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ON PARASITIC OSCILLATIONS IN WAVE DIGITAL FILTERS WITH RETRIEVAL OF REFLECTED PSEUDOPOWER

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Dans un système de communication multiplex, précisément, dans une section quatre-câbles d'une liaison de communication comprenant des transitions quatre-câbles/deux-câbles, les filtres (numeriques) sont utilisés en boucles fermés. Il est donc possible de voir apparaître des oscillations parasites même si les filtres numériques eux-mêmes sont stables.

Parmi les filtres du type IIR, les filtres numériques d'ondes (WDF) se caractérisent particulièrement bonne résistance aux oscillations parasites. Cet article présente une analyse du phénomène d'oscillations parasites en boucle contenu le filtre numérique d'onde. On a considéré deux types de filtre: le WDF classique et le WDF avec recouvrement de la pseudopuissance réfléchie. On a prouvé que les filtres avec recouvrement de la pseudopuissance sont beaucoup plus résistantes aux oscillations parasites en boucle que les classiques filtres numériques d'ondes.

In a multiplexed communication system, or more precisely, in a four-wire part of a communication link comprising four-wire/two-wire transitions, filters (e.g., digital filters) operate under looped conditions. Therefore, parasitic oscillations can occur even if the digital filters by itself are stable.

Among IIR filters, wave digital filters (WDFs) embody particularly good resistance to the parasitic oscillations. Occurrence of these oscillations in a loop containing a wave digital filter, is analysed. Two filter types are considered: classical WDFs and WDFs with retrieval of the reflected pseudopower. It is shown that the latter are much more resistive to parasitic oscillations under looped conditions than the classical WDFs.

1. INTRODUCTION

As is well known, parasitic oscillations (limit cycles) can occur in recursive digital filters due to the finite word lengths representing signal samples. Under zero input conditions essentially exist two types of limit cycles: overflow oscillations and granularity oscillations. Recently a big effort has been devoted to suppression of such oscillations in recursive filters [1-14]. The most approches, however, have been restricted to the second-order sections [1-10]. Up to now, the problem of avoiding of both these oscillations has been generally solved for wave digital and related filters only [11-13]. This may be achieved by quite simple means guaranteeing pseudopassivity of the filter under nonlinear conditions, e.g., for the fixed-point two's-complement arithmetic - by appropriate chopping operations [11].

However, suppression of parasitic oscillations in isolated digital filters is often not sufficient to design a properly working system because filters may operate under looped conditions with some positive feed-back. This situation is, e.g., typical for transmultiplexers. In fact, transmultiplexer filters are connected in a loop formed by a four-wire part of a communication link and four-wire/two-wire transitions (Fig. 1). Under such circumstances parasitic oscillations may occur even in nonrecursive filters which are obviously always stable under isolated conditions. Moreover, such oscillations need not be periodic any more. Thus, in this case, the term "parasitic oscillations" is more appropriate than the term "limit cycles".

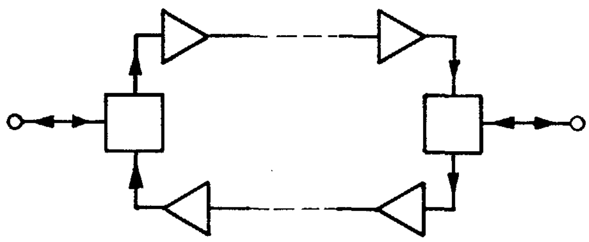

Fig. 1. Schematic illustration of the communication loop formed by a four-wire part of a communication link and four-wire/two-wire transitions

Suppression of parasitic oscillations in digital filters under looped conditions has already been analysed [13]. However, in [15] it has been shown that the results presented in [13] lack generality required in many cases, e.g., for transmultiplexers, as the given stability conditions are restricted to the digital systems with one sampling rate only, and containing no modulators in a loop.

The purpose of this paper is to formulate more general stability conditions which would be valid for multirate systems and for systems containing filters together with modulators in a loop, i.e., valid for transmultiplexers. Two types of wave digital filters (WDFs) are considered: classical WDFs [16] and WDFs with retrieval of the reflected pseudopower. The latter filters have been proposed in the generalized form in [17-19].

The results are presented in this paper in a shortened form. The full version of the paper will be published elsewhere [14].

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2. DESCRIPTION OF THE ANALYSED MODEL

In order to investigate occurrence and suppression of parasitic oscillations in wave digital filters under looped conditions, the system of Fig. 1 must be replaced by its simplified model containing a WDF, a product modulator (with a carrier signal $q(kT)$), and an outside system S (Fig. 2(a)).

The filter under consideration may be either a classical WDF (Fig. 2(b)) or that with retrieval of reflected pseudopower (Fig. 2(c)).

