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PROPAGATION OF PHASE-COHERENCE IN A SHALLOW-WATER WAVEGUIDE

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RESUME

PROPAGATION DE LA COHERENCE DE PHASE
DANS LES EAUX PEU PROFONDES
CONSTITUANT UN GUIDE D'ONDES

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RESUME

Modelisation de la propagation dans des eaux peu profondes constituant un guide d'ondes se fait habituellement en fonction des modes normaux. Un article précédent démontrait que, dans le cas d'un guide d'ondes determiné, le calcul de la cohérence verticale de phase, basé sur la théorie des modes normaux, ne pose pas de problèmes particuliers. Le présent article montre comment la cohérence verticale dans le cas d'une antenne acoustique verticale permet de calculer la cohérence dans un autre plan si l'on suppose une propagation par ondes stationnaires dans le guide d'ondes entre les deux plans verticaux. L'article présente des résultats pour une série de guides d'ondes et pour différentes distances.

SUMMARY

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SHALLOW WATER WAVEGUIDE

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ABSTRACT

Propagation in a shallow water waveguide is usually modelled in terms of normal modes. Using the normal mode solution the calculation of vertical phase-coherence for a particular waveguide was shown in a previous paper to be a straight forward matter. In this paper it is shown how the knowledge of the vertical coherence for a top-to-bottom array permits the calculation of the coherence on another plane assuming stationary waveguide propagation between the two vertical planes considered. Results for various waveguides and different ranges are given.

PROPAGATION OF PHASE-COHERENCE IN A SHALLOW WATER WAVEGUIDE

I. INTRODUCTION

The propagation through a dispersive shallow water waveguide can be described in terms of the mutual coherence function between separated receivers in an acoustic field. Fig 1 shows the general geometry where at the receiving area A the complex amplitudes are created by a source element ds. The different source elements are mutually incoherent and statistically independent with zero mean value. It was shown (1,2,5,7,8) that the superposition of the signals at the receiver area A will depend on the phase correlation and the power in this area. To calculate the power at a point P_1' from received signals, the phase properties from A to B have to be known. In general there is no difference when this phase property is cast in the format of general beamforming or a suitable propagation model.

According to Fig 1 the power at P_1' contributed from one source element ds can be expressed by:

$$dE_1' = (c_{01}c_{11}+c_{02}c_{21})(c_{01}^* c_{11}^* + c_{02}^* c_{21}^*)\,ds$$
$$= |c_{01}|^2|c_{11}|^2 ds + |c_{02}|^2|c_{21}|^2 ds$$
$$+ 2\,\mathrm{Re}\{c_{01} c_{02}^* c_{11} c_{21}^*\}\,ds$$

The total powers received at P_1, P_2 are calculated as the integral over all source elements and with:

$$E_1 = \int_S |c_{01}|^2 ds \quad \text{and} \quad E_2 = \int_S |c_{02}|^2 ds$$

the quantities at P_1' and P_2'

$$E_1' = E_1|c_{11}|^2 + E_2|c_{21}|^2 + 2\sqrt{E_1E_2}\,\mathrm{Re}\{\gamma_{12} c_{11} c_{21}^*\} \tag{1.1}$$

and

$$E_2' = E_1|c_{12}|^2 + E_2|c_{22}|^2 + 2\sqrt{E_1E_2}\,\mathrm{Re}\{\gamma_{12} c_{12} c_{22}^*\} \tag{1.2}$$

are time averaged powers at these points.

The phase coherence between the points P_1 and P_2 is now defined (5) as:

$$\gamma_{12} = (E_1E_2)^{-\frac{1}{2}} \int_S c_{01} c_{02}^*\,ds \tag{2}$$

It has been shown (7) that the waveform in a receiving area can be expressed in terms of this phase-coherence being a function of source and receiver locations, frequency and type of waveguide.

It was reported first by F. Zernicke (8) how the knowledge of the phase-coherence on the first plane permits the calculation on a distant plane. The method derived by H. Hopkins (5) will be used here, implying that the phase-coherence on the plane of departure and the propagation constants of the acoustic waveguide are known.

