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MICROWAVE HOLOGRAPHY FOR THREE-DIMENSIONAL OBJECTS

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RESUME

Une deuxième ordre simulation numérique d'hologramme avec la transformation discrète de Fresnel-Kirchhoff est indiquée pour des objets tridimensionnaux. On a choisi comme exemples trois différents plans, avec des distances différentes et avec une inclinaison différente par rapport aux coordonnés x, y, z ainsi qu'une sphère métallisée. La reconstruction numérique des images montre qu'il est possible d'avoir une identification sûre de ces objets. L'usage de l'algorithme "Fast Fourier Transform" assure la reconstruction informatique en l'espace de quelques secondes. Une détermination numérique est indiquée montrant un taux d'erreur de moins de 1 pourcent.

SUMMARY

A numerical second order simulation of holograms with the discrete Fresnel-Kirchhoff-transformation is shown for three-dimensional objects. As examples three various planes in different distances and differently inclined to the x,y,z-coordinates, as well as a metallized sphere, are choosen. The numerical reconstruction of the images shows that reliable identification of the objects is possible. The use of the Fast Fourier Transform algorithm provides the computer reconstruction within a few seconds. A numerical range determination is indicated which exhibits an error of only less than 1 percent.

1. INTRODUCTION

Microwave holography meanwhile has developped from its origin stage as an analogue of optical imaging to an extensive digital signal processing technique comprising data acquisition, processing and display, and using current methods like the Fast Fourier Transform (FFT), e.g.[1,2]. So a reliable identification of plane objects is possible, as has already been reported[3]. For three-dimensional objects, however, the problems of signal processing are not yet completely solved and the quality of the imaging is still unsatisfactory. Further, for true microwave holography investigated in this paper, in contrast to quasi-holography (e.g.[4]), where one of the coordinates is range, this information has additionally to be determined besides the hologram coordinates.

Therefore the objective of this paper is primarily to show suitable investigations to improve the imaging technique for three-dimensional objects. Hereto it is convenient to subdivide their z-dimension into appropriate parts, so that a suitable discrete representation of the diffraction integral is possible. By numerical computer simulation of the hologram of three rectangular objects inclined in the z-direction the demands according to necessary hologram expansion, to the sampling theorem, and to appropriate window functions are suitably determined.

Numerical reconstruction of the images from the numerical simulated as well as from the measured holograms of these three-dimensional objects demonstrates the obvious improvement of the obtained imaging quality. The use of standard microprocessors (e.g. TMS 9900) provides the numerical reconstruction within a few seconds. A metallized sphere serves as a second example of three-dimensional objects.
The paper further investigates the possibility to determine numerically the distance between the hologram (measuring) plane and the object, using the Fletcher-Powell optimization technique. Hereto, suitable criterions are defined, which allow to locate the range up to an error of less than 1 %.

2. THEORY

Since the relevant theoretical aspects have been already reported in[3], the description of the diffraction theory for three-dimensional objects can be abbreviated. The Fresnel-Kirchhoff's diffraction integral[3] is written in its discrete form, if the z-dimension of the object (Fig. 1) is subdivided into P planes with the dimensions $\Delta x_p \cdot M$ and $\Delta y_p \cdot N$

$$\underline{U}_{il} = j\frac{1}{2\lambda}\sum_{p=1}^{P}\Delta x_p\,\Delta y_p \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} O_{mnp}\cdot C_s \cdot \frac{e^{-jk s_{mnp}}}{s_{mnp}} \cdot \frac{e^{-jk r_{mnilp}}}{r_{mnilp}} \cdot [\cos\{\vec{v}_{mnp},\vec{r}_{mnilp}\} - \cos\{\vec{v}_{mnp},\vec{s}_{mnp}\}] \quad (1)$$

