SYSTEM IDENTIFICATION TECHNIQUES FOR ADAPTIVE SIGNAL PROCESSING

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RESUME

De nombreux problèmes en filtrage adaptatif peuvent être traités du point de vue de l'identification des systèmes. L'algorithme récursif du maximum de vraisemblance est proposé pour l'estimation des paramètres du modèle du signal. Les valeurs estimées des paramètres sont ensuite utilisées comme coefficients d'un filtre adaptatif de type ARMA. Plusieurs exemples sont presentés, en particulier: prédiction adaptative, déconvolution adaptative, suppression adaptative du bruit et estimation adaptative de retards temporels.

SUMMARY

Many problems in adaptive filtering can be approached from the point of view of system identification. The recursive maximum likelihood algorithm is proposed for estimating the parameters of the signal model. The parameter estimates are then used to form an adaptive infinite impulse response filter. Several examples are discussed including: adaptive line enhancement, adaptive deconvolution, adaptive noise cancelling and adaptive time delay estimation.

1. INTRODUCTION

Considerable progress was made in the last decade in the development and analysis of recursive parameter estimation algorithms. The major part of this work was in the area of system identification, in the context of controlling plants with unknown or slowly time varying parameters. A large number of algorithms were developed for fitting linear models to the observed data. The following ARMAX model is an example of the class of models that are typically considered.

Let u_t and y_t denote the input and output processes of the model, and v_t an unmeasurable white noise process (i.e. a "disturbance", in the control terminology). These processes are related by the following equation:

$$y_t = -\sum_{i=1}^{NA} a_i y_{t-i} + \sum_{i=1}^{NB} b_i u_{t-i} + \sum_{i=1}^{NC} c_i v_{t-i} + v_t \tag{1}$$

which can be written in polynomial form as

$$A(z^{-1})y_t = B(z^{-1})u_t + C(z^{-1})v_t \tag{2a}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{NA} z^{-NA} \tag{2b}$$

$$B(z^{-1}) = b_1 z^{-1} + \cdots + b_{NB} z^{-NB} \tag{2c}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \cdots c_{NC} z^{-NC} \tag{2d}$$

$$z^{-1} = \text{unit delay operator, i.e., } z^{-1}x_t = x_{t-1} \tag{2e}$$

The case where $B(z^{-1}) = 0$ is of special interest in signal processing applications, since the output $y_t = (C(z^{-1})/A(z^{-1}))v_t$ is then an autoregressive moving-average (ARMA) process. When $B(z^{-1}) = 0$ and $C(z^{-1}) = 0$ we have an autoregressive (AR) process $y_t = (1/A(z^{-1}))v_t$. Such processes are very common in time series analysis and statistical signal processing.

Many problems in signal processing involve signals that can be represented by linear models of this type (AR, ARMA, ARMAX), as we will show later. Knowledge of the signal model parameters makes it relatively straightforward to design filters that perform various processing functions such as: linear prediction/smoothing, deconvolution, noise/interference suppression, or spectral analysis. When the signal model is not known, the parameters of the related filter need to be adaptively adjusted. A technique that is commonly used in adaptive control problems, is to first estimate the model parameters and then design the controller as if these estimates were the true parameter values.

The same idea can be used for adaptive signal processing, as already indicated in [1]-[4]. The combination of a recursive parameter estimation algorithm with a filtering technique based on known model parameters is the main theme of this paper.

The successful application of system identification techniques in adaptive control motivated us to apply the same techniques to signal processing. Of particular interest is the fact that some commonly used parameter estimation algorithms such as recursive maximum likelihood (RML), extended least-squares (ELS) or recursive generalized least squares (RGLS) [5]-[6], are capable of estimating ARMA (or ARMAX) parameters, and not just AR parameters. As we shall see, this leads naturally to adaptive infinite impulse response (IIR) filters. Adaptive IIR filtering is considered by the signal processing community to be a difficult problem. Consequently, the overwhelming majority of the work in the area of adaptive signal processing seems to concentrate on finite impulse response (FIR) filtering. The application of system identification algorithms opens the way to the development of a whole new class of adaptive IIR filters, backed by the extensive convergence analysis that was performed in recent years [7]-[12].

