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ADDITIVE MIXTURE OF GAUSSIAN AND IMPULSIVE NOISE IN M-ARY NONCOHERENT DIGITAL SYSTEMS*

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RESUME

On présente l'évaluation des performances des systèmes de communication numériques incohérents à plusieurs états lorsque les bruits gaussien et impulsionnel sont simultanément présents.

On calcule la probabilité d'erreur pour les systèmes FSK binaires, tandis que pour l'ASK et l'FSK à plusieurs états sont présentées des estimations de la probabilité d'erreur aptes à délimiter un intervalle suffisamment étroit ("tight bounds").

Puisque les systèmes FSK sont employés lorsque il y a fading du signal, on a considéré l'influence du fading sur la performance de ces systèmes.

Les structures du récepteur considérées sont celles (habituellement employées) qui réalisent l'algorithme du maximum de vraisemblance pour un bruit blanc gaussien.

SUMMARY

The performance evaluation of multilevel noncoherent digital communication systems in simultaneous presence of Gaussian and impulsive noise is presented.

For binary FSK systems the exact expression of the bit error probability is derived, whereas for multilevel ASK and FSK systems upper and lower bounds of the character error probability are obtained.

Since the FSK systems are preferred in applications where signal fading is expected, for such systems also fading is taken into account.

The analysis is performed considering the maximum likelihood receivers for additive white Gaussian noise (AWGN).

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I. Introduction

The performance of M-ary digital communication systems operating in Gaussian noise environments has been exstensively analysed also in the presence of other types of interference (i.e., intersymbol, co-channel, etc.) However, expecially in the frequency region below UHF, the noise generated by several natural and man-made electromagnetic sources (atmospheric noise, ignition noise, etc.) exhibits impulsive characteristics [1-6].

Comparatively few authors [7-11] have studied the performance of digital systems in the presence of impulsive noise. In particular very few papers [11] have been devoted to M-ary noncoherent systems.

A more realistic noise model takes into account the simultaneous presence of Gaussian and impulsive noise because in general both types are present, though sometimes only one may be predominant. With reference to such a model the error rates of binary [12,13] and M-ary [14,15] coherent systems have been already evaluated. In [12,13,15] the effect of Rayleigh signal fading has also been considered.

In the present paper we evaluate upper and lower bounds for the error probability of M-ary noncoherent ASK and FSK systems (useful results for DPSK have not yet been obtained) in the presence of an additive mixture of Gaussian and impulsive noise.

The impulsive noise model employed here consists of a stream of delta function impulses of random areas occurring at random times which constitute a Poisson process. This model, already considered in previous papers [7-15], has been chosen because it gives [1] a good description of some real physical situations (e.g., atmospheric and ignition noise) in which the bandwidth of the incoming noise is larger than that of the receiver's front-end stages.

The present analysis is performed considering the well-known receiver structures for additive white Gaussian noise because such receivers are in common use and, on the other hand, the derivation and the implementation of the optimum receiver for M-ary signaling schemes in the presence of Gaussian and impulsive noise are very difficult.

Since the error probability of digital systems for purely Gaussian noise is strongly dependent on the presence of signal fading and its parameters [16], we take into account here the channel fading characteristics assuming that such a fading is non-selective and slow in comparison with the signaling duration. The signal fading is not considered for the ASK systems which, as well-known [16], are not preferred in applications where it is expected.

Let us mention finally that, for the numerical computations, the areas of the noise impulses are assumed to follow a bilateral Rayleigh distribution. However the results obtained hold practically also for different distribution laws. In fact, as shown in [17], the performance of ASK systems is practically unaffected by this choice over a wide range of the signal to noise ratio (SNR), whereas for fading FSK signals the bilateral Rayleigh distribution gives a slightly pessimistic estimation of the error probability obtained with reference to the Gaussian and exponential distributions which have been frequently assumed in the previous analyses [7, 11,14].

In Section II the noise model is stated, whereas bounds for the character error probabilities of M-ary ASK and FSK systems are obtained and discussed in Sections III and IV, respectively. Also some numerical results are presented.

II. Noise model

The noise $n(t)$ at the receiver input is modeled as an additive mixture of two statistical independent processes:

$$n(t) = n_g(t) + n_i(t) \qquad (1)$$

where $n_g(t)$ is a zero-mean white Gaussian noise process and $n_i(t)$, referred to as the Poisson or generalized shot noise model, consists of sample functions

$$n_i(t) = \sum_{r=-\infty}^{\infty} a_r\, \delta(t - t_r) \qquad (2)$$

where a_r is the random area of the rth impulse $\delta(t-t_r)$ which occurs at random instant t_r. The number of impulses occurring in any observation interval is assumed to obey a Poisson distribution.

The areas a_r are supposed to be statistically independent of one another and of the occurrence times and to have the same even probability density function (pdf) $p_a(\cdot)$. Moreover in the following we shall consider the most interesting case of highly impulsive noise for which the average number γ of noise impulses occurring in the signaling duration is very small ($\gamma \ll 1$).