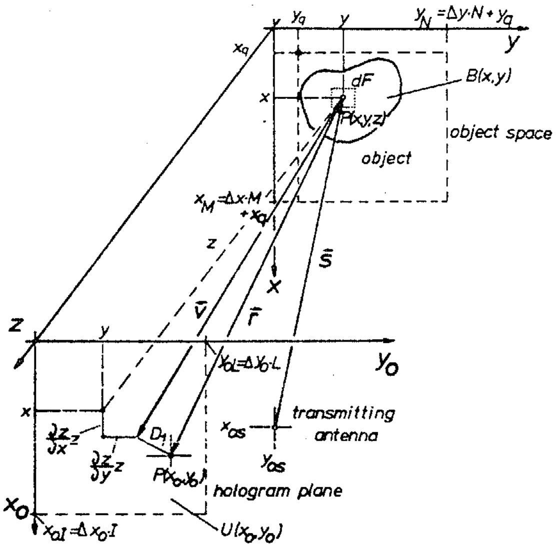


Fig. 1a

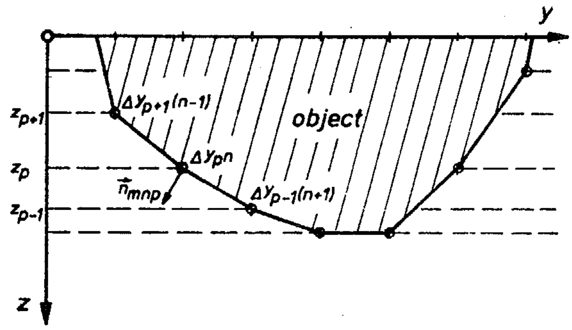


Fig. 1b

Fig. 1 Object and hologram plane
a Arrangement and related coordinates
b Subdivision of the three-dimensional object space into P planes perpendicular to the z-coordinate

where

$\underline{U}_{il} = \underline{U}(x_0, y_0) \sim \underline{E}_y(x_0, y_0)$ = complex hologram function of the diffracted wave at the point $P_0(x_0, y_0)$ in the hologram plane, (Fig. 1),

λ = wave-length,

$k = 2\pi/\lambda$,

$O_{mnp} \cdot C_s$ = field distribution of the diffracting interface (object planes)

O_{mnp} = object function of the reflecting object (e.g. for a rectangular plate $O_{mnp} = 1$),

C_s = constant of the radiating set (it can be for instance chosen to be 1m since only relative results are of interest),

$$s_{mnp} = [(x_{os} - x_q - \Delta x_p m)^2 + (y_{os} - y_q - \Delta y_p \cdot n)^2 + z_p^2]^{1/2}$$

$$r_{mnilp} = [(\Delta x_0 \cdot i - x_q - \Delta x_p \cdot m)^2 + (\Delta y_0 \cdot l - y_q - \Delta y_p \cdot n)^2 + z_p^2]^{1/2}$$

$$\vec{v} = \vec{n} \cdot v$$

$$v = [1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2]\cdot z$$

$$\cos\{\vec{n},\vec{r}\} = \cos\{\vec{v},\vec{r}\} = \frac{r^2+v^2-D_1^2}{2rv}$$

$$\cos\{\vec{n},\vec{s}\} = \cos\{\vec{v},\vec{s}\} = -\frac{s^2+v^2-D_2^2}{2rv}$$

$$D_1^2 = (x_0 - x - \frac{\partial z}{\partial x} z)^2 + (y_0 - y - \frac{\partial z}{\partial y} z)^2$$

$$D_2^2 = (x_{0s} - x - \frac{\partial z}{\partial x} z)^2 + (y_{0s} - y - \frac{\partial z}{\partial y} z)^2$$

$\vec{n}$ = direction of the area dF.

Equation (1) allows to simulate for three-dimensional objects numerically amplitude and phase of the hologram function $\underline{U}_{il}$ at a point x_i, y_l in the hologram plane. Note that this result is a second-order approximation, since the theory is based on the Fresnel-Kirchhoff-integral.