In spite of the natural interconnections between system identification and adaptive signal processing very little work seems to have been done to transfer algorithms from one area to the other. The objective of this paper is to report on some recent work in which a version of the RML algorithm was successfully applied to a number of problems including: adaptive line enhancement, adaptive deconvolution, adaptive noise cancelling and time-delay estimation. A brief description of these applications is presented in Section 3.

In Section 2 we present the RML algorithm and discuss its properties. Some of the special characteristics of the signal processing problem (when compared to the control problem) and their effect on the behavior of the algorithm are also discussed. Finally, in Section 4 we outline some alternative algorithms for adaptive processing and areas for further investigation. We hope that our work will stimulate further research into the numerous potential applications of parameter estimation algorithms to adaptive signal processing.

2. THE RECURSIVE MAXIMUM LIKELIHOOD ALGORITHM

The RML algorithm provides a recursive estimate of the parameters of the ARMAX model in equation (1). For a detailed derivation of this algorithm see [13] [14]. Here we present only a summary of the recursions.

Let θ denote the vector of model parameters

$$\theta = [a_1,\ldots,a_{NA}, b_1,\ldots,b_{NC}, c_1,\ldots,c_{NC}]^T, \qquad (3a)$$

and ϕ_t, ψ_t denote the data vector and the filter data vector, respectively.

$$\phi_t - [-y_{t-1},\ldots,-y_{t-NA}, u_{t-1},\ldots,u_{t-NB}, \bar{e}_{t-1},\ldots,\bar{e}_{t-NC}]^T \qquad (3b)$$

$$\psi_t = [-\bar{y}_{t-1},\ldots,-\bar{y}_{t-NA}, \bar{u}_{t-1},\ldots,\bar{u}_{t-NB}, \bar{e}_{t-1},\ldots,\bar{e}_{t-NC}]^T \qquad (3c)$$

Denote by $\hat{\theta}_t$ the parameter estimates at time t, and by $\hat{C}(z^{-1})$ the filter whose coefficents are the estimates $\hat{c}_i$. Then,

$$\varepsilon_{t+1} = y_{t+1} - \phi_{t+1}^T\hat{\theta}_t = \text{prediction error} \qquad (4a)$$

$$P_{t+1} = [P_t - P_t\psi_{t+1}^T\psi_{t+1}P_t/(\lambda_{t+1}+\psi_{t+1}^T P_t\psi_{t+1})]/\lambda_{t+1} =$$

$$= \text{error covariance matrix} \qquad (4b)$$

$$\hat{\theta}_{t+1} = \hat{\theta}_t + P_{t+1}\psi_{t+1}\varepsilon_{t+1} \qquad (4c)$$

$$e_{t+1} = y_{t+1} - \phi_{t+1}^T\hat{\theta}_{t+1} = \text{residual} \qquad (4d)$$

$$\left.\begin{aligned} \bar{y}_t &= (1/\hat{C}(kz^{-1}))y_t \\ \bar{u}_t &= (1/\hat{C}(kz^{-1}))u_t \\ \bar{e}_t &= (1/\hat{C}(kz^{-1}))e_t \end{aligned}\right\} \text{filtered quantities} \qquad (4e)$$

with initial conditions

$P_o = \alpha I$, α = initial estimate of the covariance

$\hat{\theta}_o = \theta_o$, initial estimate of θ (typically $\theta_o = 0$).

The parameter λ_t is an exponential weighting on the data. Typically λ_t is a constant close to unity, or

$$\lambda_{t+1} = \lambda\,\lambda_t + (1-\lambda)(\lambda = .99,\ \lambda_o = 0.95) \qquad (5)$$

The parameter k is used to "pull in" the roots of the polynomial $\hat{C}(kz^{-1})$ into the unit circle, when $\hat{C}(z^{-1})$ has roots near the unit circle, as is often the case in signal processing applications. This parameter affects the convergence rate of the algorithm as discussed in [15]. In fact, for k=0 this algorithm becomes the so-called Extended Least-Square algorithm described in [16], [17]. In most cases the choice of k is not critical and in the following we assume that k is close or equal to unity. To ensure convergence, the stability of $\hat{C}(z^{-1})$ needs to be monitored. If unstable, the parameter estimates need to be projected into a region of stability [7],[8].