It is worthwhile mentioning at the beginning that often the use of the words 'coherence' and 'correlation' is based on identical formulations as indicated in the Appendix. The word phase coherence is used here concurrently with (5) because the calculated quantity is governed by the phase correlation. For the sake of consistency with most of the papers referenced the word 'coherence' is used according to this Appendix.

II. PHASE-COHERENCE PROPAGATION

Using the complex amplitude C_{01}, C_{02}, Fig 1, for the calculation of the coherence on the receiver area A it is demonstrated how the coherence known on any given surface permits the calculation on another surface. This has been shown first by Zernicke (8) and a simpler method based on this has been derived by Hopkins (5). The latter method will be used here to obtain the desired quantity on the plane distant from the first. To calculate the phase-coherence for a given set of parameters the propagation model formula derived in (7) is rewritten here as

$$\gamma_{12} = C \int_S \sum_{nm} A_{12}^{n,m} \exp\{i\,\phi^{n,m}\} \cdot \exp\{i\,\Theta_{12}^{n,m}\}\,ds \tag{3}$$

where $A_{12}^{nm} = A^n(z_1)\,A^m(z_2)/\sqrt{r_1 r_2}$

and $\Theta_{12}^{nm} = k^n r_1 - k^m r_2$

are the amplitudes and phase at points P_1 and P_2.

The mode-dependent phase Θ_{12}^{nm} according to the Clay-model (1) for a slightly irregular sea-surface has been included.

Let $d\Sigma$ and $d\Sigma'$ be elements on the receiving area A and suppose the transmission functions from one area A to another area B are expressed in terms of a general transmission function and the summation over all the contributing elements

$$c_{11}' = \int_\Sigma c_{01}\,T_{01}\,d\Sigma \tag{4.1}$$

$$c_{22}' = \int_{\Sigma'} c_{02}\,T_{02}\,d\Sigma' \tag{4.2}$$

where T_{01} is a general transmission function, depending on particular waveguide parameters.
Using the definition of eq (2) for the coherence calculations on area B, yields

$$\gamma_{12}' = \int_S c_{11}'\,c_{22}'^*\,ds \tag{4.3}$$

$$= \iint_{\Sigma\Sigma} \left[\int_S c_{01} c_{02}^*\,ds\right] T_{01}\,T_{02}^*\,d\Sigma\,d\Sigma$$

$$= \iint_{\Sigma\Sigma'} \gamma_{12}\, T_{01}\, T_{02}^{*}\, d\Sigma\, d\Sigma'$$

The coherence γ'_{12} on area B is now expressed by the coherence γ_{12} on area A and the transmission function between those areas. Inserting in (4.3) the normal mode solution permits

(4.4) $$\gamma'_{12} =$$

$$(E_1 E_2)^{-\frac{1}{2}} \int_S \left\{ \frac{ik^n}{2\pi} \int_\Sigma \frac{C_{01}}{\sqrt{\Delta r_1}} \exp\left[k^n \Delta r_1 - \pi/4\right] d\Sigma \right\} *$$

$$* \left\{ -\frac{ik^m}{2\pi} \int_{\Sigma'} \frac{C_{02}^{*}}{\sqrt{\Delta r_2}} \exp\left[k^m \Delta r_2 - \pi/4\right] d\Sigma' \right\} ds$$

The factor $ik/2\pi$ has to be inserted to identify a plane wave result, when no waveguide propagation is considered. Rearranging the integration in eq (4.4) yields:

$$\gamma'_{12} = C_1 \iint_{\Sigma\Sigma'} \left\{ \int_S C_{01} C_{02}^{*}\, ds \right\} \frac{1}{\sqrt{\Delta r_1 \Delta r_2}} *$$

$$* \exp\left\{ -i\left[k^n \Delta r_1 - k^m \Delta r_2\right] \right\} d\Sigma\, d\Sigma'$$

$$\text{where} \quad C_1 = (E_1' E_2')^{-1/2} \frac{k^n k^m}{4\pi^2} (E_1 E_2)^{1/2}$$

The inner integral is the phase-coherence on the first plane modified by the normalizing factor C_1. Rewriting the phase-coherence on the second plane in terms of the phase coherence of the first plane, gives