For the computer reconstruction, however, the simpler Fresnel-integral has turned out to be sufficient[3]:

$$\underline{B}_{mn} = -j\frac{\Delta x_0 \Delta y_0}{\lambda z} \exp(jkz) \cdot \exp[\{(x_q+\Delta x \cdot m)^2 + (y_q+\Delta y \cdot n)^2\}\frac{jk}{2z}] \qquad (2)$$
$$\cdot \sum_{i=0}^{I-1} \sum_{l=0}^{L-1} \underbrace{\underline{U}_{il} \cdot \exp\{j\frac{\pi}{\lambda z}[(\Delta x_0 \cdot i)^2 + (\Delta y_0 \cdot l)^2]\}}_{\underline{G}_{il}}$$
$$\cdot \exp\{-j\frac{2\pi}{\lambda z}[(x_q+\Delta x \cdot m)\Delta x_0 \cdot i + (y_q + \Delta y \cdot n)\Delta y_0 \cdot l]\}$$

The hologram plane is subdivided into I, and L sections: $\Delta x_0 \cdot I$, $\Delta y_0 \cdot L$. Equation (2) can be interpreted as a two-dimensional discrete Fourier-transform of the discrete complex function $\underline{G}_{il}$ ($i=0...I-1$, $l=0...L-1$). If I and L are suitably chosen, standard two-dimensional FFT-routines can be used.

3. RESULTS

As first example to demonstrate the computer simulation of holograms and the computer reconstruction of three-dimensional objects, an arrangement of three various rectangular planes, in different distances and differently inclined to the x,y,z-coordinates, is investigated. Figs. 2a,b show the arrangement of the planes.

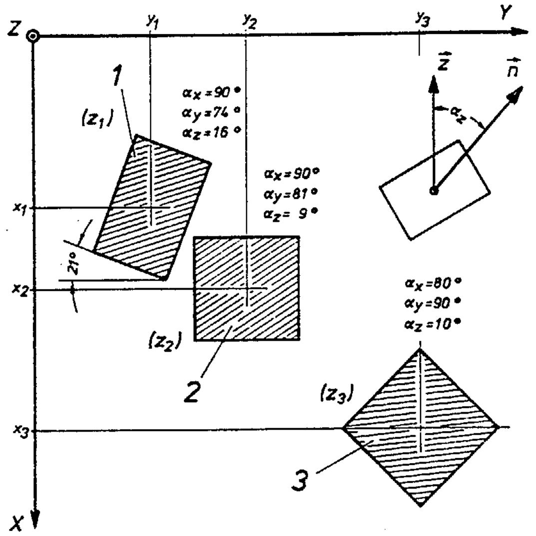


Fig. 2a

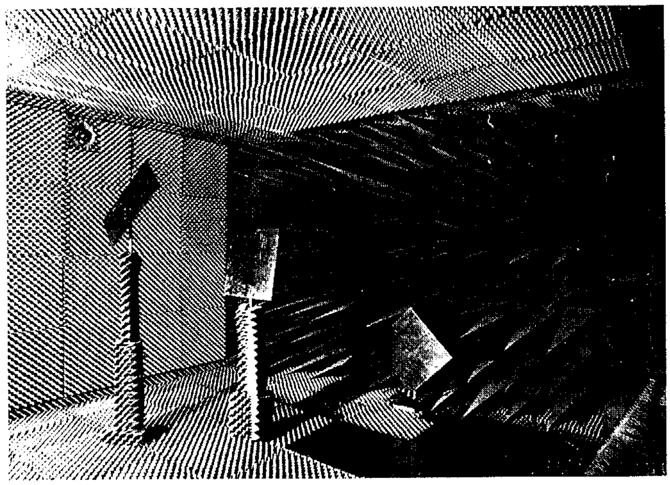


Fig. 2b

ARRANGEMENT AND RELATED COORDINATES

PLANE 1 (A1=0.51 M ; B1=0.51 M):
X1=1.82 M ; Y1=1.79 M ; Z1=5.17 M

PLANE 2 (A2=0.48 M ; B2=0.48 M):
X2=1.17 M ; Y2=0.99 M ; Z2=4.55 M

PLANE 3 (A3=0.37 M ; B3=0.57 M):
X3=0.79 M ; Y2=0.55 M ; Z3=3.93 M

Fig. 2 Three rectangular planes (a x b) placed at different distances and inclined to the x,y,z-coordinates