The product modulator is of a general type. We assume only that the carrier signal $q(kT)$, $k=\ldots-1,0,1,2,\ldots$, is real and periodic with the period

$$T_o = N\,T \quad , \qquad (1)$$

where N is an arbitrary positive integer and T is the sampling period. The signal $q(kT)$ may be fully determined by an N element carrier sequence $\{q(kT)\}$, $k=0,1,\ldots,N-1$. This sequence should contain some zeros in order to make the retrieval of the reflected pseudopower possible [19].

Two processes are of particular importance: multiplier-free modulation with sinusoidal carrier signals and sampling rate alteration (interpolation or decimation) [19]. In the latter case the carrier sequence contains one nonzero element (e.g., equal to 1) and N-1 zeros. Sampling rate decrease consists in omitting zero samples after modulation with this sequence. Sampling rate increase by a factor N of a signal $x(\kappa T)$, $\kappa=\ldots-1,0,1,2,\ldots$ (i.e., filling in zeros at empty places) may be interpreted as modulation with this sequence of an artificially introduced signal

$$x^*(kT) = \begin{cases} x(\kappa T_o) & \text{for } kT = \kappa T_o \\ \text{irrelevant} & \text{for } kT \neq \kappa T_o . \end{cases}$$

The outside system S in Fig. 2 is in general nonlinear but it is reasonable, on the basis of the discussion given in Section III-C of [13], to approximate it by a linear system with the transfer function H'.

3. SUPPRESSION OF THE PARASITIC OSCILLATIONS

3.1. General considerations

A general WDF is shown in Fig. 3. The n-port N is a static system, i.e., containing only multipliers, adders, and branch nodes. We define the pseudoenergy $\varepsilon(t_k)$ stored in delays and the pseudoenergy (or in other words - the instantaneous pseudopower) absorbed by N at the instant $t_k=kT$, by the following expressions

$$\varepsilon(t_k) = \sum_{\nu=3}^{n} G_\nu b_\nu^{\,2}(t_k) \qquad (2)$$

and

$$p(t_k) = \sum_{\nu=1}^{n} G_\nu [a_\nu^{\,2}(t_k) - b_\nu^{\,2}(t_k)] \qquad (3)$$

where $a_\nu(t_k)$ is the incident wave, $b_\nu(t_k)$ is the reflected wave, and G_ν is the port conductance corresponding to port ν.

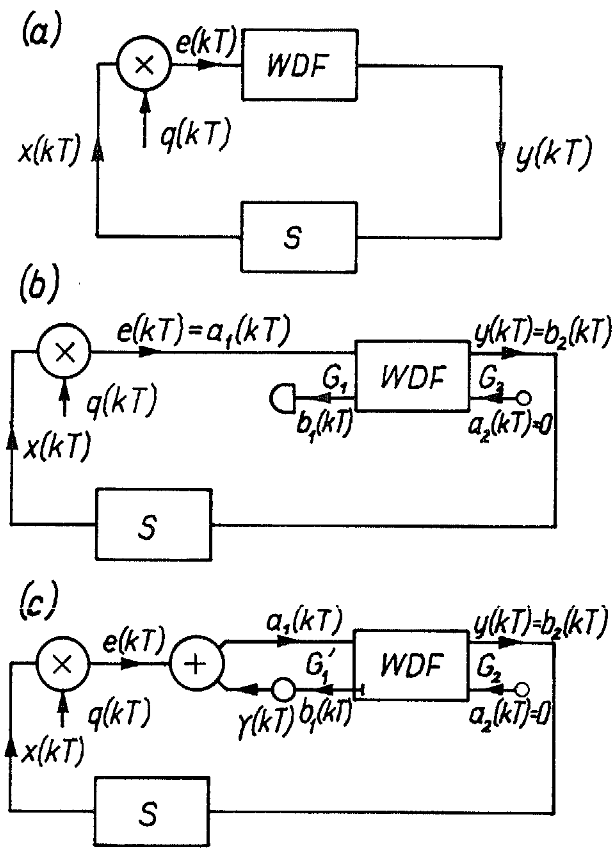

Fig. 2. Analysed models of the communication loop: (a) general scheme, (b) system with classical WDF, (c) system with retrieval of reflected pseudopower

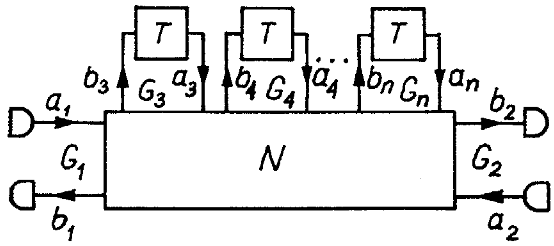

Fig. 3. General scheme of a WDF

It is obvious that the inequality

$$\varepsilon(t_k) \geqq 0 \qquad (4)$$

holds for all t_k.

