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APPLICATION DES TECHNIQUES D'IDENTIFICATION DE MODELES ARMA A LA SYNTHESE DES FILTRES NUMERIQUES

APPLICATION OF TECHNIQUES OF ARMA-MODELS IDENTIFICATION TO THE DESIGN OF DIGITAL FILTERS

MM. A. HANAFY, J. LE ROUX, J. PRADO.

ECOLE NATIONALE SUPERIEURE DES TELECOMMUNICATIONS DEPARTEMENT SYSTEMES ET COMMUNICATIONS
46 RUE BARRAULT 75634 PARIS CEDEX 13

RESUME

Les techniques globales d'identification des signaux et de systèmes permettent de calculer les coefficients d'une fonction de transfert linéaire $\frac{B(z)}{A(z)}$ à partir de l'autocorrélation d'un signal. L'autocorrélation se déduisant de la densité spectrale ces mêmes techniques sont utilisées pour la synthèse des filtres numériques à partir du spectre du filtre idéal recherché.

Les recherches nouvelles présentées dans la communication portent principalement sur la comparaison des différents algorithmes possibles pour le calcul du numérateur et du dénominateur de la fonction de transfert (parties MA et AR) : minimisation de l'erreur de prédiction ; "blanchiment" de cette erreur ; identification après factorisation spectrale ; identification successives des parties AR et MA après déconvolution (dans le domaine spectral).

Ces algorithmes sont implantés (langage FORTRAN) dans un programme conversationnel permettant la visualisation graphique des résultats. Les comparaisons pratiques entre les différentes méthodes portent sur l'écart séparant le spectre désiré et le spectre calculé, qui est fonction de la forme du spectre original et de l'ordre des fonctions de transfert. Il apparaît que même pour des gabarits de formes assez complexes, la réponse en fréquence du filtre trouvé est très proche de celle de l'original.

En conclusion, on montre que ces méthodes, très systématiques et utilisant des techniques mathématiques et informatiques simples (résolution de systèmes linéaires, factorisation spectrale, transformées de Fourier) permettent d'aboutir à des résultats assez différents mais aussi intéressants que ceux obtenus par les méthodes de programmation fondées sur la définition d'un gabarit en bande passante et en bande atténuée. Elles ont de plus l'avantage de permettre la synthèse de formes plus complexes.

SUMMARY

The global techniques for identification of signals and systems are useful for calculating the coefficients of the transfer function $\frac{B(z)}{A(z)}$, knowing its autocorrelation function. The same techniques are used here for the design of digital filters knowing its spectral density characteristic.

Different algorithms are proposed in the present paper for calculating coefficients of the numerator (MA-part) and denominator (AR-part) : minimization of error of prediction, whiting this error, identification after spectral factorization, successive identification of the AR and MA-parts after deconvolution (in the spectral domain).

All the algorithms are implemented in a FORTRAN program which permits the graphical visualization of the results. The practical comparisons between the resulting and the ideal spectral density characteristic allow us to choose that filter which has smallest peak pass band ripple (PPBR), and highest minimum stop band attenuation (MSBA). Moreover, these techniques can be used for the design of multiband-multigain digital filters.

In conclusion, we indicate that these techniques are systematic and use simple mathematical and programming techniques (solution of systems of Linear equations, spectral factorization, Fourier transforms) compared to the classical methods of design.

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I. INTRODUCTION

The problem of designing ARMA-digital filter can be considered as an approximation problem, specially in calculating the coefficients of the moving average (MA) part /1/ - /4/.

Three approches are proposed in the present paper for the solution of the MA-part : 1) Spectral factorization of the inverse filter correlation, 2) Deconvolution of the MA-spectral density after identification of the AR-part, 3) Spectral factorization of the ideal correlation for getting the impulse response to be used with the correlation for calculating the zeros' coefficients.