Fig. 3a shows the simulated, Fig. 3b the measured hologram. The related data are given also in Fig. 3. The sampling theorem for spatial spectra[3] is held. Since the discrete computer simulation provides periodically repeating holograms, a suitable window-function is chosen, which reduces all amounts outside of $\frac{\lambda \cdot z}{\Delta x}$, $\frac{\lambda \cdot z}{\Delta y}$ to zero. As can be stated by comparison of the measured hologram with the computer simulated hologram (Fig. 3) their coincidence is quite good. The slight displacement of the measured amplitude hologram is due to a corresponding displacement of the measuring plane in relation to the simulated hologram plane. This can also be stated at the reconstructed images (cf. Fig. 4). Such a simulation of holograms (together with related reconstruction) of three-dimensional objects allows one to investigate accurately the necessary conditions for good image reconstruction. The data given in Fig. 3 and 4 provide an appropriate compromise between accuracy related time and storage requirements due to the capability of the TI 990 computer.

Fig. 4 shows the computer reconstruction of the three inclined planes (Fig. 2) from the simulated hologram together with the related reconstruction from the measured hologram. To facilitate the corresponding identification, the three reconstructed images of each plane are presented separately. For this purpose, Equation (2) is so written, that the dimensions and the initial coordinates x_q, y_q of the object planes (Fig. 1) can appropriately be chosen. For this the discrete Fourier transform (DFT) is used. Comparison with the original images (Fig. 2) shows that clear identification of the three inclined planes is possible.

