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USE OF PULSE TRAINS TO MEASURE DOPPLER SCATTERING FUNCTIONS

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RESUME

La magnitude carrée de la fonction d'ambiguité croisée doppler entre la entré et la sortie d'un système linéaire et variant avec temps est une convolution double modifiée entre la fonction d'eparpillment doppler du système et la magnitude carrée de la fonction de'ambiguite doppler de la entrée. Trains d'impulsions sont commodes pour measure la fonction d'eparpilliment parce que, comme inscrit dans ce conférence, les leurs fonctions de'ambiguité sont coussins pour ésingles et la banc de filtres assortis qui s'exigent pour déterminer la fonction d'ambiguité croisée peut être approximé par passer la sortie á travers un filtre assorti á l'impulsion et additionner avec déplacements convenables de temps les sorties de ce filtre qui correspondent à chaque impulsion du train.

SUMMARY

The magnitude squared of the cross-ambiguity function between the input and output of a time-varying linear system is a modified double convolution between the doppler scattering function of the system and the magnitude squared of the input doppler ambiguity function. Pulse trains are convenient for measuring the scattering function because, as shown in this paper, their ambiguity functions are pin cushions and the matched filter bank required to determine the cross-ambiguity function can be approximated by passing the output through a filter matched to the pulse and adding with appropriate time shifts the outputs of this filter corresponding to each pulse of the train.

INTRODUCTION

Any time-varying linear system can be represented by its spreading function, which tells how the system spreads any input signal in time and frequency [1]. Often--for example, in sound propagation in the ocean--this spreading is stochastic in nature and further is uncorrelated in both time and frequency; in this case the spreading function is replaced by the scattering function [1]. Typically in active sonar and radar a received signal is processed first by a bank of filters matched to the transmitted signal and frequency-shifted versions of the transmitted signal [2].

By definition the output of a bank of filters matched to frequency-shifted versions of a reference signal is the cross-ambiguity function between the reference signal and the input signal to the bank of matched filters. (When the reference and input signals are the same the term ambiguity function is used. Rihaczek [2] has investigated the ambiguity functions of a number of useful signals.) Søstrand [1] has shown that the magnitude squared of the cross-ambiguity function between the input and the output of a stochastic time-varying linear system with a scattering function is a modified double convolution in time and frequency between its scattering function and the magnitude squared of the ambiguity function of the input.

Pulse trains are convenient input signals for measuring the scattering function of a system since, as Rihaczek [2] has shown, their ambiguity function is a pin cushion. In particular their ambiguity function has a sharp central spike on the order of $1/B$ wide in time and $1/[(n - 1)\Delta T]$ wide in frequency surrounded by a clear rectangle $2\Delta T$ wide in time and $2/\Delta T$ wide in frequency, where ΔT is the time between pulses in the train, n is the number of pulses in the train and B is the bandwidth of the pulse. Hence, with such an input the magnitude squared of the cross-ambiguity between the input and output of a system with a scattering function will reproduce the scattering function with high resolution provided it lies inside a rectangle ΔT wide in time and $1/\Delta T$ wide in frequency. Furthermore, Costa and Hug [3] have shown that the matched-filter bank required to compute this cross-ambiguity function can be approximated by passing the system output through a filter matched to the pulse and adding with appropriate phase shifts the output of this filter corresponding to each pulse of the train.

In many situations--for example, with moving objects--the effect of time-varying linear systems is a doppler shift rather than a frequency shift. Although a doppler shift may be approximated for narrow-band signals by a frequency shift, for wide-band signals (such as required in measuring scattering functions) a doppler shift must be represented by the time stretch it naturally is. Some systems--for example, a moving, turning submarine--may be modelled as spreading signals in time and time stretch in a stochastic manner that is uncorrelated in them [4]. The purpose of this paper is to extend the results summarized above to such systems. References [4,5] give in detail both the conventional result summarized above and the novel results described herein.

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The Fundamental Relationship

The doppler cross-ambiguity function γ_{xy}^D between two signals x and y is defined by

$$\gamma_{xy}^D(\tau,\alpha) = \int_{-\infty}^{\infty} x^*[\alpha(t-\tau)]\; y(t)\; dt \qquad (1)$$

where τ is time shift, α is time stretch and * denotes complex conjugate. Note that it is a cross-correlation between y and time-shifted-and-stretched versions of x.

For a time-varying linear system describable by a doppler spreading function S^D, the output y is related to the input x via

$$y(t) = \int_{-\infty}^{\infty}\int_{0}^{\infty} \alpha\; S^D(\tau,\alpha)\; x[\alpha(t-\tau)]\; d\alpha\; d\tau\ . \qquad (2)$$

We assume that

$$E[S^{D*}(\tau',\alpha')\; S(\tau,\alpha)] = \mathscr{S}^D(\tau,\alpha)\;\delta(\tau'-\tau)\;\delta(\alpha'-\alpha), \qquad (3)$$

where E[ ] denotes expected value, $\mathscr{S}^D$ is the scattering function and δ is the Dirac delta function.

