A MICROWAVE DIGITAL FILTER OR A DIGITAL MICROWAVE FILTER ?

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ABSTRACT. In this paper a new type of wave digital filter is described. The filter may be called a true wave digital filter, because the state variables of the filter are true waves propagating in opposite directions on transmission lines in the reference filter, which is a microwave interdigital coupled transmission line filter. The signal flow diagram for this filter has the form of a digital filter flow diagram ! The corresponding digital filter might be called a digital interdigital filter (a DIF).

The DIF will have the same properties as the reference filter, and the number of multipliers will be minimum. The reference filter is a bandpass filter, but simple changes in the flow diagram change the digital filter to e.g. a lowpass filter.

Analysis and synthesis "tools", developed by the author, for the microwave filter may be used also for the digital filter.

I. INTRODUCTION.

All the so called wave digital filters have analog passive filters as reference filters. The structure of the wave digital filter is such, that the digital filter constants correspond to physical parameters in the analog passive filters. This means, that the digital filter will have some of the same properties as the analog filter. It will always be stable, it will have the same sensivities to variations in filter constants etc.

The waves in a wave digital filter are not really waves. The name stems from the fact, that scattering- or transmission parameter models for the analog filter (the reference filter) components together with a suitable frequency transformation lead to the flow diagram for the digital filter.

The filter type (DIF) described in this paper, however, may be called a true wave digital filter, because the state variables of the filter are travelling waves (propagation modes) on the transmission lines, which form the reference filter. A frequency transformation is not necessary, the flow diagrams for the reference filter and the digital filter are identical !

The filter type thus belongs to a class of filters, which may be called "State Space Wave Digital Filters".

II. THE REFERENCE FILTER.

In fig.1 is shown schematically an even order microstrip interdigital coupled transmission line filter. The filter consists of N coupled transmission lines in the form of e.g. microstrips. In the general case (inhomogeneous dielectric), one must find the N different propagation modes which may exist on the coupled lines [1] - [4], but here homogeneous dielectric is supposed, which makes the flow diagram for the filter simple, because all the phase velocities for the different propagation modes are identical. All the transmission lines have the same length l, which means that all signal delays are mul tipla of l/v , where v is the phase velocity.

The equations (3) to (8) lead to the signal flow diagram in fig.2, which shows the relation between vectors $\underline{A}$ and $\underline{B}$ representing the travelling waves and the voltage vectors $\underline{V}_g$, $\underline{V}_1$ and $\underline{V}_2$ (1) representing the generator and the voltages at the two ends of the coupled strips.

The flow diagram will look exactly like the flow diagram for a digital filter, but the constants will be matrices (7), (8) and (10) expressed in terms of the capacitance matrix $\underline{\underline{C}}$ for the coupled strips and the terminating impedance matrices $\underline{\underline{Z}}_1$ and $\underline{\underline{Z}}_2$ (2).

The N propagation modes, which may exist on the coupled strips, can be chosen arbitrarily [1] - [4]. In order to make the flow diagram simple, they are chosen as "one mode per strip".

The equations (5) and (6) may be combined into one equation, which expresses the vector $\underline{A}$ as the sum of the input vector $\underline{V}_g$ multiplied by a matrix and a delayed version of $\underline{A}$ multiplied by another matrix. This is a so called state-variable equation for the filter. The filter is thus a State Space Wave Digital Filter.

The input vector $\underline{V}_g$ has only one element (the generator voltage V_g) and only the element (1,1) in the matrix $0.5\cdot\{\underline{\underline{1}}-\underline{\underline{\rho}}_1\}$ is used.

The additional equation giving the output voltage vector $\underline{V}_2$ is simple. The output voltage V_o is the last element in the vector $\underline{V}_2$

because the filter shown in fig.1 is supposed to be an even order filter. In the odd order filter, the output voltage V_o will be the last element in the vector $\underline{V}_1$.

Because coupling between nonadjacent strips is presumed nonexistent, and because of the way the strips are terminated, the matrices $\underline{P}_1$ and $\underline{P}_2$ will be bandmatrices (10), that is, only about 2N elements will be nonzero in each matrix, and about half of these will be 1 or -1. This means that the number of multipliers in the digital filter will be minimum !

Each element in these matrices relate to physical parameters for the filter, thus ensuring, that the digital filter with the same flow diagram will always be stable, have low sensitivities and have low noise.

Simple changes in the flow diagram, which can not be realized in the physical filter, make the flow diagram into the representation for a digital lowpass-, highpass- or bandstop filter.