(4.5)

$$\gamma'_{12} = C_1 \iint_{\Sigma\Sigma'} \frac{\gamma_{12}}{\sqrt{\Delta r_1 \Delta r_2}} \exp\left\{ -i\left(k^n \Delta r_1 - k^m \Delta r_2\right) \right\} d\Sigma\, d\Sigma'$$

where the integration is extended over the first plane. In other words, the points P_1, P_2 are made to explore the surface A independently. Clearly the new phase-coherence γ'_{12} depends on the paths differences Δr_1 and Δr_2 and the propagation constants k^n and k^m. For two vertically separated planes the distance Δr_1 equals Δr_2 simplifying the integral eq (4.5) to

(4.6)
$$\gamma'_{12} = C_1 \iint_{\Sigma\Sigma'} \frac{\gamma_{12}}{\Delta r} \exp\left\{ -i\left(k^n - k^m\right)\Delta r \right\} d\Sigma\, d\Sigma'$$

III. RESULTS

The normal mode program (9) has been used to calculate for a given set of waveguide parameters the amplitudes and phases of a given vertical array extending from top to bottom. The modulus and phase for a shallow water waveguide are shown in Fig 3 for three different bottom types. Density, attenuation and compressional speed are indicated on the right. A single source at depth SD = 50 m insonifies a vertical plane distanced at 2 km. A waveheight of $\sigma = 1$(m) rms according to the Clay model (1) has been used to simulate the irregularities of the sea surface.

The modulus and phase for the coherence are shown in contoured levels, ranging from 0 to 1 for the modulus and from -1 to 1 (normalized to π) for the phase.

The scales on the squares are the depth of the waveguide. Picking one level at a point in the square, gives the two sensor locations at the depths shown on the two scales; one square is for the modulus and one for the phase. In general, Fig 3 demonstrates that a point source is not perceived as a point source anymore otherwise the modulus of coherence would be constantly $\gamma = 1$ indicating complete coherence. It has been shown (2) that the spatial uncertainties caused by the rough boundaries of the waveguide are a spatial spread of the sound field, hence a point source is received as a source of finite extent. In the case where the sound field is described in terms of normal modes, each mode travelling with a different group velocity and having various interference wavelength, the receiving directional array will 'see' a source of finite extent because of the modal spread of energy.

In Fig 3 coherent areas below the mid-depth of the watercolumn indicate how to receive the signals best, in other words, where to place an array. A relatively low frequency has been selected to make sure that the modal spread among the modes is not negligible and a short range was chosen to have a small loss of coherence of one mode relative to the other. This actually means all the generated modes are contributing to the simulated sound field at this distance.

Fig 4 shows the vertical coherence for three ranges 2.5, 5 and 7.5 km calculated for the parameters indicated on the graph. The same vertical coherence at the range of 5 km and 10 km is now calculated according to eq 4.5 using the coherence of the plane 2.5 km distant from the source. The integration is now extended to twice the coherence area. As in fact the numerical integration is carried out as a summation over only 21 receiving points the results for the two methods of calculating the vertical coherence are slightly different. By increasing the number of gridding points the theoretical identity will become better approximated by numerical summations.

APPENDIX

RELATIONSHIP BETWEEN EQUATIONS USED IN OPTICS AND THOSE USED IN ACOUSTICS

(Taken from 'Systems and Transforms with applications in Optics'
A. Papoulis, McGraw-Hill, pages 259 & 260)

EQUATION	DESCRIPTION OPTICS	ACOUSTICS
$R_{xx}(P_1,P_2)=E\{x(P_1)x^*(P_2)\}$ $R_{xy}(P,Q)=E\{x(P)y(Q)\}$	Not Clear; sometimes use the same as given below	Autocorrelation function Cross-correlation function
$C_{xx}(P_1,P_2)$ $=E\{[x(P_1)-E\{x(P_1)\}]\cdot[x^*(P_2)-E\{x^*(P_2)\}]\}$ $=R_{xx}(P_1,P_2)-E\{x(P_1)\}E\{x^*(P_2)\}$	Self-coherence function	Auto-covariance function
$C_{xy}(P,Q)$ $=R_{xy}(P,Q)-E\{x(P)\}E\{y^*(Q)\}$	Mutual-coherence function	Cross-covariance function
$r_{xy}=\dfrac{C_{xy}(P,Q)}{\sqrt{C_{xx}(P,Q)\,C_{yy}(P,Q)}}$	Complex degree of coherence	Correlation coefficient