We assume that the N-port N is pseudolossless (or even pseudopassive) under ideal conditions. Then, by proper realization (mentioned in Introduction) of arithmetical operations of the N-port N, the following inequality

$$p(t_k) \geqq 0 \qquad (5)$$

will also be fulfilled under nonlinear conditions for all t_k.

Consider now a spectrum $E(e^{j\omega T})$ of the input signal $e(t_k)$ to the WDF. The signal $e(t_k)$ is given by

$$e(t_k) = q(t_k)x(t_k) \quad . \qquad (6)$$

According to (1) we can write

$$E(e^{j\omega T}) = \sum_{n=0}^{N-1} Q_n X\left(e^{j(\omega-n\Omega_o)T}\right) \qquad (7)$$

where

$$Q_n = \frac{1}{N}\sum_{k=0}^{N-1} q(t_k)e^{-jn\Omega_o kT} \qquad (8)$$

and

$$\Omega_o = 2\pi/T_o \quad . \qquad (9)$$

From (7) it follows that in order to avoid aliasing, the spectrum $X(e^{j\omega T})$ of the signal $x(t_k)$ must be reduced to one of the bands

$$\omega \in \left(\nu\frac{\pi}{T_o}, (\nu+1)\frac{\pi}{T_o}\right), \quad \nu = 0,1,\ldots,N-1. \qquad (10)$$

Moreover the WDF passband length should not be greater than π/T_o. Thus, it is very probable that the spectrum of parasitic oscillations will also be reduced to the band of length π/T_o. Such oscillations are called in this paper "passband parasitic oscillations".

If a parasitic oscillation would occur in a system of Fig. 2 then

$$\lim_{k\to\infty} \varepsilon_{sx}(t_k) = \infty \qquad (11)$$

where ε_{sx} is the signal-pseudoenergy defined as

$$\varepsilon_{sx} = \sum_{\nu=0}^{k} x^2(t_\nu) \quad . \qquad (12)$$

It may be shown [14] that for passband parasitic oscillations the following relation holds

$$\lim_{k\to\infty} \frac{\varepsilon_{se}(t_k)}{\varepsilon_{sx}(t_k)} = P_{sq} \qquad (13)$$

while for general parasitic oscillations inequality

$$\frac{\varepsilon_{se}(t_k)}{\varepsilon_{sx}(t_k)} \leq N P_{sq} \qquad (14)$$

is fulfilled, where

$$\varepsilon_{se}(t_k) = \sum_{\nu=0}^{k} e^2(t_\nu) \qquad (15)$$

is the signal-pseudoenergy of $e(t_k)$ and

$$P_{sq} = \frac{1}{N}\sum_{\nu=0}^{N-1} q^2(t_\nu) \qquad (16)$$

is the average carrier-sequence pseudopower.

3.2. Systems with classical WDFs

According to (2), (3), and (5), increase of the pseudoenergy $\varepsilon(t_k)$ is given by

$$\varepsilon(t_k) - \varepsilon(t_{-1}) = \sum_{\nu=0}^{k}\left[-p(t_k) + G_1 a_1^2(t_k) - G_1 b_1^2(t_k) - G_2 b_2^2(t_k)\right] \leq \qquad (17)$$

$$\leq G_1\varepsilon_{sa}(t_k) - G_2\varepsilon_{sb}(t_k)$$

where

$$\varepsilon_{sa}(t_k) = \sum_{\nu=0}^{k} a_1^2(t_\nu) \qquad (18a)$$

and

$$\varepsilon_{sb}(t_k) = \sum_{\nu=0}^{k} b_2^2(t_\nu) \quad . \qquad (18b)$$

Now we assume that parasitic oscillations arise. Using (13), (14), (17), and taking into account that $a_1(t_k) = e(t_k)$ for the system of Fig. 2(b), it may be shown [14] that

$$\lim_{k\to\infty}\frac{\varepsilon_{sb}(t_k)}{\varepsilon_{sx}(t_k)} \leq \begin{cases} N\dfrac{G_1}{G_2}P_{sq} & \text{for general parasitic oscillations} \\[2mm] \dfrac{G_1}{G_2}P_{sq} & \text{for passband parasitic oscillations} \end{cases} \qquad (19)$$

From (19) and from Parseval's theorem the following conditions sufficient for suppression of parasitic oscillations follow

$$|H'|_{max} < \begin{cases} \sqrt{\dfrac{G_2}{N G_1 P_{sq}}} & \text{for general parasitic oscillations} \\[2mm] \sqrt{\dfrac{G_2}{G_1 P_{sq}}} & \text{for passband parasitic oscillations .} \end{cases} \qquad (20)$$