In the first approach we approximate the correlation of the MA-filter by the inverse filter correlation resulted from the minimization of error of prediction, then easily we factorize this mirror image polynomial, we get the required MA-parameters.

In the second approach we get a correlation corresponds to an AR-filter, when identified we get directly the required coefficients.

In the third approach we get the impulse response of the ARMA-filter, then either using it for calculating both poles' and zeros' coefficients (time domain design of digital filter /5/ -/8/),or using the correlation for calculating poles' coefficients and then both correlation and impulse response for calculating the b- coefficients. The use of first and second-order informations is prefered for the purpose of matching spectra /3/, /9/.

As the spectral factorization is used in two approaches we introduce in sec. II the principle of LE ROUX algorithm /10/ which is used for the factorization process.

The design parameters are, number of bands, pass (stop) - band edges, and attenuation in each band. With these parameters we can design low (high) - pass, band - pass (stop), and multiband - multigain ARMA(N,M) frequency matching digital filters.

The experimental results at the end of this paper show that, the optimum filter specifications resulting from the design with our approaches can be compared with those resulting from the design using either the statistical method of design /3/, or the modified ARMA methods of design /4/.

II. FIRST-ORDER INFORMATION THROUGH SPECTRAL FACTORIZATION

Given the spectral density characteristic $R(z)$, we have to find its impulse response $\{g(n)\}_0^L$ or the L - coefficients of the polynomial :

$$G(Z) = \sum_{n=0}^{L} g(n)\, Z^{-n} \quad , \quad g(0) = 1 \qquad (1)$$

where $G(Z)\, G(Z^{-1})$ is an approximation to the spectral density $R(Z)$.

The solution to such a problem can be obtained using the roots finding technique, but finding the roots of a polynomial of an order greater than 36, leads to a higher error of calculation (practical limitation). Another technique which is equivalent to the previous one, and having the ability to calculate the roots of a polynomial of an order up to 256 with good accuracy, is that which uses LE ROUX spectral factorization algorithm. This algorithm at its convergence, extracts the first - order information sequence $\{g(n)\}_0^L$ from the second - order information sequence $\{r'(n)\}$ ($r'(n)$ is the inverse discrete Fourier transform of $R(Z)$, $Z = e^{jw}$).

The principle idea of this algorithm depends on the calculation of the partial correlation coefficients K_m , m = 1, 2, ..., L used in Linear prediction technique /11/ - /13/.
The spectral factorization process starts with the following initialization :

$$e_{o,\,i} = \begin{cases} r'(i) & , i = 0, \pm 1, \pm Z, \dots , \pm Z \\ 0 & \text{elsewhere} \end{cases} \qquad (2)$$

where $e_{m,\,i}$ is the ith intermidite variable at the mth iteration, meanwnile for i negative and at convergence, it represents the required impulse response. This variable is calculated from the following Levinson-type recursive algorithm :

$$e_{m+1,i} = e_{m,i} + K_m\, e_{m,m+1-i} \qquad (3)$$

$$e_{m+1,0} = e_{m,0}\,(1-K_m^2) \qquad (4)$$

$$K_m = -e_{m,m+1}/e_{m,0} \qquad (5)$$

As a result, we get $e_{m,-i}$ and from which me calculate $G^m(Z)$ as follows :

$$G^m(Z) = \sum_{i=0}^{L} e_{m,-i}\, Z^{-i} = \sum_{n=0}^{L} g(n)\, Z^{-n} \qquad (6)$$

III. POLES' AND ZEROS'COEFFICIENTS CALCULATION

The problem of designing an ARMA (N,M) digital filter can be stated as follows : given the ideal spectral density characteristic $R(Z)$, it is required to find the N-poles' coefficients and the M-zeros' coefficients of the transfer function $H(Z)$, such that its spectral density characteristic matches that of the ideal one. This transfer function is given by :

$$H(Z) = \frac{\sum_{i=0}^{M} b_i\, Z^{-i}}{\sum_{i=0}^{N} a_i\, Z^{-i}} \quad , \; a_0 = b_0 = 1 \qquad (7)$$

In all the three approaches, the poles' coefficients are to be calculated firstly by the approximation of the ideal correlation $\{r(n)\}_{-K}^{K}$ ($r(n)$ = IDFT $\{R(Z)\}$). The details of this calculation will be given in the following subsection.

