\partial c_k}\right)^T R_n^{-1}(y-\sum_{i=1}^{N} H_i X_i) + \frac{c_k-\bar{c}_k}{\sigma_{c_k}^2} = 0 \qquad (15)$$

Thus the MAP estimators of X_k, c_k $k=1,N$ satisfy the following equations:

$$X_k = \bar{X}_k + R_{X_k} H_k^T R_n^{-1}(y-\sum_{i=1}^{N} H_i X_i) \qquad k=1,N \qquad (16)$$

$$c_k = \bar{c}_k + \sigma_{c_k}^2\left(\sum_{i=1}^{N}\frac{\partial H_i}{\partial c_k} X_i\right)^T R_n^{-1}(y-\sum_{i=1}^{N} H_i X_i) \qquad k=1,N \qquad (17)$$

(16), (17) have a recursive form and they can be used for a recursive calculation of X_k, c_k (Picard iteration) [9]. When c_k $k=1,N$ is known, (16) reduces to a system of equations:

$$\begin{bmatrix} R_{X_1}^{-1}+H_1^T R_n^{-1} H_1 & H_1^T R_n^{-1} H_2 \ldots H_1^T R_n^{-1} H_N \\ H_2^T R_n^{-1} H_1 & R_{X_2}^{-1}+H_2^T R_n^{-1} H_2 \ldots H_1^T R_n^{-1} H_N \\ \vdots & \vdots \\ H_N^T R_n^{-1} H_1 & H_n^T R_n^{-1} H_2 \ldots R_{X_N}^{-1}+H_N^T R_n^{-1} H_N \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} = \begin{bmatrix} R_{X_1}^{-1}\bar{X}_1 + H_1^T R_n^{-1} y \\ R_{X_2}^{-1}\bar{X}_2 + H_2^T R_n^{-1} y \\ \vdots \\ R_{X_N}^{-1}\bar{X}_N + H_N^T R_n^{-1} y \end{bmatrix} \qquad (18)$$

When the model consists only of one layer, (18) degenerates to:

$$X_1 = (R_{X_1}^{-1} + H_1^T R_n^{-1} H_1)^{-1} (R_{X_1}^{-1}\bar{X}_1 + H_1^T R_n^{-1} y) \qquad (19)$$

When the apriori mean $\bar{X}_1$ is zero, (19) becomes:

$$X_1 = (R_{X_1}^{-1} + H_1^T R_n^{-1} H_1)^{-1} H_1^T R_n^{-1} y \qquad (20)$$

This is the classical Wiener filter [6]. It has already been proposed in [11] to be used for noise suppression in migration.
The solution of the system (18) is usually very tedious because the dimensions of the matrices and the vectors are very large in practical cases. Thus we must find other methods for the maximization of (13). Such a method is the conventional steepest descent algorithm [9]. Its basic iteration at the step (p) is the following:

1. Calculate $\nabla_{X_k} L_{(p)}, \frac{\partial L^{(p)}}{\partial c_k}$ $\quad k=1,N.$

2. If $|\nabla_{X_k} L^{(p)}| < \varepsilon$, $|\frac{\partial L^{(p)}}{\partial c_k}| < \varepsilon$ $\quad$ stop.

   (Where ε is a small positive number).

3. Optimize $L(X_1^{(p)},..,X_N^{(p)}, c_1^{(p)} - a_p \frac{\partial L^{(p)}}{\partial c_1},..,c_N^{(p)} - a_p \frac{\partial L^{(p)}}{\partial c_N})$

   with respect to the scalar a_p.

4. $c_i^{(p+1)} = c_i^{(p)} - a_p \frac{\partial L^{(p)}}{\partial c_i}$ $\quad i=1,N \qquad (21)$

5. Optimize $L(X_1^{(p)} - b_p \nabla_{X_1} L^{(p)},..,X_N^{(p)} - b_p \nabla_{X_N} L^{(p)}, c_1^{(p)}, \ ,c_N^{(p)})$

   with respect to the scalar b_p.