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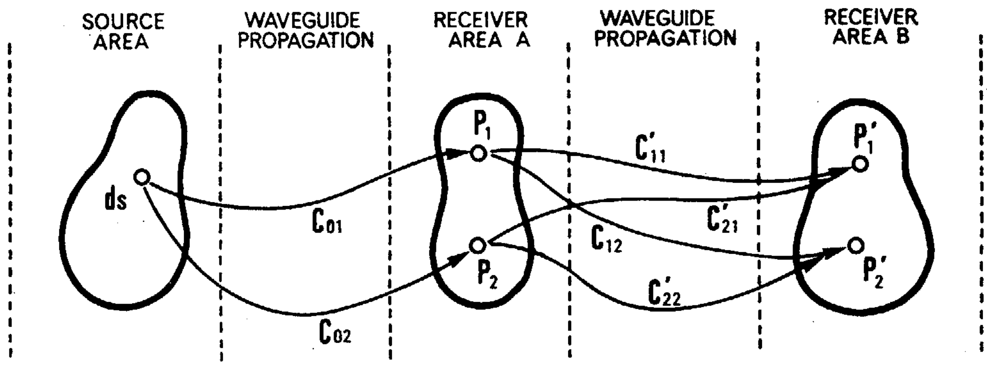

FIGURE 1

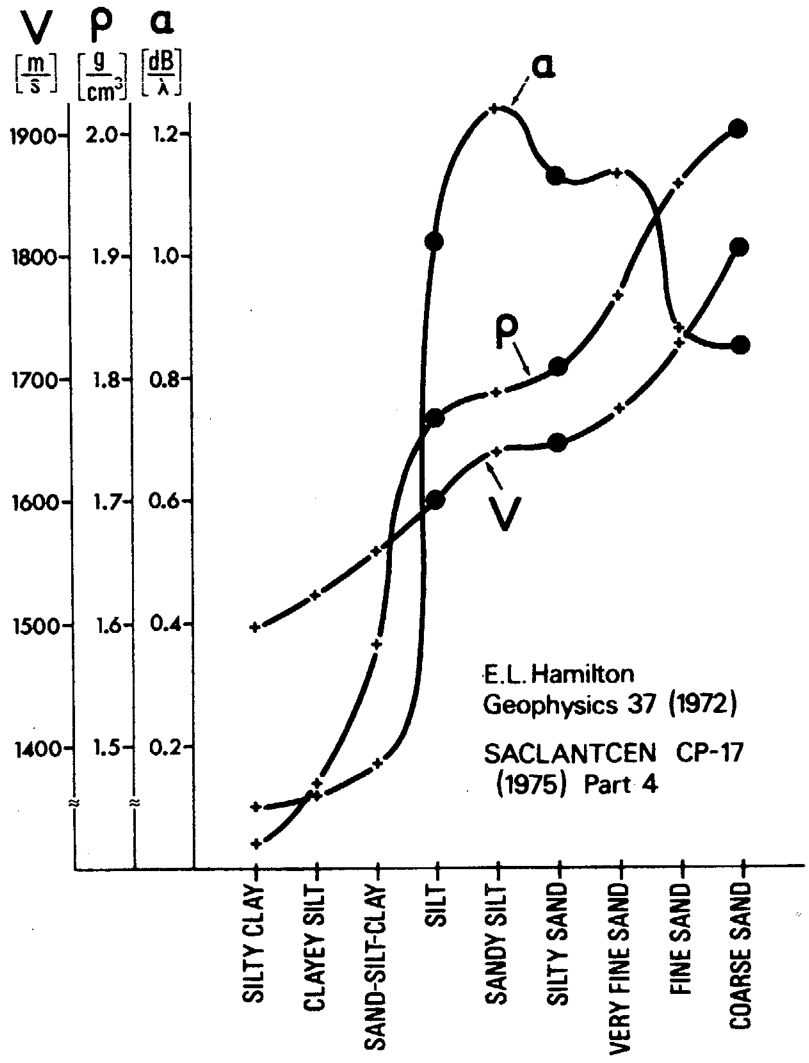

FIGURE 2

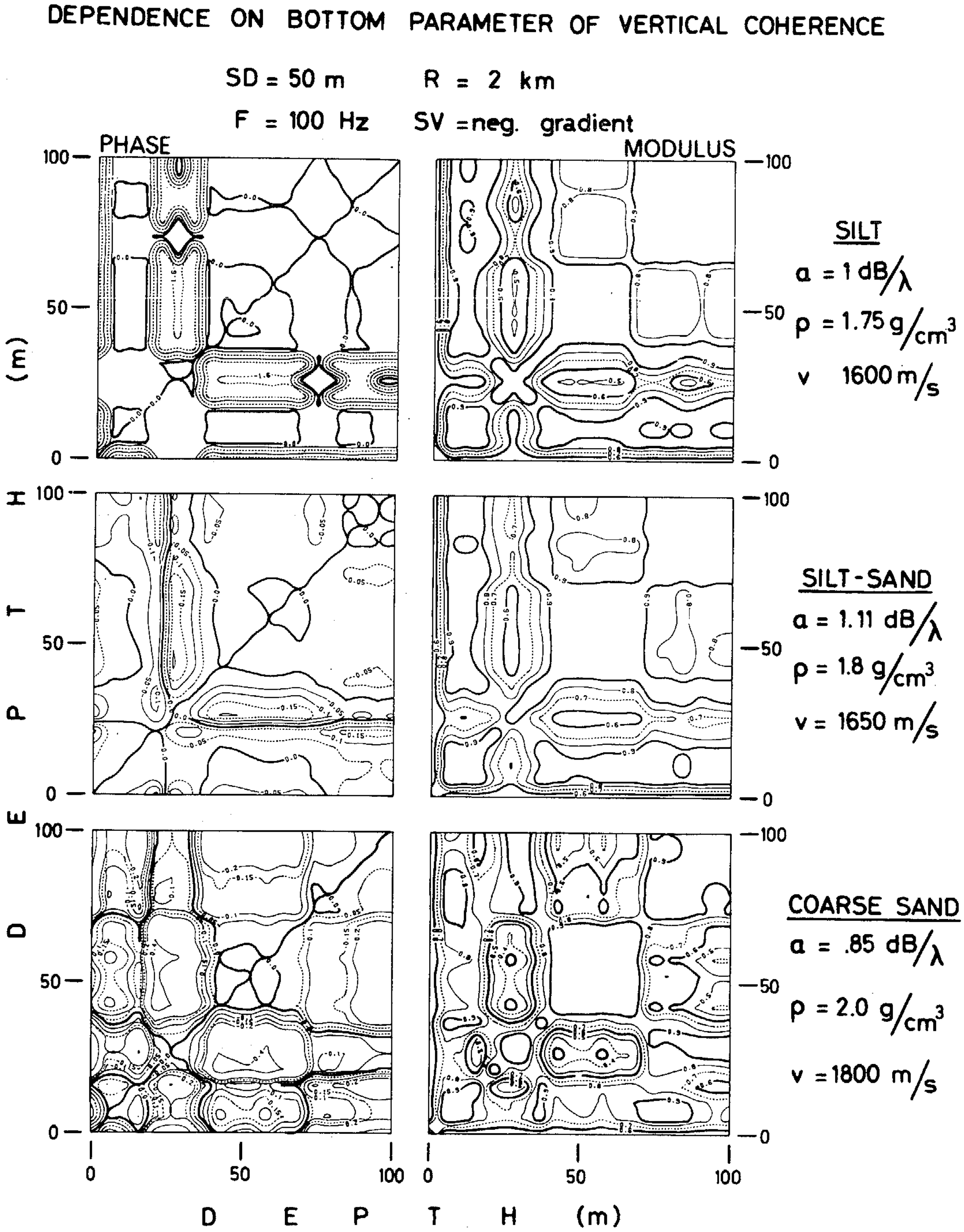
DEPENDENCE ON BOTTOM PARAMETER OF VERTICAL COHERENCE
SD = 50 m
R = 2 km
F = 100 Hz
SV = neg. gradient
PHASE
MODULUS
DEPTH (m)
SILT
a = 1 dB/λ
ρ = 1.75 g/cm³
v 1600 m/s
SILT-SAND
a = 1.11 dB/λ
ρ = 1.8 g/cm³
v = 1650 m/s
COARSE SAND
a = .85 dB/λ
ρ = 2.0 g/cm³
v = 1800 m/s
D E P T H (m)