III. 1 POLES' COEFFICIENTS CALCULATION

Using the transfer function (7) and its difference equation, we can write the following very important relation of the ARMA (N,M) digital filter :

$$\sum_{i=0}^{N} a_i\; c\,(n-i) = c\,(n), \quad n \geq M+1 \qquad (8)$$

where $\{c(n)\}_{-k}^{k}$ is the correlation sequence of the filter $H(Z)$. Using (8), the poles' coefficients can be obtained such that $r(n) = c(n)$, n = M+1, M+2, ..., M+N. Hince (8) becomes :

$$\sum_{i=1}^{N} a_i\, r(n-i) = -\, r(n) \;, \quad n = M+1,\; M+2, \dots, M+N \qquad (9)$$

Solving the system of equations (9), we get the poles' coefficients $\{a_i\}_1^N$. This method of calculation can be interpreted as the exact fitting of the ideal correlation sequence $\{ r(n)\}_{M+1}^{M+N}$.

III. 2 STABILITY

The condition of stability of the resulting filter is that all the poles lie inside the unit circle in the Z - plain. As the spectral density polynomial of the autoregressive filter (AR) is symetric, we can reflect the poles which are outside the unit circle to be inside it.

III. 3 ZEROS' COEFFICIENTS CALCULATION

The calculation of the coefficients of the MA-part was always an approximation solution. In this paper we propose three approaches which are related to the previous techniques. In the three methods we have to calculate the zeros' coefficients using the calculated poles' coefficients.

III. 3.1 SPECTRAL FACTORIZATION OF THE INVERSE FILTER CORRELATION (M1).

This method is similar to that proposed by L. Scharf in /1/, but it has no restriction over the order of the MA-part.

As the pole coefficients were calculated, we can calculate the invers filter correlation as :

$$r_{inv}(n) = \mathrm{IDFT}\ \{R_{inv}(Z)\} \tag{10}$$

where $R_{inv}(Z)$ is given by :

$$R_{inv}(Z) = A(Z)\ R(Z)\ A(Z^{-1}) \tag{11}$$

and

$$R(Z) = \frac{B(Z)\ B(Z^{-1})}{A(Z)\ A(Z^{-1})} \tag{12}$$

Therefore using (12) we can write :

$$B(Z)\ B(Z^{-1}) \simeq R_{inv}(Z) = A(Z)\ R(Z)\ A(Z^{-1}) \tag{13}$$

Using the correlation coefficients we have :

$$B(Z)\ B(Z^{-1}) = \sum_{n=-q}^{q} r_{inv}(n)\ Z^{-n} \tag{14}$$

Applying LE ROUX algorithm over $\{r_{inv}(n)\}$, we get at its convergent the required zeros' coefficients.

III. 3.2 POLE-ZERO DECONVOLUTION (M2)

In this method we calculate the AR-filter, and its spectral density, then we separate a spectral density corresponding to an AR-filter, when identified we get the required MA-coefficients. This method can be considered as pole-zero decomposition technique as proposed by YEGNANARAYANA /14/, but the computations in our method are four-times smaller.