The asymptotic properties of the RML algorithm have been investigated in considerable detail. It was shown that asymptotically the recursive maximum likelihood technique has the same properties as the corresponding off-line version. Thus nothing is sacrificed by going to a recursive implementation, provided that enough data is available. The maximum likelihood estimator has all the desirable properties one may expect from a parameter estimator:

- Asymptotic consistency [18], i.e., $\hat{\theta}_N \to \theta$ as the number of data points N goes to infinity
- Asymptotic efficiency [19], i.e., the estimation error covariance approaches the Cramer-Rao lower bound
- The estimation error distribution is asymptotically normal [19].

The convergence properties of the RML algorithm were studied by Ljung [7],[8] and others [5],[12]. It was shown that this algorithm will always converge to a local maximum of the likelihood function. In some situations there may exist "false" maxima which can cause difficulties. However, for ARMA processes it was shown that all the local maxima coincide with the global maximum [20] (provided that the orders (NA,NC) of the estimated ARMA model are equal to or larger than the true model orders).

Relatively little is known about the convergence rate of the RML algorithm for different types of processes. Hardly any analytical units are available and most of the results are based on extensive simulation studies [11]. However, most of these studies were related to control problems involving stable plants with poles and zeroes well inside the unit circle. Models arising in signal processing applications typically involve narrowband signals, which are represented by poles and zeroes on or very near the unit circle. Our own experience indicates that pole and zero locations have a significant effect on convergence rates. When the poles and zeroes are well separated, fast convergence was observed. Clusters of poles and zeroes, especially when they are near the unit circle, often lead to much slower convergence. The issue of convergence rate is crucial in adaptive signal processing since signals are often nonstationary and time-varying and it is important to know how well the adaptive filter will track the changing parameters. In adaptive control problems the parameters are often very slowly time varying (compared to the time constants of the plant). We are currently investigating the convergence rate of the RML algorithm for ARMA models with poles and zeroes near the unit circle.

The properties of the maximum likelihood estimator described above make it very attractive for adaptive IIR filtering, as illustrated in the next section.

3. ADAPTIVE SIGNAL PROCESSING: SOME EXAMPLES

3.1 The Adaptive Line Enhancer (ALE)

The ALE is an adaptive filter for narrowband signals in additive noise [21]-[23]. The ALE can be interpreted as an adaptive predictor, i.e., its output is $\hat{y}_{t|t-1}$: the estimate of y_t based on data up to time t-1. A narrowband (autoregressive) signal in white noise can be represented as an ARMA process [24]. Thus, the optimal predictor is given by

$$\hat{y}_{t|t-1} = -\sum_{i=1}^{NA} a_i y_{t-i} + \sum_{i=1}^{NC} c_i \varepsilon_{t-i} \tag{6}$$

which is depicted as a tapped delay line filter in Figure 1. The parameters of the filter will be adjusted using the RML algorithm (with NB=0). Note that the resulting filter has an infinite impulse response while most ALE-s discussed in the literature are of the FIR type. In [24] we discuss in detail the advantages of the IIR-ALE and its superior performance at low signal-to-noise ratios.