III. SYNTHESIS OF A DIF.

The flow diagram representing the state-variable equations for the filter look simple (fig.2). Even in its "unfolded" form (fig.4) it looks simple, because of the repetitious nature of the flow diagram. However, for synthesis purposes an equivalent diagram must be found, in which the elements can be found in terms of the specifications for the filter.

III.1 An equivalent diagram.

In fig.3 is shown an exact equivalent diagram for the bandpass DIF or for the interdigital micro strip filter with homogeneous dielectric [1] - [4]. This shows, that the filter consists of N parallel resonators with admittance converters in between. The resonator constants and the admittance converter constants are expressed in terms of the self- and coupling capacitances for the coupled strips as shown in the equations (11).

A filter with the equivalent diagram shown in fig.3 (the flow diagram in fig.2) is a bandpass filter with no zeroes in the stopband. Only at the Nyquist frequency and of course at the zero frequency will the bandpass filter have zeroes. This means that double terminated ladder type all poles lowpass filters may be used as basic filters.

III.2 Synthesis method

The synthesis method, which is described in [2] and [3] is simple. Given the filter type (e.g. Chebyshev 0.5 dB ripple) and the bandwidth, the capacitance matrix for the coupled transmission lines is found. The method works best in the case of small bandwidths, because the number of zeroes does not correspond to the number of zeroes for the ladder type basic filter. For filters with larger bandwidths (>20%), the method can still be used with some modifications. If the bandwidth is very large, only an iterative procedure will give an exact solution for the capacitance matrix $\underline{C}$. The filter may of course also be synthesized from the knowledge of the desired filter poles in the z-plane. See later.

There will be an infinity of solutions for the capacitance matrix for the coupled strips, but while not all solutions are physically realizable, there are no such constraints on the digital equivalent to the microstrip filter!

A synthesis program for microwave interdigital micro strip filters, developed by the author, can, with minor modifications, be used for the synthesis of the digital interdigital filter (DIF). (The program actually finds the strip dimensions by iteration.)

A sensible solution must of course be chosen, But in the digital filter case there are other considerations than the practical problems of making coupled micro strips with a given capacitance matrix.

IV. SENSITIVITIES, ROUNDOFF NOISE AND DYNAMIC RANGE

A solution must be chosen where the filter constants for the digital filter have values so that: 1) They can be represented with reasonable accuracy using the number representation for the actual filter (number of bits). 2) The sensitivity of the filter response to errors in these constants will be as low as possible. 3) The roundoff noise for the filter will be as low as possible. 4) The dynamic range for the input signal will be as high as possible.

For a DIF, as for most other types of wave digital filters, it is no great problem to achieve these goals, because the prototype filter is a passive filter with low sensitivities and little or no internal amplification.

It is possible to choose a solution where there is no internal amplification in the filter. (The state variables A_n and B_n will at no frequency become larger then the input quantity V_g).

V. FLOW DIAGRAMS FOR DIGITAL INTERDIGITAL FILTERS.

V.1. The bandpass filter.

The signal flow diagram for this type of filter has already been given in fig.2.

To make the amplification equal to 1 at the center frequency, the matrix on the first path is multiplied by 2. The center frequency will always lie at half the Nyquist frequency.

V.2. The lowpass filter.

The center frequency for the bandpass filter lies at the freuency, where z is equal to j, or where $z \cdot z$ is equal to -1. By replacing one of the delays by a constant -1 (a unity matrix with a negative sign), the center frequency will now be 0. See fig.5. The cutoff frequency for the lowpass filter will be the same as the bandwidth for the bandpass filter.

The lowpass filter has no physical equivalent, but the synthesis is made as if it was a bandpass filter. The output vector for the filter is found (as before) at one or the other "end" of the filter dependant on, whether the filter is an even or an odd order filter.

V.3. The highpass filter.

A highpass filter is made by replacing one of the delay elements by the unity matrix. The difference between the Nyquist frequency and the highpass filter cutoff frequency is equal to the bandwidth for the bandpass filler with the same filter constants. this means that in this case also, the filter is synthesized as a bandpass filter. Like the lowpass filter the highpass filter has no physical equivalent.

V.4. The bandstop filter.

A bandstop filter is made by changing the sign of either $\underline{\underline{P}}_1$ or $\underline{\underline{P}}_2$. Which sign to change depends on the order of the filter. The stopband bandwidth for this filter is equal to the Nyquist frequency minus the bandwidth for the corresponding bandpass filter. Narrowband bandstop filters can therefore only be synthesized by iteration.

Like the lowpass- and highpass filters the bandstopfilter has no physical equivalent.

VI. POLES AND ZEROS FOR A DIF.

VI.1. The bandpass filter.