The performance analysis of M-ary non coherent digital systems, as at will be evident in the following sections, is carried out starting from the characteristic function (CF) of the noise vector $\underline{N} = \underline{N_i} + \underline{N_g}$ at the output of the matched filters of the receivers at sampling instant $t=T$.

The CF of the impulsive noise component $\underline{N_i}$ at the output of a parallel bank of k orthogonal matched filters, on the assumption $\gamma \ll 1$, is given by [7]:

$$\Phi_{\underline{N_i}}(\lambda_1,\dots,\lambda_k) = 1-\gamma+2\gamma\int_0^{\infty} p_a(a)\int_0^{T}\cos\left[a\sum_{m=1}^{k}\lambda_m h_m(t)\right]\frac{dt}{T}\,da \qquad (3)$$

where $h_m(t)$ is the impulse response of the mth filter.

III. ASK performance analysis

For noncoherent ASK systems the received signal in the signaling duration T is of the form:

$$r(t) = \sqrt{\frac{2E_i}{T}}\cos(\omega_o t+\alpha) + n(t) \qquad i = 1,2,\dots,M \qquad (4)$$

where: E_i is the energy content of the ith transmitted signal; $\omega_o = 2\pi n_o/T$ with n_o some fixed integer; α is a random variable (rv) uniformly distributed over a 2π interval.

According to [18], let us assume that

$$\sqrt{E_i} = (i - 1)\Delta \qquad i = 1,2,\dots,M \qquad (5)$$

where, if the transmitter is subject to the average power limitation E/T, Δ is given by

$$\Delta = \sqrt{\frac{6E}{(M-1)(2M-1)}} \qquad (6)$$

The receiver calculates

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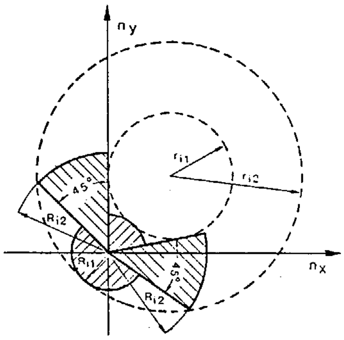

Fig.1 Integration regions for the lower bound of the conditional probability of a correct decision in ASK systems.

$$x = \int_0^T r(t)\sqrt{\frac{2}{T}}\cos\omega_0 t\, dt = \sqrt{E_i}\cos\alpha + n_x \qquad (7)$$

$$y = \int_0^T r(t)\sqrt{\frac{2}{T}}\sin\omega_0 t\, dt = -\sqrt{E_i}\sin\alpha + n_y \qquad (8)$$

computes $\sqrt{x^2+y^2}$ and, for equally likely signals, chooses the signal minimizing

$$\left|\sqrt{x^2+y^2} - \sqrt{E_j}\right| \qquad j = 1,2,\ldots,M \qquad (9)$$

Such decision rule is in common use because, as well-known [18], it approaches the optimum rule for AWGN in the range of high SNR's and, moreover, it is easy to implement.

Bounding techniques for the error probability

An analytical expression of the error rate can be obtained. Its complexity however suggests (already in the presence of purely Gaussian noise) to derive bounds of the character error probability. The bounding techniques employed here are an extension of those proposed in [18] for AWGN.

Since the pdf of the noise vector $\underline{N}$ possesses circular symmetry [15], the conditional probability of a correct decision for the ith transmitted signal is given by the probability that $\underline{N}$ falls in the ring-shaped region bounded by the concentric circumferences of radii $r_{i1} = (\sqrt{E_{i-1}} + \sqrt{E_i})/2$ and $r_{i2} = (\sqrt{E_i} + \sqrt{E_{i+1}})/2$. The center of such circumferences lies on any point of the circumference of radius $\sqrt{E_i}$, centered at the origin of the axes (Fig.1). This probability is lower [upper] bounded by the probability that $\underline{N}$ lands inside the shaded regions of Fig.1 [Fig.2].

The circular structure of these regions is particularly suitable for the above mentioned circular symmetry. In fact the probability P_{R_o} that $\underline{N}$ lands inside a circle of arbitrary radius R_o centered at the origin of the axes is given [17] by:

$$P_{R_o} = \int_0^{2\pi} d\phi \int_0^{R_o} R\, p_{\underline{N}}(R)\, dR = R_o \int_0^{\infty} \Phi_{\underline{N}}(\rho) J_1(R_o\rho)\, d\rho \qquad (10)$$

where $J_1(\cdot)$ is the Bessel function of the first kind of first order and R and ϕ are polar coordinates defined by

$$n_x = R\cos\phi$$
$$n_y = R\sin\phi \qquad (0 \le R < \infty\ ,\ 0 \le \phi < 2\pi) \qquad (11)$$