To illustrate the behavior of the IIR-ALE we depict in Figure 2 the input and output of the filter for a single sinusoid in noise, at SNR = 0 dB. The spectra were obtained by a 512 point FFT. The ALE

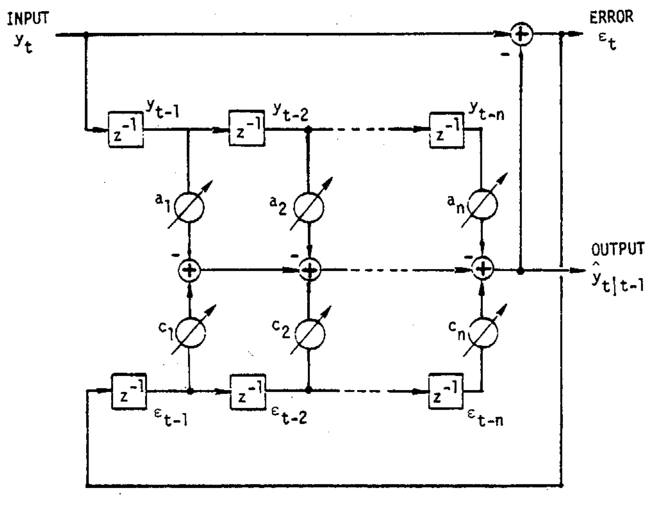

Figure 1 The IIR Adaptive Line Enhancer

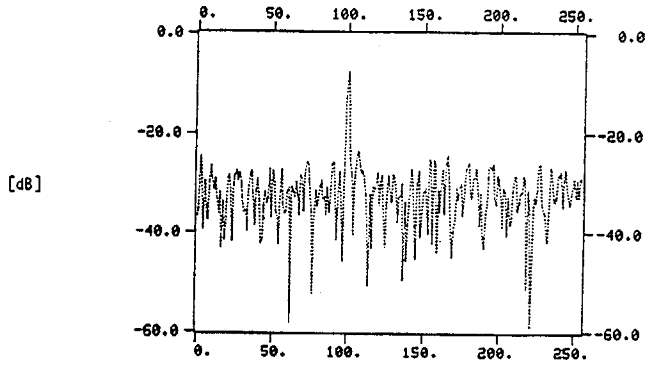

Figure 2a Spectrum of the ALE Input y_t

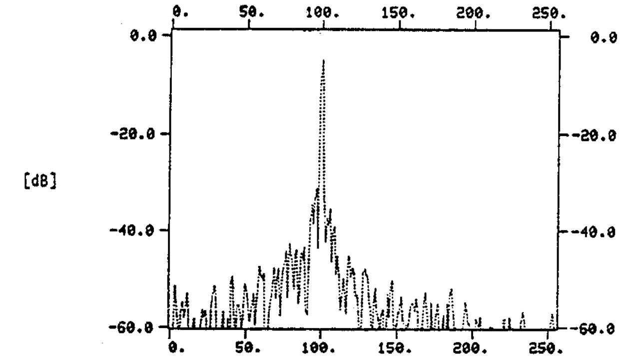

Figure 2b Spectrum of the ALE Output $\hat{y}_{t|t-1}$

output data corresponds to the last 512 samples in a total of 2048 data points. Note the significant noise reduction. Further noise suppression is achieved if the filter is allowed to continue its convergence. See [24],[25] for more results.

The parameters of the ARMA model $C(z^{-1})/A(z^{-1})$ which serve as the coefficients of the IIR-ALE can also be used to estimate the spectrum of the observed signal. In [26] we present some comparisons of the ARMA spectra obtained by the RML with corresponding estimates obtained by the maximum entropy method.

3.2 Adaptive Deconvolution

The need to extract a signal, given a filtered version of the signal arises in many situations including: (i) speech analysis/synthesis by linear predictive coding, and pitch estimation, (ii) estimation of the reflectivity sequence in seismic data processing, (iii) channel equalization for the removal of the intersymbol interference caused by convolution of the message sequence with the channel impulse response.