Taking the logarithm of (12) we get :

$$R_{dB}(Z) = B_{dB}(Z) + A_{dB}(Z) \tag{15}$$

Since A(Z) was calculated we can calculate $B_{dB}(Z)$, and the better is to calculate $-B_{dB}(Z)$ which corresponds to an AR-filter which can be identified as follows :

$$\sum_{i=0}^{M} b_i\ r_{IAR}(n-i) = -r_{IAR}(n),\ n = 1, 2, \ldots, M \tag{16}$$

where

$$r_{IAR}(n) = \mathrm{IDFT}\ \{10^{-0.1\ B_{dB}(Z)}\big|_{Z=e^{jw}}\} \tag{17}$$

III. 3.3 FIRST AND SECOND-ORDER INFORMATION IN THE DESIGN OF ARMA-DIGITAL FILTER (M3)

In this method we use both first and second-order informations for calculating the coefficients of MA-part. This approach is similar to that proposed by L. Scharf /3/, but in our approach we factorize a mirror image polynomial (corresponding to the ideal correlation) of an order 64, for getting the approximate impulse response $\{ g(n)\}_0^{63}$. while Scharf solved a system of 256 linear equations for the same purpose.

Using (7) we can write the following relation :

$$b_n = \sum_{i=0}^{n} a_i\ h(n-i) \quad ,\quad n=0, 1, \ldots, M \tag{18}$$

Now we can calculate the b's - coefficients such that $\{h(n)\}_0^M = \{g(n)\}_0^M$. Substituting the calculated optimum poles' coefficients $\{a_i^*\}_0^N$ and the deduced impulse response into (18), it becomes :

$$b_n = \sum_{i=0}^{n} a_i^*\ g(n-i) \quad ,\quad n = 0, 1, \ldots, M \tag{19}$$

It is clear from (19), that the effort given for the solution of the MA-part is transfered to the spectral factorization process, which is more easier than the solution of a system of 256 linear equations as proposed by L. Scharf in /3/.

IV. STEPS OF DESIGN

The steps of design are summarized below as follows :

1- specify an ideal filter spectral density

2- choose number of FFT points

3- choose N, and M

4- choose method of zero's coefficients calculation

5- calculate poles' coefficients

6- find poles' locations and check stability

7- calculate zeros' coefficients

8- calculate resulting spectral density

A FORTRAN program was implemented for fulfilling the above design steps. This program was used to design all of the examples that follow in section V.

V. EXAMPLES

In this section we introduce the design of three kinds of ARMA-digital filters, with the three methods of design.

In first example we design a -50 dB rejection band low-pass filter. The resulting spectral density of the ARMA (10,6) -50 dB low-pass filter of M2, is shown in Fig. 1 along with the ideal one. For the purpose of comparison, the specifications of the filter resulted from the design with the three methods are given in Table. 1. From this table, it is clear that the filter designed with M2, and M3 has better specification than that designed with M1.

In Fig. 2 we give the resulting spectral density of the -40 dB rejection band-ARMA (12,8) - band - stop filter designed with M3, via the ideal characteristic. The resulting specifications of this filter corresponding to each method of design is given in Table. 1.

The most important advantages of our method is that, we are able to design digital filters with complex transfer function (multiband-multigain). Fig. 3 shows the ideal spectral density of the three-bands-three gains (0 dB, -10 dB, -20 dB) digital filter along with the ARMA (6,5) three bands-three gains optimum filter designed with M1. The specifications of this filter for the three methods of design is given in Table. 1.

Table.1. Specification of the optimum filter

Ex.No	M.No	PPBR(dB)	MSBA(dB)	Remarks
1	1	+0.37	49.02	-
	2	-0.22	49.92	-
	3	+0.19	49.89	-
2	1	+0.47	39.08	higher transi. region
	2	+0.35	39.57	-
	3	+0.18	39.98	-
3			MSBA1(dB)	MSBA2(dB)
	1	+0.15	9.79	19.63
	2	-0.21	9.84	19.69
	3	-0.31	9.83	19.60