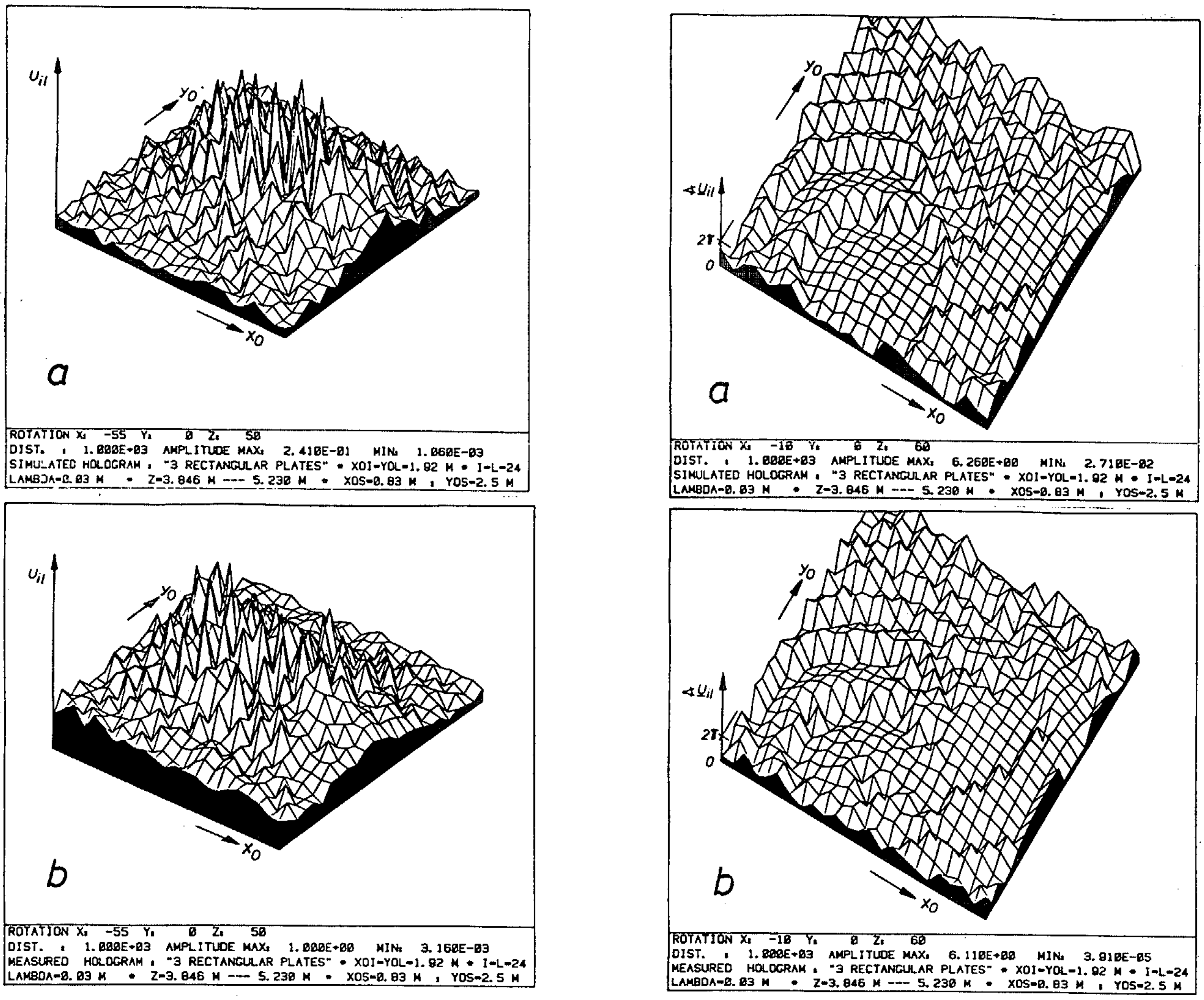


Fig. 3 Hologram of the three planes (Fig. 2)

a Computer simulated

b Measured

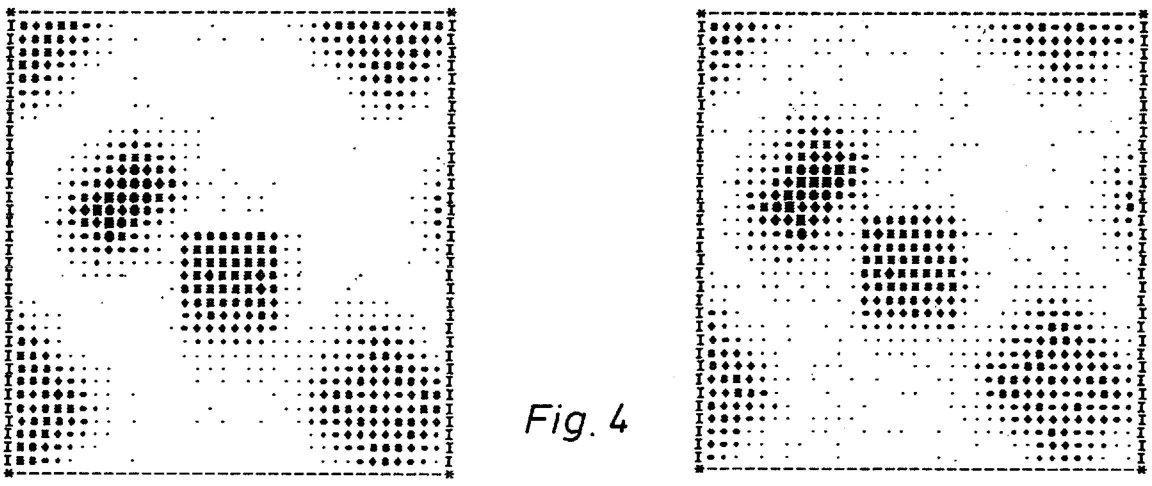
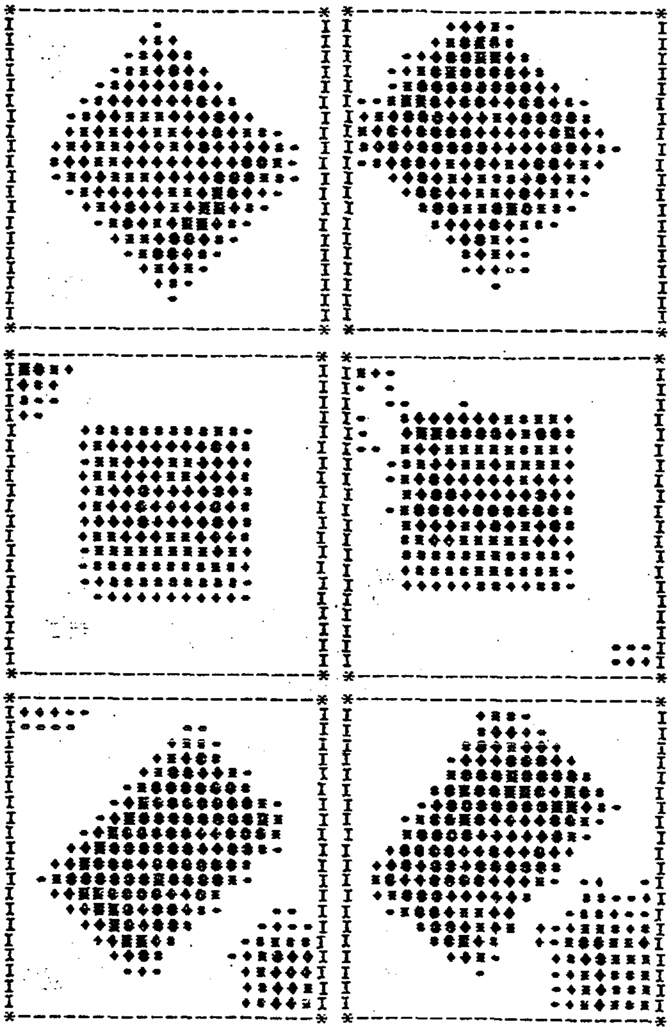