Consider the case where a white signal process passes through an IIR filter $C(z^{-1})/A(z^{-1})$ (stable and minimum phase). The RML algorithm can be used to estimate the filter parameters, and deconvolution will be achieved by passing the data through the estimated inverse filter $\hat{A}(z^{-1})/\hat{C}(z^{-1})$. Note that most current deconvolution techniques are limited to the case where the convolving filter has only poles $(1/A(z^{-1}))$ while the RML can handle the pole-zero case $(C(z^{-1})/A(z^{-1}))$. In Figure 3 we present a comparison between IIR and FIR deconvolution. The data y_t was generated in this case by passing a train of impulses through the filter

$$\frac{C(z^{-1})}{A(z^{-1})} = \frac{1 + z^{-2}}{1 + .606z^{-1} + .93z^{-2}} \tag{7}$$

and adding measurement noise. The signal to noise ratio was 20 dB. Note that the IIR filter restores the

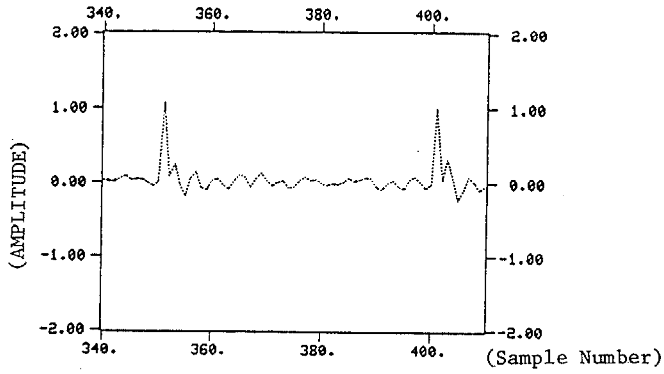

Figure 3a The Deconvolved Signal ε_t at the Output of an IIR Filter (NA=2, NC=3)

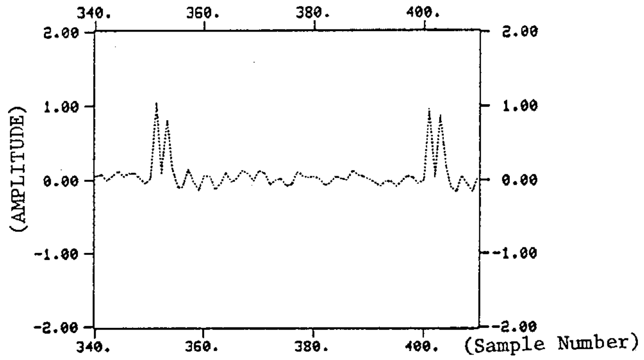

Figure 3b The Deconvolved Signal $_t$ at the Output of an FIR Filter (NA=2,NC=0)

original impulse sequence while the FIR filter gives pairs of impulses. This is caused by the fact that the moving average part of the convolving filter is not identified by the algorithm and therefore the deconvolved signal is $A(z^{-1})y_t = C(z^{-1})v_t = (1+z^{-2})v_t$. For a more detailed discussion of adaptive deconvolution see [27], [28].

3.3 Adaptive Noise Cancelling (ANC)

The ANC and its applications are discussed in [29] [30]. Here we present only a very brief description. In the ANC problem we are provided with a noisy measurement y_t of the signal s_t

$$y_t = s_t + z_t \tag{8}$$

and also with a reference input u_t which contains information about the noise process z_t. From the side information (u_t) an estimate $\hat{z}_t$ of the noise process is obtained and then subtracted from the primary input y_t to "cancel out" the noise, i.e.,

$$s_t = y_t - \hat{z}_t \tag{9}$$

under the assumptions that u_t and z_t are related by a linear model and that s_t is an ARMA process it can be shown [31] that y_t is an ARMAX process of the form

$$y_t = z_t + s_t = \frac{B(z^{-1})}{A(z^{-1})} u_t + \frac{C(z^{-1})}{A(z^{-1})} u_t \tag{10}$$

where u_t is the reference input and v_t is a white noise process. The noise estimate can be obtained by the following IIR filter.

$$\hat{z}_t = -\sum_{i=1}^{NA} a_i z_{t-i} + \sum_{i=1}^{NB} b_i u_{t-i} . \tag{11}$$

Using the RML algorithm to estimate the ARMAX model parameters and then estimating the "clean" signal by equstions (9), (11) gives an IIR-ANC. Figure 4 depicts the noise cancellation achieved for a narrowband signal in the presence of narrowband interference. See [31] for more results.