The poles for the bandpass filter are found from the matrix $\{\underline{\underline{1}}\cdot z\cdot z - \underline{\underline{P}}_1\cdot\underline{\underline{P}}_2\}$. The poles are the values of z which makes the determinant for this matrix zero. That is, the poles squared are found as eigenvalues for the matrix $\underline{\underline{P}}_1\cdot\underline{\underline{P}}_2$.

There are only two zeroes for the bandpass filter, at f = 0 and at the Nyquist frequency.

VI.2. The lowpass filter.

The poles for the lowpass filter are found as eigenvalues for the matrix $-\underline{\underline{P}}_1\cdot\underline{\underline{P}}_2$. The filter has only one zero at the Nyquist frequency.

VI.3. The highpass filter.

The poles for the highpass filter are found as the eigenvalues for the matrix $\underline{\underline{P}}_1\cdot\underline{\underline{P}}_2$. It has one zero at 0.

VI.4. The bandstop filter.

For the bandstop filter, the poles squared are found as the eigenvalues for the matrix $-\underline{\underline{P}}_1\cdot\underline{\underline{P}}_2$. There is one zero at half the Nyquist frequency.

VII. THE "UNFOLDED" FLOW DIAGRAM FOR A DIF.

In fig.4 the signal flow diagram for a bandpass filter has been "unfolded", showing that the flow diagram is repetitious in its nature.

In the flow diagrams fig.2 and fig.5 the input- and output parameters are vectors, but of course, as shown in fig.1 and fig.4, the input and output parameters are the vol-tages V_g and V_o.

VIII. THE FILTER REALIZATION.

The flow diagram in fig.4 is repetitious. The same operation is repeated N times for an N.th order filter (an N strip filter), This means, that the filter may be realized in an economical way using this flow diagram and using existing signal processors.

It may be possible to design a fast signal processor especially suited to the band-matrix vector multiplications shown in the matrix flow diagrams for a DIF or to the "unfolded" flow diagram in fig. 4. Thus, e.g. high frequency high order digital bandpass filters may be realizable as DIF filters.

IX. ANALYSIS METHOD FOR A DIF.

As mentioned earlier, an analysis method (program) already exists for digital interdigital filters, because the signal flow diagram for the digital bandpass filter is identical to the signal flow diagram for a microwave interdigital transmission line bandpass filter.

The analysis method described in [1] - [4] can thus be used with only a few modifications.

Of course programs for the analysis of digital filters exist, which may be used as well.

X. A 7.TH ORDER 0.5 dB RIPPLE CHEBYSCHEV FILTER.

In fig.6 is shown the normalized 7th. order 0.5 dB ripple Chebyshev lowpass filter, which forms the starting point in the synthesis of an interdigital microwave filter or a DIF. The filter coefficient matrices $\underline{\underline{P}}_1$ and $\underline{\underline{P}}_2$ for the resulting 7.th order 0.5 dB ripple Chebyshev bandpass filter are given in (11) and (12). The filter is designed with a 10% relative bandwidth.

In the matrix $\{\underline{\underline{1}}-\underline{\underline{P}}_1\}$ only the element (1,1) (= 0.0866) (two times the value for the equivalent microwave filter) need be known. The amplification at the center frequency will be 1. For the equivalent microwave filter the amplification will be 0.5.

In fig.7 is shown the output from a digital filter analysis program, where the flow diagram shown in fig.4 was used. Two curves are shown, one with the exact filter coefficients, and one with quantized (5 bit) filterkoefficients. (two's complement 6 bit signed fraction, or 5 bit unsigned fraction).

XI. CONCLUSION.

A new type of wave digital filter, a digital interdigital filter (DIF), or a "true" wave digital filter, is described. The flow diagram is simple and a special signal processor may be designed which can utilize this flow diagram, thus making it possible to construct very fast high order digital filters.

Synthesis methods and analysis tools already exist, because the flow diagram is exactly the same as the flow diagram for a microwave interdigital transmmission line filter.

Sensitivities will be low, roundoff noise will be low and dynamic properties will be fine. because the prototype filter is a passive filter with low sensitivities.

The bandpass DIF contains the minimum number of multipliers in relation to the filter order.