Fig. 4

3 PLANES , DATA OF RECONSTRUCTION

		X-DIR.	Y-DIR.
OBJECT PLANE	:	2.00 M	2.00 M
SAMPLE POINTS	:	34	34
SAMPLE DISTANCE	:	0.059M	0.059M
TRANSM. ANTENNA	:	0.83 M	2.50 M
X0,Y0 - (FIG.1)	:	0.00 M	0.00 M

WAVELENGTH : 0.03 M ; MEAN DISTANCE : 4.55 M



A) PLANE 1 , DATA OF RECONSTRUCTION

		X-DIR.	Y-DIR.
OBJECT PLANE	:	0.80 M	0.80 M
SAMPLE POINTS	:	20	20
SAMPLE DISTANCE	:	0.04 M	0.04 M
TRANSM. ANTENNA	:	0.83 M	2.50 M
XQ,YQ - (FIG.1)	:	1.42 M	1.39 M

WAVELENGTH : 0.03 M ; MEAN DISTANCE : 5.17 M

B) PLANE 2 , DATA OF RECONSTRUCTION

		X-DIR.	Y-DIR.
OBJECT PLANE	:	0.80 M	0.80 M
SAMPLE POINTS	:	20	20
SAMPLE DISTANCE	:	0.04 M	0.04 M
TRANSM. ANTENNA	:	0.83 M	2.50 M
XQ,YQ - (FIG.1)	:	0.77 M	0.59 M