6. $X_i^{(p-1)} = X_i^{(p)} - b_p \nabla_{X_i} L^{(p)}$ $\quad i=1,N \qquad (22)$

The partial derivatives are given by (14),(15).
The variables X_i, c_i $i=1,N$ are updated simulaneously in each step.
Another approach is to update only c_i $i=1,N$ until $|\frac{\partial L^{(p)}}{\partial c_i}|$ $i=1,N$ are less than the threshold ε and they update X_i $i=1,N$ until $|\nabla_{X_i} L^{(p)}|$ $i=1,N$ are less than the threshold. We shall use the second approach in our simulations.

IV. SIMULATION EXAMPLE

The estimation method described in the previous section will be illustrated by a simulation the physical experiment of Figure 1. A subsurface explosion is made and the downgoing wave is diffracted by two diffractor points located in 80m and 150m depth respectively. The diffractors have equal strength as it is shown in Figure 2. The diffracted wave is recorded by the geophones placed on the surface of the earth. The simulated data corrupted by white additive Gausian noise are shown in Figure 3. The two hyperbolas are easily recognized. The wavelet is supposed to be a delta function. The hyperbolas though are smoother since they have been created by using a bandlimited operator [2]. The experiment is described by the following equations;

$$y = H_1 X_1 + H_2 X_2 + n \qquad (23)$$

The form of H_1, H_2 is given by (6), (7) for $z_1 = 80$m, $z_2 = 150$m. The velocities c_1, c_2 are equal to 2400m/s and 3000m/s respectively. Our aim is to estimate the reflector strengths X_1, X_2 at the depths z_1, z_2 respectively and the correct velocities c_1, c_2. Our initial guess on these velocities is 1500m/s and 2400m/s respectively. This guess is far away from the actual velocities. The only apriori information available for the velocities is that their apriori means are 2500m/s and 3000m/s respectively.

Having only this limited information available we shall try to estimate X_1, X_2, c_1, c_2 in the presence of white Gaussian noise, and to compare it to the results of the conventional techniques. The results of the conventional migration-imaging algorithm employing the correct velocities (usually unknown) are shown in Figure 4. Extraneous reflectors are presesent. The flector amplitudes are not equal to each other. The MAP estimation of $X_1, X_2 c_1, c_2$ has been performed by using the iterative algorithm described in section IV. First only the velocities c_1, c_2 have been estimated iteratively. The good estimates 2359m/s and 3017m/s have been obtained in only 12 iterations. The reflector strengths X_1, X_2 have been estimated in 9 iterations on the basis of the velocity estimates. The results are shown in Figure 5. They are must better than the results of the convevtional technique. They are also very close to the ideal results of Figure 2. Note that the two estimated reflector strengths are equal, as they should be. The various convolutions of the form $H_i X_i$, $\frac{\partial H_i}{\partial c_k} X_i$, $H_i^T X_i$ $i=1,2$ used in the MAP estimation have been implemented by using two-dimensional Fast Fourier Transform algorithms [1,2].

V. CONCLUSIONS

The Bayesian estimation has been proposed to be used in seismic migration. The algorithms derived combine estimation theory and the deterministic approach based on wave theory. Their results are proven to be much better than the results of the conventional iterative migration-imaging algorithm. The superiority of our approach lies in the fast that it estimates the velocity distribution of the earth and that it takes into account all the appriori information available. The proposed algorithms are iterative ones. This fact gives an additional flexibility in our method.

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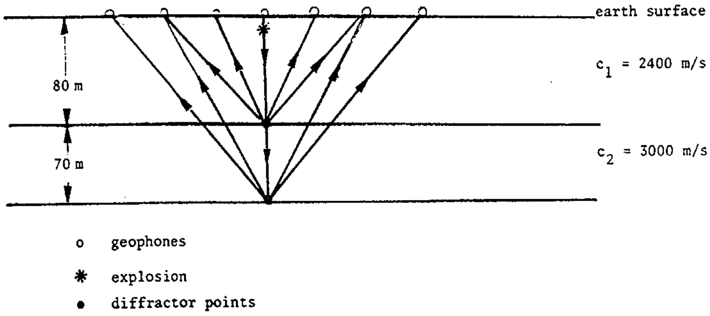


Figure 1: Wave diffraction by two subsurface diffractor points.