Substitution of (2) into (1) and use of (3) and (1) yields the fundamental relationship:

$$|\gamma_{xy}^D(\tau,\alpha)|^2 = \int_{-\infty}^{\infty}\int_{0}^{\infty} \mathscr{S}^D(\tau',\alpha')\; |\gamma_{xy}^D[x'(\tau-\tau'),\ \tfrac{\alpha}{\alpha'}]|^2\; dx'\; d\tau'\ , \qquad (4)$$

where $|a|^2 \triangleq E(a^*a)$. Note that this relationship is a modified double convolution.

The Doppler Ambiguity Function of a Pulse Train

Consider the following pulse train

$$x_{PT}(t) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} x_p[t - (i - \tfrac{n+1}{2})\Delta T] \qquad (5)$$

where the pulse $x_p(t)$ is assumed to be normalized to have unit total energy (integral of $|x_p|^2$ over t). In the appendix it is shown that substitution of (5) into (1) for $y = x = x_{PT}$ yields for the ambiguity function γ_{PT}^D of x_{PT}:

$$\gamma_{PT}^D(\tau,\alpha) = \sum_{i=-n+1}^{n-1} \gamma_i^D(\tau - i\Delta T,\alpha), \qquad (6)$$

where

$$\gamma_i^D(\tau,\alpha) = \frac{1}{n}\sum_{k=1}^{n-|i|} \gamma_p^D[\tau + \frac{\alpha-1}{\alpha}(\frac{n+1}{2} - k + \frac{i-|i|}{2})\;\Delta T,\ \alpha] \qquad (7)$$

and $\gamma_p^D(\tau,\alpha)$ is the doppler ambiguity function of $x_p(t)$.

For $\alpha = 0$, $\gamma_i^D = [(n-|i|)/n]\;\gamma_p^D$ and γ_{PT}^D is just a shaded pulse train in τ. If the spread of γ_p^D in τ is small compared to ΔT the pulses in this pulse train do not overlap. Since the spread of γ_p^T is on the order of 1/B where B is the bandwidth of x_p, this requirement becomes $B\Delta T \gg 1$. It is shown in [4] that for $|\alpha-1|/\alpha < 1/n$, the pulses γ_i^D do not overlap in τ for γ_{PT}^D; therefore for $B\Delta T \gg 1$ and $|\alpha-1|/\alpha < 1/n$,

$$|\gamma_{PT}(\tau,\alpha)|^2 \simeq \sum_{i=-n+1}^{n-1} |\gamma_i^D(\tau - i\Delta T,\alpha)|^2\ . \qquad (8)$$

To get some idea of how $|\gamma_{PT}|^2$ behaves with respect to α, we determine the total energy in γ_i^D for a given α by integrating γ_i^D over t for a pulse x_p, such as a linear FM pulse, whose fourier transform is uniform in magnitude over a bandpass B centered upon f_o. In [4] it is shown that for this pulse,

$$\int_{-\infty}^{\infty} |\gamma_i^D(\tau,\alpha)|^2 d\tau \simeq \frac{1}{\mu}\int_{-(\mu-|\alpha-1|)/2}^{(\mu-|\alpha-1|)/2} \left(\frac{\sin[\pi\xi(x+\frac{\alpha+1}{1})(n-|i|)]}{n\,\sin[\pi\xi(x+\frac{\alpha+1}{2})]}\right)^2 dx \qquad (9)$$

where $\xi = [(\alpha-1)/\alpha]f_o\Delta T$ and $\mu = B/f_o$. This integral is readily evaluated since it can be written as a sum of cosine functions. In Figure 1 this integral is plotted versus ξ for $\mu = 0.06$, $|\alpha-1| \ll \mu$, i=0 and n=5. Note that the ordinary ambiguity function for a pulse train, which is derived in [2], takes the same form as (6) with frequency shifts ν replacing α. For comparison in Figure 1 the total energy for given ν for the same type of pulse is plotted versus $\xi = \nu\Delta T$ for $\mu = 0.06$, $\nu/f_o \ll \mu$, i=0 and n=5. Observe that the functions plotted in Figure 1 are just wide-band and narrow-band beam patterns respectively.

From the foregoing it follows that the doppler ambiguity function of a pulse train consists of a central spike surrounded by a rectangular clear area. From (8) this clear rectangle has dimension along the τ-axis of roughly $2\Delta T$ and from (9) it has dimension along the α-axis of roughly $2/(f_o\Delta T)$; hence the area of the rectangle is rough $1/f_o$. As mentioned previously the width of the spike in τ is roughly 1/B. The width of the spike in ξ from Figure 1 is the same order as that of the ordinary ambiguity function. But the width of the latter is on the order of $1/[(n-1)\Delta T]$; hence the width of the central spike of the doppler ambiguity function in α is on the order of $1/[(n-1)\Delta T f_o]$.