FIGURE 3

MODULUS AND PHASE OF VERTICAL COHERENCE

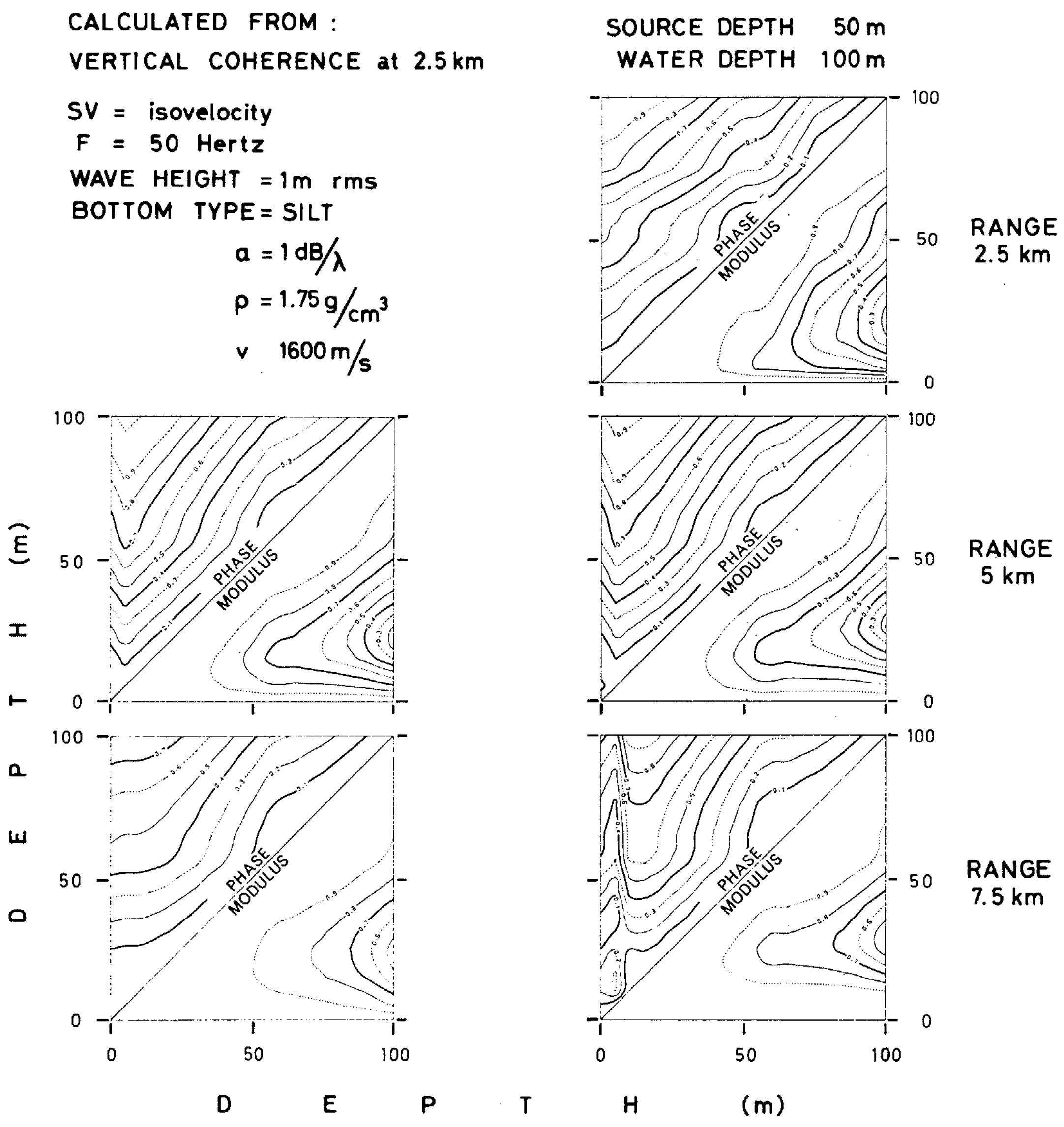

FIGURE 4