WAVELENGTH : 0.03 M ; MEAN DISTANCE : 4.55 M

C) PLANE 3 , DATA OF RECONSTRUCTION

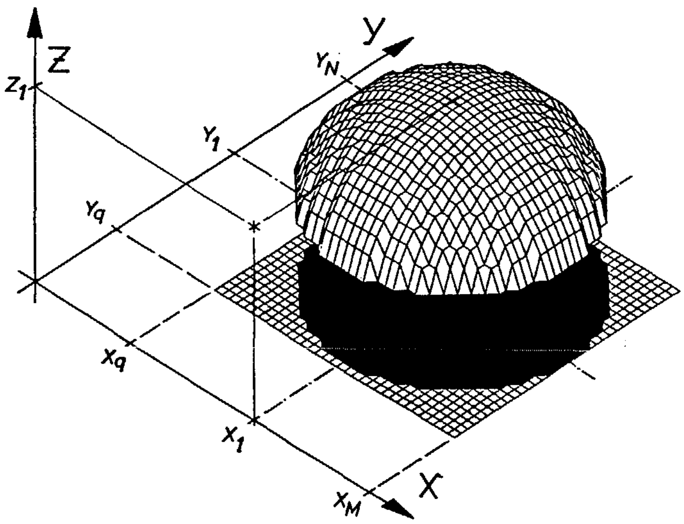
		X-DIR.	Y-DIR.
OBJECT PLANE	:	0.74 M	0.74 M
SAMPLE POINTS	:	20	20
SAMPLE DISTANCE	:	0.037M	0.037M
TRANSM. ANTENNA	:	0.83 M	2.50 M
XQ,YQ - (FIG.1)	:	0.42 M	0.18 M

WAVELENGTH : 0.03 M ; MEAN DISTANCE : 3.93 M

Fig. 4 Computer image reconstruction of the three inclined planes (Fig. 2)

(S = from the computer simulated hologram)
(M = from the measured hologram)

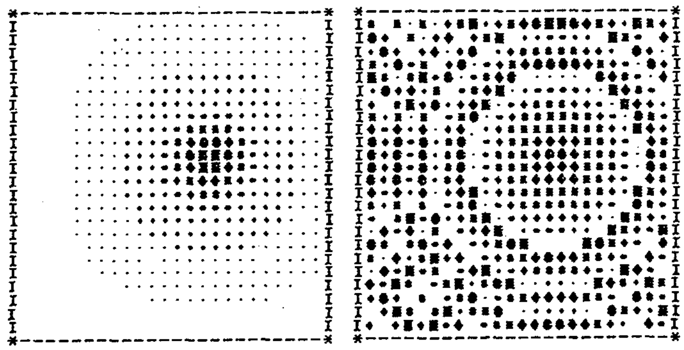
The second three-dimensional object investigated in this paper is a metallized sphere (Fig. 5). The object function is assumed to be $O_{mnp}=1$ everywhere on the surface of the sphere. The computer simulated hologram (amplitude and phase) is shown in Fig. 6. The computer image reconstruction (Fig. 7) of the metallized sphere from the simulated hologram shows also -like the three inclined planes- that a reliable identification is possible. Note that related to the limited hologram dimension only the image of a spherical calotte is obtained. The contour of the whole sphere is indicated by the dotted circle.



ARRANGEMENT AND RELATED COORDINATES

SPHERE (DIAMETER = 0.78 M):
XQ=0.406M ; X1=0.808M ; XM=1.21 M
YQ=0.756M ; Y1=1.158M ; YN=1.56 M
Z1=4.16 M

Fig. 5 Metallized sphere. Projection of the hemisphere



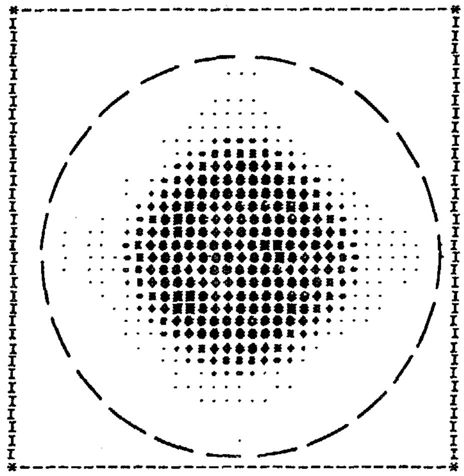
DATA OF THE 3D - SIMULATION

		X-DIR.	Y-DIR.
HOLOGRAM PLANE	:	1.92 M	1.92 M
SAMPLE POINTS	:	24	24
SAMPLE DISTANCE	:	0.08 M	0.08 M
TRANSM. ANTENNA	:	0.83 M	1.16 M