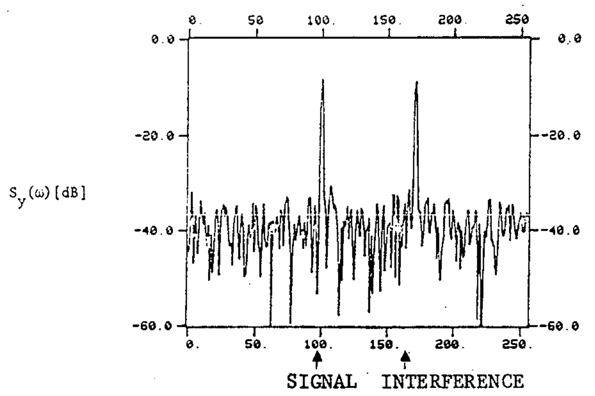

Figure 4a Spectrum of Primary Input y_t, Narrowband Interference

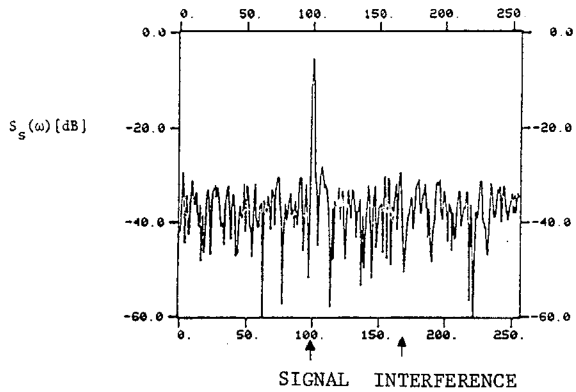

Figure 4b Spectrum of ANC Output s_t

Note that in addition to obtaining the predictor $B(z^{-1})/A(z^{-1})$ of the noise process z_t, the RML algorithm automatically provides us with an ARMA model $C(z^{-1})/A(z^{-1})$ for the signal process s_t. If the signal s_t is a narrowband process corrupted by white noise (uncorrelated with the noise process z_t measured by the reference input), additional noise suppression can be obtained by narrowband filtering. Thus, the ARMA parameters (C,A) can be used to form an IIR-ALE as was discussed in Section 3.1, i.e.,

$$\hat{s}_t = -\sum_{i=1}^{NA} a_i s_t + \sum_{i=1}^{NC} c_i \varepsilon_{t-i} . \tag{12}$$

In otherwords, the RML algorithm leads naturally to an adaptive filter (equations (9), (11), (12)) that can be interpreted as an IIR-ANC followed by an IIR-ALE. In [31] we present a much more detailed discussion of this filter.

3.4 Adaptive Time Delay Estimation

The need to estimate time delay between two signals arises in many applications such as target localization by sonar systems, and position estimation by radio navigation systems. The problem is usually formulated as follows: let us assume that two sensors (recivers, microphones, geophones) receive time shifted and scaled versions of some signal x_t: $u_t = x_t + n_t$, $y_t = dx_{t-\tau} + m_t$, where m_t, n_t are independent measurement noise processes. Many different techniques for estimating the delay τ have been proposed in the literature. These techniques typically involve filtering the signals and then cross-correlating [32]-[34]. To do this in an optimal way (in the least-squares or maximum likelihood sense) requires knowledge of the statistics of both the signal and noise. Using the system identification approach we were able to derive an adaptive technique for time delay estimation which requires no prior information [27],[35]. Here we present only the simplest version of the algorithms presented in [27],[35].

Note that two processes that are delayed versions of the same underlying signal are related by a moving-average filter whose coefficients contain the delay information. In the example mentioned above

$$y_t = B(z^{-1})u_t + v_t \tag{13}$$

where

$$B(z^{-1}) = dz^{-\tau}$$

$$(b_i=0 \text{ for } i\neq\tau \text{ and } b_i=d \text{ for } i=t)$$

$$v_t = m_t - dn_{t-\tau} = \text{a white noise process}$$

Using the RML algorithm to estimate the coefficients of $B(z^{-1})$ will provide an estimate of the time delay by looking at the index of the largest coefficient of $B(z^{-1})$. If the delay is not an integer multiple of the sampling interval, it is necessary to perform a simple interpolation to get at the true delay [27]. The RML algorithm is, of course, capable of handling the case where v_t is not white.