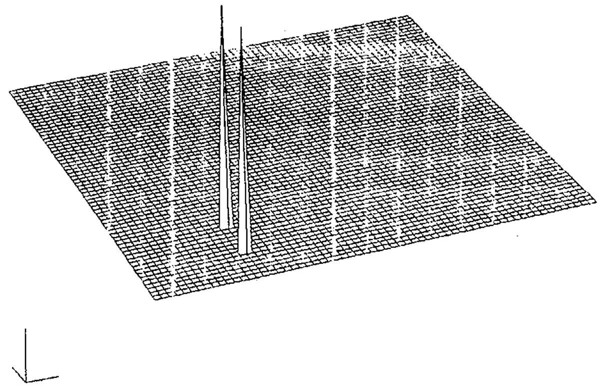
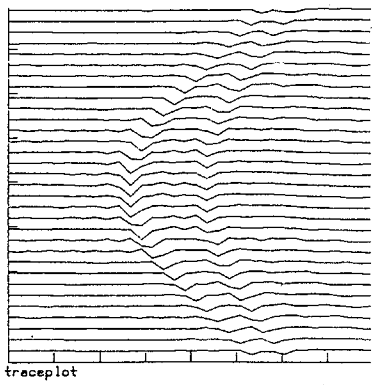
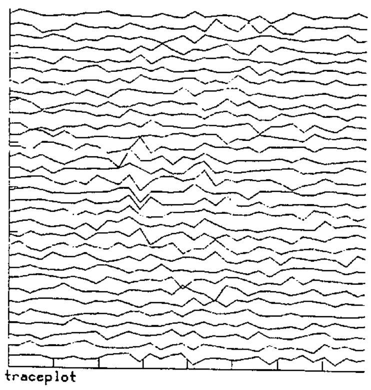


Figure 2: Ideal diffraction amplitudes.



(a)



(b)

Figure 3: (a) Noiseless recorded data
(b) Recorded data corrupted by white additive Gaussian noise.

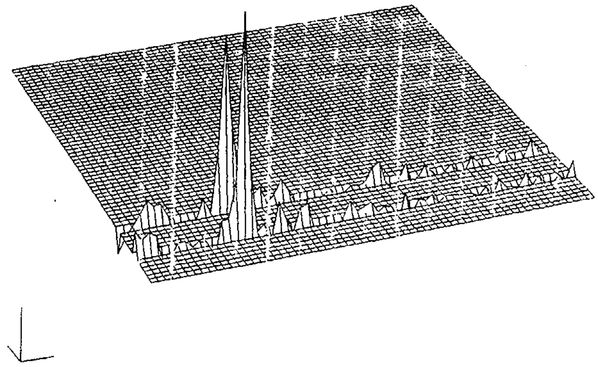


Figure 4: Result of the migration imaging by employing correct velocities.

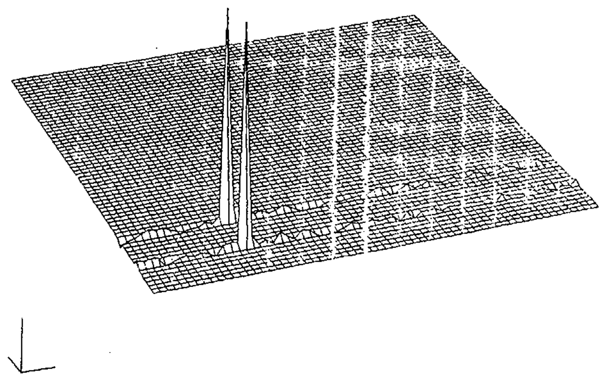


Figure 5: Result of the MAP estimation.