We define now the minimum WDF attenuation, α_o, and the minimum guaranteed loop attenuation, α''_{oo}, by

$$\alpha_o = \ln|S_{21}|^{-1}_{max} \geq 0 \quad , \qquad (21)$$

$$\alpha''_{oo} = \ln|H|^{-1}_{max} + \ln|H'|^{-1}_{max} \qquad (22)$$

where

$$|S_{21}| = \sqrt{\frac{G_2}{G_1}}\;\frac{|H|}{|Q_n|} \quad , \qquad (23)$$

$$H = Y/X = B_2/X \quad , \qquad (24)$$

Q_n is the appropriate value given by (8) (that for $\omega + n\Omega_o$ lying in the WDF passband), and X, $Y = B_2$ are complex constants corresponding to the signals $x(t_k)$, $y(t_k) = b_2(t_k)$, respectively, under sinusoidal steady-state conditions.

Finally, from (20)-(24) we obtain the following condition sufficient for suppression of the passband parasitic oscillations

$$\alpha''_{oo} > \alpha_o + \ln\left(\sqrt{P_{sq}}/|Q_n|\right) \quad . \qquad (25)$$

3.3. Systems with retrieval of pseudopower

The varying coefficient $\gamma(kT)$ in the system of Fig. 2(c) is given by

$$\gamma(kT) = \begin{cases} \gamma_o & \text{if } q(kT) \neq 0 \\ 1 & \text{if } q(kT) = 0 \end{cases} \qquad (26)$$

where $-1 < \gamma_o < 1$ [17-19]. The conductance G_2 is an arbitrary positive constant but the conductance G_1' is given by

$$G_1' = Z_1^{-1}(1) \qquad (27)$$

where $Z_1 = Z_1(\psi) = \hat{Z}_1(p)$ is the input impedance of the reference filter of the WDF [18]. By ψ and p complex frequencies are denoted: of the reference filter and of the WDF, respectively.

$$\psi = \tanh(pT/2) = \frac{z-1}{z+1}, \quad z = e^{pT}. \tag{28}$$

Assume that parasitic oscillations occur. Analogical analysis to that of Section 3.2 leads to the following inequalities

$$\lim_{k\to\infty} \frac{\varepsilon_{sb}(t_k)}{\varepsilon_{sx}(t_k)} \leqq \begin{cases} \dfrac{N\,G_1\,P_{sq}}{G_2(1-\gamma_o)^2} & \text{for general parasitic oscillations} \\ \dfrac{G_1\,P_{sq}}{G_2(1-\gamma_o)^2} & \text{for passband parasitic oscillations}. \end{cases} \tag{29}$$

From (29) and from Parseval's theorem the following conditions sufficient for suppression of parasitic oscillations result

$$|H'|_{max} < \begin{cases} (1-\gamma_o)\sqrt{\dfrac{G_2}{N\,G_1\,P_{sq}}} & \text{for general parasitic oscillations} \\ (1-\gamma_o)\sqrt{\dfrac{G_2}{G_1\,P_{sq}}} & \text{for passband parasitic oscillations}. \end{cases} \tag{30}$$

Taking into account that now

$$\alpha_o = \ln|H_{21n}|^{-1}_{max} \geqq 0 \tag{31}$$

where

$$|H_{21n}| = (1-\gamma_o)\sqrt{\frac{G_2}{G_1\,P_{sq}}}\,|H| \tag{32}$$

is the appropriate conversion function [19], n=0,1,..., N-1, the following condition sufficient for suppression of the passband parasitic oscillations may be derived

$$\alpha''_{oo} > \alpha_o . \tag{33}$$

4. CONCLUSIONS

In the paper the so called "passband parasitic oscillations" are distinguished. That is why conditions (20) and (30) for suppression of parasitic oscillations have two versions: for general parasitic oscillations and for the passband parasitic oscillations. Important are only the latter versions. The former bring practically no restriction because the acceptable stopband (or transition band) loss in a loop is much greater than that following from these conditions.

Furthermore, it is worth notice that the conditions (25) and (33) are sufficient but not necessary for suppression of the passband parasitic oscillations. Thus, in practice, somewhat weaker, experimentally selected conditions might be allowed. It is, e.g., permissible to approximate α''_{oo} in (25) and (33) by the minimum loop attenuation

$$\alpha''_o = \ln|H\,H'|^{-1}_{max} \geqq \alpha''_{oo} .$$

Comparison of (25) and (33) shows that WDFs with retrieval of reflected pseudopower are more resistive to parasitic oscillations in a loop than the classical WDFs provided that both filter types are characterized by the same attenuation α_o. However, for WDFs with retrieval of reflected pseudopower, minimization of α_o needs involved optimization [18]. Moreover, the optimized α_o may occur to be greater than that obtainable by classical WDFs.

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