An Approximate Matched Filter Bank

For ease of presentation a uniformly spaced train x_{PT} of identical linear FM pulses x_{FM} is assumed; however the same analysis applies in an obvious manner if the pulses are not linear FM pulses and are shifted in time and phase in the train. Let y be the signal to be passed through a bank of filters matched to x_{PT} and time stretched versions of x_{PT}. Substitution of (5) and the approximate identity

$$x_{FM}(\alpha t) \simeq e^{-2\pi j\frac{(\alpha-1)f_o^2}{k}}\; x_{FM}[t + (\alpha-1)f_o/k]\ , \qquad (10)$$

where k is the rate of change of frequency with time of x_{FM}, into (1) yields:

$$\begin{aligned}\gamma_{x_{PT}y}^D(\tau,\alpha) &= \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\int_{-\infty}^{\infty} x_{FM}^*[\alpha(t-\tau) - (i-\tfrac{n+1}{2})\Delta T]\,y(t)\,dt \\ &\simeq \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\int_{-\infty}^{\infty} e^{2\pi j\frac{(\alpha-1)f_o^2}{k}}\; x_{FM}\left[t + \frac{(\alpha-1)f_o}{k} - \tau - \frac{(i-\frac{n+1}{2})\Delta T}{\alpha}\right] y(t)\,dt \qquad (11)\\ &= \frac{e^{2\pi j\frac{(\alpha-1)f_o}{k}}}{\sqrt{n}}\sum_{i=1}^{n}\gamma_{x_{FM}y}^D\left[\tau - \frac{(\alpha-1)f_o}{k} - \frac{(i-\frac{n+1}{2})\,T}{\alpha},\ 1\right].\end{aligned}$$

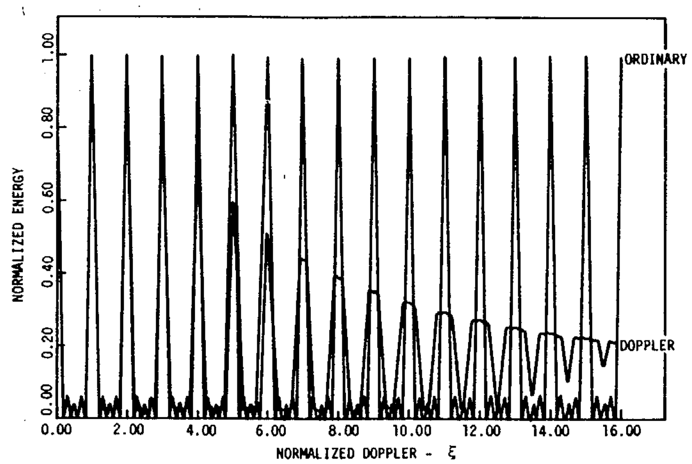

Figure 1 Normalized Energy in Ambiguity Function for i=0, n=5.

The quantity $\gamma^D_{x_{FM}y}(\ ,1)$ is the output of y passed through a filter matched to x_{FM}; therefore the first step in the approximate matched filter bank is to pass the signal through a filter matched to the input pulse. The summation in (11) is just a sum of suitably time-shifted versions of the outputs of this matched filter corresponding to the various pulses in the train. As such it is the temporal analog of a broad-band beam former. For $|\gamma_{x_{PT}y}|^2$ the phase term in (11) drops out. In [3,4] it was shown that the bank of filters matched to x_{PT} and frequency-shifted versions of x_{PT} could be approximated by a filter matched x_P followed a summation of suitably phase-shifted versions of the outputs of this matched filter corresponding to the various pulses in the train. This summation is the temporal analog of a narrow-band beam former.

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APPENDIX

Substitution of (5) into (1) yields

$$\gamma^D_{PT}(\tau,\alpha) = \frac{1}{n}\sum_{i=1}^{n} \Big\{ \sum_{k=1}^{n} \int_{-\infty}^{\infty} x_P^*[\alpha(t-\tau) - (k-\frac{n+1}{2})\Delta T]$$

$$x_P[t-(i-\frac{n+1}{2})\Delta T]dt \Big\} = \frac{1}{n}\sum_{i=1}^{n} \Big[\sum_{k=1}^{n} \int_{-\infty}^{\infty} x_P^* \Big(\alpha\Big\{ t-\tau$$

$$+ [i-\frac{k}{\alpha}-\frac{\alpha-1}{\alpha}(\frac{n+1}{2})]\Delta t \Big\}\Big) x_P(t)dt = \frac{1}{n}\sum_{i=1}^{n}\Big(\sum_{k=1}^{n} \gamma_P^D \Big\{ \tau$$

$$- [i-\frac{k}{\alpha}-\frac{\alpha-1}{\alpha}(\frac{n-1}{2})]\Delta T, \alpha \Big\}\Big)$$

on utilization of (1) with $y = x = x_P$. Rearrangement of the double summation yields (6) and (7).