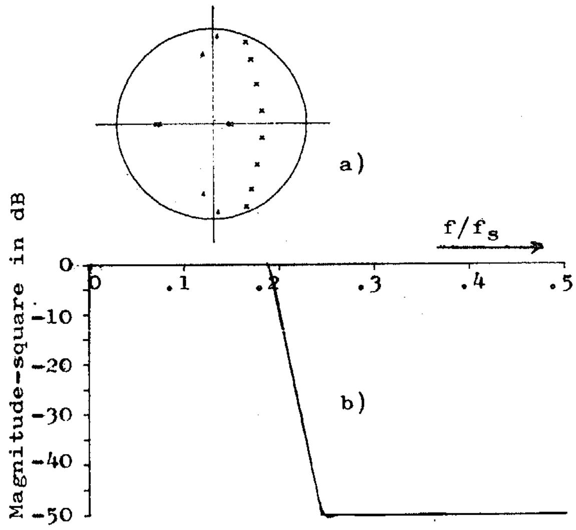

Fig.1.-50 dB rejection band low-pass filter,

a) poles'-zeros'locations

x-pole , ▲-zero

b) ARMA(10,6) approximation of M2

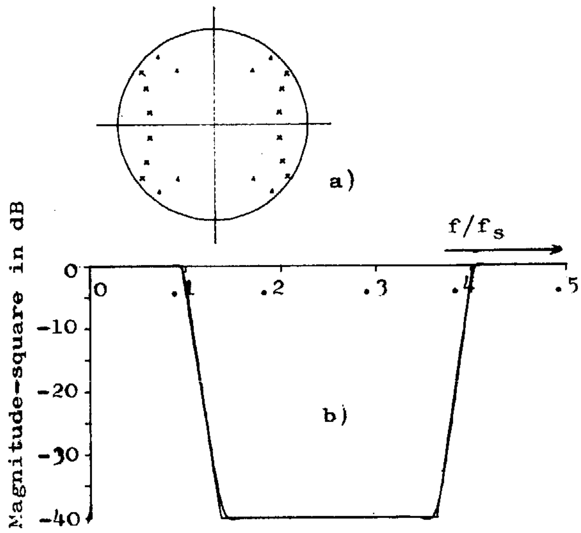

Fig.2.-40 dB rejection band stop-band filter,

a) poles'-zeros'locations.

b) ARMA(12,8)approximation of M3.

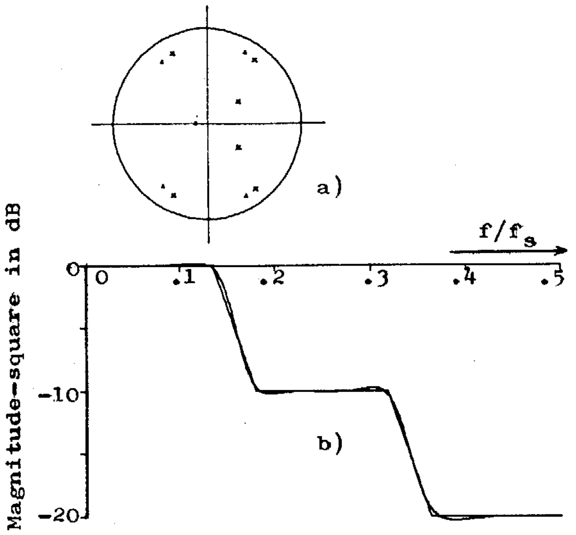

Fig.3.Three bands-three gains filter,

a) poles'-zeros' locations

b) ARMA(6,5) approximation of M1.

VI. CONCLUSION

The results presented here show that our methods are useful in designing low (high) - pass, band-stop (pass), and multiband-multigain digital filters. Several note can be stated. 1) there is no need to begin the design with a rational analog filter (classical method of design), 2) the procedures presented here permit the designer to rechoose the order of poles and zeros, 3) the stability of the resulting filter is checked, 4) the use of both first, and second-order informations leads to a better spectral matching, 5) increasing the number of correlation coefficients used in factorization process, L, results in a better spectral matching. We hope that the proposed techniques may provide useful methods for modeling ARMA digital filters.

For the moment we study the question of optimum order determination.

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