WAVELENGTH : 0.03 M ; MEAN DISTANCE : 4.16 M --- 4.45 M

Fig. 6 Computer simulated hologram of the metallized sphere (Fig. 5)

a Amplitude hologram
b Phase hologram



DATA OF THE RECONSTRUCTION

		X-DIR.	Y-DIR.
OBJECT PLANE	:	0.804M	0.804M
SAMPLE POINTS	:	34	34
SAMPLE DISTANCE	:	0.024M	0.024M
TRANSM. ANTENNA	:	0.83 M	1.16 M
X0,Y0 - (FIG.1)	:	0.406M	0.756M

WAVELENGTH : 0.03 M ; MEAN DISTANCE : 4.16 M

Fig. 7 Computer image reconstruction of the metallized sphere (Fig. 5) (from the computer simulated hologram)

4. RANGE DETERMINATION

Since the sum of the amount of the complex object function B_{mn} (Equation (2)) over m=0...M-1 and n=0...N-1 yields an optimum if the variable distance to the hologram plane z_p in Equation (1) equals the real distance of the measuring plane, this fact can be used as a criterion for range determination. The real or imaginary parts, or the phase of B_{mn} cannot be utilized since these values are ambiguous. The error function

$$EF = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \left(1-\left(\frac{B_{mn}}{B_{mn\,MAX}}\right)^{EXX}\right)^{EXP} \qquad (3)$$

yields a minimum if z_p is equal to the real distance. For a quadratic plane as example, Fig. 8 shows this error function vs. the variable distance z_p. If the exponents in Equation (3) are suitably chosen, an unambigous global minimum can be obtained. For the given exampel, the exponents are chosen to be EXX=3, EXP=2. Fig. 8 shows this minimum at z_p=8m, the real distance for this example.Also there are incidated two computed reconstructed images of the quadratic plane. The reconstruction at the real distance provides the exact image, whereas the reconstruction at about z_p=13m for instance yields a very poor image.
Note, that the application of the FFT algorithm yields a size expansion of the reconstructed area of $x_M = y_N = \frac{\lambda \cdot z}{\Delta x_0}$.

An optimizing computer program is used[5], which varies the parameter z_p until the desired minimum is obtained. The computer optimization has to be appropriately programmed so, that the local minima (Fig. 8) are omitted. For this example the real distance of z_p=8m was found within 4 iteration steps with an error of less than + 1 %.

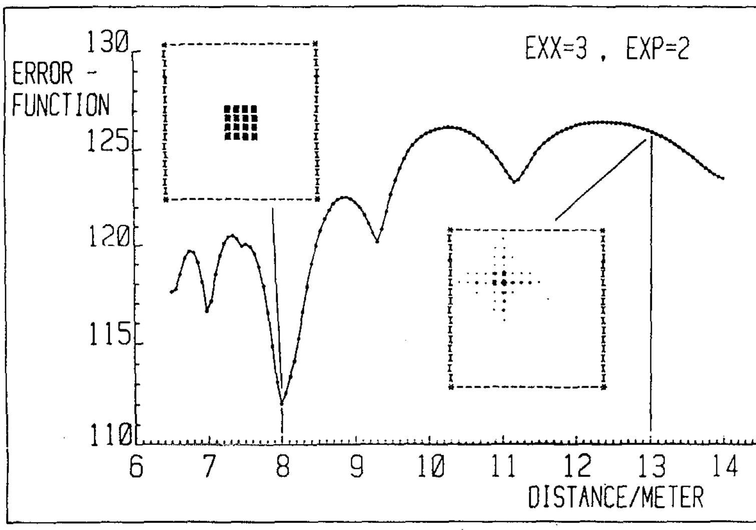


Fig. 8 Range determination.
Error function vs. hologram plane distance z_p (Equation (3)) showing an unambiguous minimum for the real distance

CONCLUSION

Computer simulation of three-dimensional objects allows to investigate accurately the necessary conditions for reliable image reconstruction (sampling theorem, hologram plane dimension, window function etc.). The examples of three different planes in different distances and differently inclined to the coordinates x,y,z, as well as a metallized sphere, show that a clear identification of the objects is possible. Using the FFT algorithm, computer reconstruction is possible within a few seconds. A suitable error function provides the possibility of numerical range determination.

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