HUITIEME COLLOQUE SUR LE TRAITEMENT DU
SIGNAL ET SES APPLICATIONS

NICE du 1er au 5 JUIN 1981

CONVOLUTION AND DECONVOLUTION WHEN ONLY THE LOWER ORDER MOMENTS OF THE CONVOLUTION FUNCTION ARE KNOWN.

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RESUME

La deformation transmise à un signal par l'appareil de mesure peut être tres souvent decrite par l'operation de convolution. Le sujet de cette communication est de prêsenter une technique permettant d'obtenir le signal originale à partir du signal observê. Cette technique n'utilise ni iteration ni transformation de Fourier mais emploie uniquement les moments d'ordre les plus bas du noyau de convolution.

SUMMARY

The transformation a signal suffers by measuring apparatus is often given by the proces of convolution. A technique is presented for obtaining the original signal from the measured, a technique which requires neither iteration nor Fourier transformation. The method only makes use of the lower order moments of the convolution kernel.

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INTRODUCTION

A simple one-dimensional convolution and deconvolution method has recently been developed which makes a very modest demand on computing capacity and storage [1,2].

An original but unknown function $f(x)$ is supposed to be convoluted with a distribution function $h(x)$ to give a measured function $g(x)$. The connection between g,f and h is supposed to be:

$$g(x)=\int_{-\infty}^{\infty} h(u)f(x-u)du \qquad (1)$$

It was shown [1] that we can write for convolution:

$$g(x)=(2\pi)^{1/2}\{H(0)f(x)+H'(0)f'(x)(-i)+\cdots$$

$$+H^{(n)}(0)f^{(n)}(x)(-i)^n\frac{1}{n!}+\cdots\} \qquad (2)$$

where $H(\alpha)$ is the Fourier transform of $h(x)$.

If we define the function $H_1(\alpha)=1/H(\alpha)$ we can similarly write for deconvolution:

$$f(x)=(2\pi)^{-1/2}\{H_1(0)g(x)+H_1'(0)g'(x)(-i)+\cdots$$

$$+H_1^{(n)}g^{(n)}(x)(-i)^n\frac{1}{n!}+\cdots\} \qquad (3)$$

It was demonstrated [1,2] how eq.(3) could be used in a simple way to make deconvolution for a measured Compton spectrum to estimate multiple Compton scattering from a large plastic scintilation detector and to extract information from average evoked response EEG signals. In these works it was assumed that the convolution function $h(x)$ was either Gaussian or box shaped.

METHOD OF MOMENTS

If the convolution function $h(x)$ is not a simple function - where it is easy to find the Fourier transform - eq.(3) may be difficult to use. The convolution function $h(x)$ may e.g. be the measured spectrum from a monoenergetic γ-ray incident on a solid state detector. In such cases it is proposed to use the method described below.

If in eq.(1) we insert

$$f(x-u)=f(x)-u\cdot f'(x)+\cdots+(-1)^n u^n f^{(n)}(x)\frac{1}{n!}+\cdots \qquad (4)$$

and define the moment of $h(x)$ as

$$\mu_n=\int_{-\infty}^{\infty} u^n h(u)du \qquad (5)$$

we get:

$$g(x)=\sum_{n=0}^{\infty}(-1)^n f^{(n)}(x)\mu_n\frac{1}{n!} \qquad (6)$$

From eqs.(2) and (6) we see:

$$H^{(n)}(0)=(2\pi)^{-1/2}(-i)^n\mu_n$$

For simplicity we assume the convolution and deconvolution to be area preserving i.e. $\mu_0=1$.

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If we use:

$$H_1(\alpha)H(\alpha)=1 \quad , \tag{8}$$

differentiate eq.(8) four times implicitly, and put $\alpha=0$, we can show that:

$$H_1(0)=(2\pi)^{1/2} \;,\; H_1'(0)=(2\pi)^{1/2}\mu_1 i$$

$$H_1''(0)=(2\pi)^{1/2}(\mu_2-2\mu_1^2) \quad , \tag{9}$$

$$H_1^{(3)}(0)=(2\pi)^{1/2}(6\mu_1\mu_2-6\mu_1^3-\mu_3)i \quad \text{and}$$

$$H_1^{(4)}(0)=(2\pi)^{1/2}(24\mu_1^4-36\mu_1^2\mu_2+8\mu_1\mu_3+6\mu_2^2-\mu_4)$$

These are the terms to be inserted in eq.(3). If the convolution function h(x) can be chosen so that $\mu_1=0$ (mean equal to zero) the deconvolution formula is especially simple:

$$f(x)=g(x)-1/2\mu_2 g''(x)+1/6\mu_3 g'''(x)+ 1/24(6\mu_2^2-\mu_4)g^{(4)}(x)+\cdots \tag{10}$$

CONCLUSION

The above mentioned equations (6) and (10) for convolution and deconvolution gives simple possibilities to make spectral deconvolution, image recunstruction ect in small computer systems. The method has the advantage of using only the observed function and its derivatives multiplied by constants. For practical use one must be very careful when chosing a differentiating routine if the data to be deconvoluted are contaminated with noise.

REFERENCES

(1) Sjøntoft,E.,Nucl.Inst.Meth. 163(1979), 519-522.

(2) Sjøntoft,E.,IEEE Trans.Biomed.Eng. BME-27,NO.4,1980,227-230.