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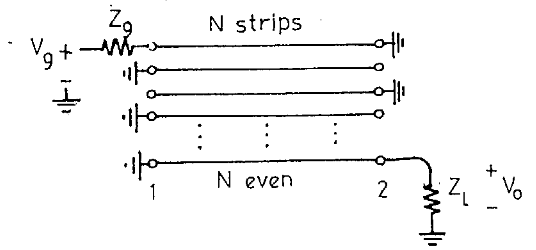

Fig.1 An even order interdigital transmission line filter shown schematically.

$$\underline{V}_g = \{V_g, 0, \cdots 0\}^T, \qquad \underline{V}_1 = \{V_{11}, V_{21}, \cdots V_{N1}\}^T$$

$$\underline{V}_2 = \{V_{12}, V_{22}, \cdots V_{N2}\}^T, \qquad V_o = V_{N2}. \tag{1}$$

$$\underline{\underline{Z}}_1 = \begin{bmatrix} Z_g & & & & & \\ & 0 & & & & \\ & & \infty & & & \\ & & & 0 & & \\ & & & & \infty & \\ & & & & & \cdot \\ & & & & & & 0 \end{bmatrix}, \quad \underline{\underline{Z}}_2 = \begin{bmatrix} 0 & & & & & \\ & \infty & & & & \\ & & 0 & & & \\ & & & \infty & & \\ & & & & \cdot & \\ & & & & & 0 \\ & & & & & & Z_1 \end{bmatrix}, \tag{2}$$

$$\underline{V}_1 = \underline{A}+\underline{B} = \underline{V}_g - \underline{\underline{Z}}_1 \cdot \underline{I}_1 , \quad \underline{V}_2 = \underline{A}\cdot z^{-1} + \underline{B}\cdot z = \underline{\underline{Z}}_2 \cdot \underline{I}_2 \tag{3}$$

$$\underline{I}_1 = v\cdot\underline{\underline{C}}\cdot(\underline{A}-\underline{B}) , \qquad \underline{I}_2 = v\cdot\underline{\underline{C}}\cdot(\underline{A}\cdot z^{-1} - \underline{B}\cdot z) \tag{4}$$

$$\underline{A} = (\underline{\underline{Z}}_1\cdot v\cdot\underline{\underline{C}}+\underline{\underline{1}})^{-1}\cdot\underline{V}_g + (\underline{\underline{Z}}_1\cdot v\cdot\underline{\underline{C}}+\underline{\underline{1}})^{-1}\cdot(\underline{\underline{Z}}_1\cdot v\cdot\underline{\underline{C}}-\underline{\underline{1}})\cdot\underline{B} \tag{5}$$

$$\underline{B}\cdot z = (\underline{\underline{Z}}_2\cdot v\cdot\underline{\underline{C}}+1)^{-1}\cdot(\underline{\underline{Z}}_2 v\cdot\underline{\underline{C}}-1)\cdot\underline{A}\cdot z^{-1} \tag{6}$$

$$\underline{\underline{\rho}}_1 = (\underline{\underline{Z}}_1\cdot v\cdot\underline{\underline{C}}+\underline{\underline{1}})^{-1}\cdot(\underline{\underline{Z}}_1\cdot v\cdot\underline{\underline{C}}-\underline{\underline{1}}) \tag{7}$$

$$\underline{\underline{\rho}}_2 = (\underline{\underline{Z}}_2\cdot v\cdot\underline{\underline{C}}+1)^{-1}\cdot(\underline{\underline{Z}}_2\cdot v\cdot\underline{\underline{C}}-\underline{\underline{1}}) \tag{8}$$

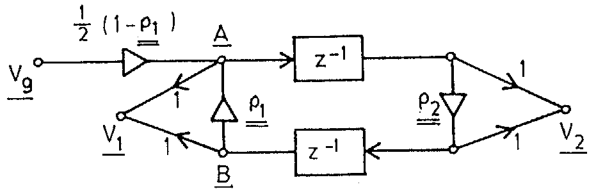

Fig.2 The flow diagram for an interdigital transmission line filter or a digital interdigital filter.

$$2\cdot\underline{V}_2 = (\underline{\underline{1}}+\underline{\underline{\rho}}_2)\cdot(\underline{\underline{1}}-\underline{\underline{\rho}}_1\cdot\underline{\underline{\rho}}_2\cdot z^{-2})^{-1}\cdot z^{-1}\cdot(\underline{\underline{1}}-\underline{\underline{\rho}}_1)\cdot\underline{V}_g \tag{9}$$

$$\underline{\underline{\rho}}_1 = \begin{bmatrix} \frac{Z_g\cdot v\cdot C_{11}-1}{Z_g\cdot v\cdot C_{11}+1} & \frac{2\cdot Z_g\cdot v\cdot C_{11}}{Z_g\cdot v\cdot C_{11}+1} & 0 & 0 \cdots\cdots & 0 \\ 0 & -1 & 0 & 0 \cdots\cdots & 0 \\ 0 & \frac{2\cdot C_{23}}{C_{33}} & 0 & \frac{2\cdot C_{34}}{C_{33}} \cdots & 0 \\ 0 & 0 & 0 & -1 \;\cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ddots\; -1 \end{bmatrix} \tag{10}$$