A much more sophisticated approach is based on the idea of fitting a multichannel model to a vector of sensor measurements y_t, e.g.,

$$y_t = -\sum_{i=1}^{NA} A_i y_{t-i} + \sum_{i=1}^{NC} C_i v_{t-i} \tag{14}$$

where y_t is a $p \times 1$ vector and A_i, C_i are $p \times p$ matrices. In [27], [36] we have shown that the resulting multi-input multi-output (MIMO) adaptive filter can be interpreted as a combination of an adaptive beamformer and a time delay estimator. The interesting point brought out in [36] is that this filter is capable of handling simultaneously several (up to p) targets! Here we only note that the use of MIMO signal models opens the way for developing new classes of MIMO adaptive filters which have numerous applications in array processing, beamforming, and processing of multisensor data. Current adaptive filtering techniques are almost entirely devoted to single input single output filters. We believe that perhaps the most important contribution of our modeling approach is that it provides a systematic framework for handling multichannel problems.

4. CONCLUSIONS

We presented an approach for developing adaptive filters for various signal processing problems. While the RML algorithm described in this paper is well known, its application to the class of problems described in Section 3 is apparently new. The RML algorithm was presented here in one particular form. It is important to note that alternative forms can be used to derive other implementations of these adaptive filters with similar asymptotic properties. Some examples:

Square-Root Form

The update equation (4) for the error covariance matrix P_t suffers from numerical problems when the number of estimated parameters n = NA + NB + NC is large (e.g., n > 10). A much better implementation is obtained by using the square root form in which $P_t^{1/2}$ is propagated rather than P_t (where $P_t^{1/2} P_t^{T/2} = P_t$). The advantages of square-root algorithms were discussed in deail by Bierman [37].

"Fast" Implementation

The RML algorithm presented in equation (4) requires in the order of $4n^2 + 5n$ multiplications and additions per data point. For higher order models (i.e., large values of n) the amount of computation may become excessively large. Thus, it is important to search for more efficient estimation techniques. Using the idea of "shift low rank" developed by Morf led to implementations of the RML requiring proportional to n rather than n^2 multiplications and additions [38]. This technique provides an efficient way of computing the gain vector $P_t\psi_{t+1}$. The rest of the algorithm remains unchanged. The detailed update equations can be found in [38] and will not be repeated here.

Recursive Lattice Forms

The recently developed square-root normalized lattice forms [39] combine the good numerical behavior of square-root implementation with computational efficiency. They furthermore provide a computational technique that is recursive both in time and in model order. In fact, the lattice form provides simultaneous parameter estimates corresponding to filters of all orders up to a maximum order! This is very sueful for addressing the order determination problem, which is one of the more difficult aspects of signal modeling.

Symmetric $A(z^{-1})$ Polynomial

In some situations the parameters of the ARMA model are interrelated in some way. For example, the case of sinusoids in white noise can be shown to have a symmetric $A(z^{-1})$ polynomial, i.e., $a_i = a_{n-i}$. In [40] we presented a way of incorporating this constraint in the parameter estimation algorithm. Several high resolution spectral estimation techniques are implicitly "Symmetrizing" the predictor coefficients. In general, whenever the problem has some special structure that can be used to reduce the number of estimated parameters, one should explore the possibility of using that structure in the parameter estimation algorithm.

Finally we should note that the results presented here are only preliminary. Much work remains to be done on the analysis and performance evaluation of the RML and related algorithms in adaptive signal processing applications. Some specific issues that need to be addressed are: the convergence rate of the RML for different classes of signals, the tradeoff between parameter tracking capability (i.e., the value of λ) and filter performance, and the development of robust techniques for ARMA order determination in real-time.

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