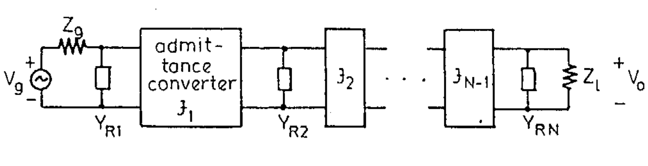

Fig.3 An exact equivalent diagram for a bandpass digital interdigital filter.

$$Y_{R1} = -j\cdot v\cdot C_{11}\cdot\cot(\theta) , \quad J_1 = v\cdot C_{12}\frac{1}{\sin(\theta)}$$

$$Y_{R2} = -j\cdot v\cdot C_{22}\cdot\cot(\theta) , \quad J_2 = v\cdot C_{23}\frac{1}{\sin(\theta)} \tag{11}$$

$$\theta = (\omega\cdot l)/v , \qquad v = 3\cdot 10^8 \text{ m/sec.}$$

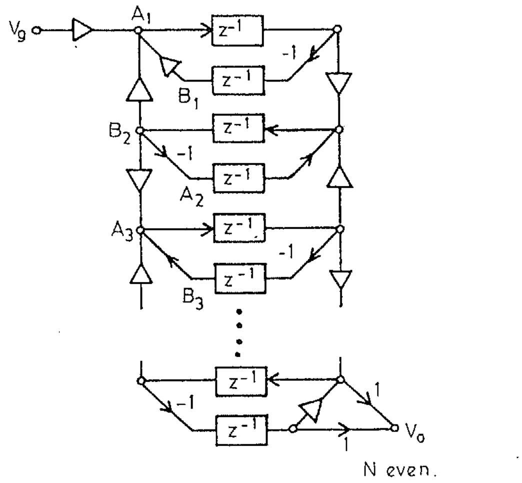

Fig.4 The unfolded flow diagram for an even order digital bandpass interdigital filter.

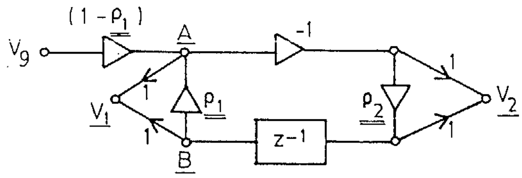

Fig.5 The lowpass filter flow diagram.

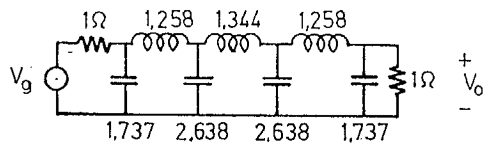

Fig.6 The normalized lowpass ladder type filter.

$$\underline{\underline{\rho}}_1 = \begin{bmatrix} .913 & -.10 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.087 & 1 & -.0842 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.0842 & 1 & -.087 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.10 & .913 \end{bmatrix} \tag{11}$$

$$\underline{\underline{\rho}}_2 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -.1083 & 1 & -.0861 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -.084 & 0 & -.084 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -.0861 & 1 & -.1083 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{12}$$

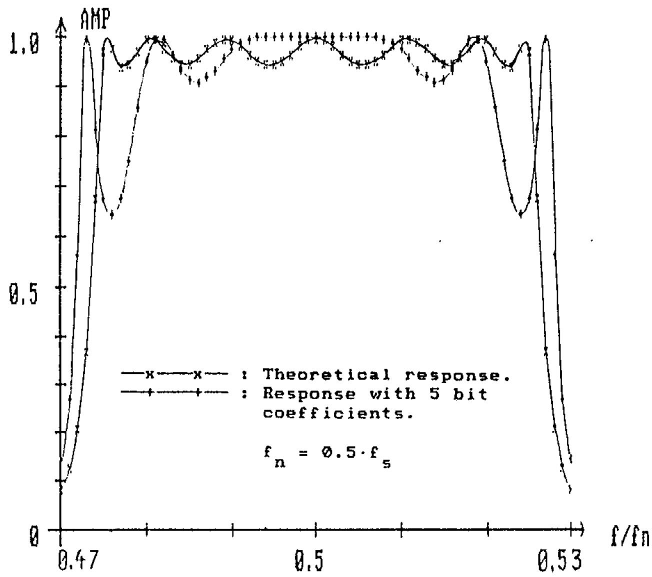

Fig.7 The response for a 7th order 0.5 dB ripple Chebyshev digital bandpass filter. Bandwidth = $0.05\cdot f_n$. Centerfrequency = $0.5\cdot f_n$.