For the statistical independence of the impulsive and Gaussian noise the CF of $\underline{N}$, taking into account the well-known results for the Gaussian processes and (3), is given by

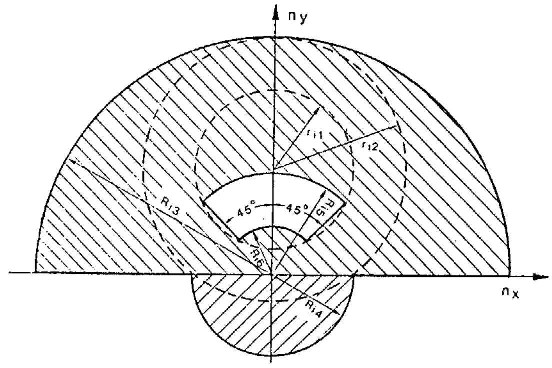

Fig.2 Integration regions for the upper bound of the conditional probability of a correct decision in ASK systems.

$$\Phi_{\underline{N}}(\lambda_1,\lambda_2) = \Phi_{\underline{N}}(\rho) = e^{-\rho^2\frac{\sigma_g^2}{2}}\left[1-\gamma+2\gamma\int_0^{\infty} p_a(a) J_0\!\left(a\sqrt{\frac{2}{T}}\rho\right)da\right] \qquad (12)$$

where: $\rho = \left(\lambda_1^2+\lambda_2^2\right)^{\frac{1}{2}}$; σ_g^2 is the Gaussian noise power spectral density at the input of the receiver; $J_o(\cdot)$ is the Bessel function of the first kind of zero order.

Finally the bounds of the character error probability P(E) can be expressed in terms of the lower $P_L(i)$ and the upper $P_U(i)$ estimations for the conditional probabilities of a correct decision:

$$1-\frac{1}{M}\sum_{i=1}^{M} P_U(i) \le P(E) \le 1-\frac{1}{M}\sum_{i=1}^{M} P_L(i) \qquad (13)$$

where, from the previous observations and from easy geometric considerations on the Figg.1 and 2, $P_L(i)$ and $P_U(i)$ are given by

$$P_L(i) = \begin{cases} P_{R_{i1}} & i = 1 \\ \frac{3}{4}P_{R_{i1}} + \frac{1}{4}P_{R_{i2}} & i = 2,3,\ldots,M-1 \\ \frac{1}{2}P_{R_{i1}} + \frac{1}{2} & i = M \end{cases} \qquad (14)$$

$$P_U(i) = \begin{cases} P_{R_{i1}} & i=1 \\ \frac{1}{2}P_{R_{i3}} + \frac{1}{2}P_{R_{i4}} & i=2 \\ \frac{1}{2}P_{R_{i3}} + \frac{1}{2}P_{R_{i4}} - \frac{1}{4}(P_{R_{i5}} - P_{R_{i6}}) & i=3,\ldots,M-1 \\ 1 & i=M \end{cases} \qquad (15)$$

In (14) and (15) $P_{R_{ij}}$ (j=1,2,...,6) denotes the probability that $\underline{N}$ lands inside the circle of radius R_{ij} centered at the origin of the axes. Such probability can be calculated by (10). The values of R_{ij} can be easily derived by simple geometric considerations on the Figg. 1 and 2; in particular $R_{i1} = \Delta/2$ for any value of i.

Numerical results and discussion

Assuming for the areas of noise impulses the bilateral Rayleigh distribution

$$p_a(a) = \frac{|a|}{\sigma_a^2}\exp(-a^2/\sigma_a^2) \qquad (16)$$

from (10) and (12) it follows that the probabilities $P_{R_{ij}}$ (i=1,2,...,M ; j=1,2,...,6) are given by

$$P_{R_{ij}} = (1-\gamma)\left[1-\exp\left(-\frac{R_{ij}^2}{2\sigma_g^2}\right)\right] + \gamma\left\{1-\exp\left[-\frac{R_{ij}^2}{2(\sigma_g^2+\sigma_i^2/\gamma)}\right]\right\} \qquad (17)$$

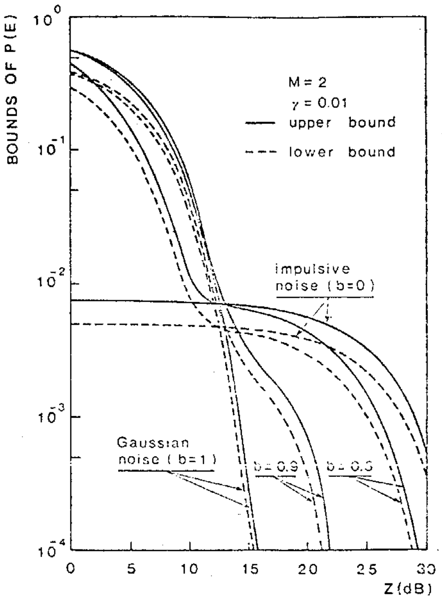

Fig.3 ASK systems: bounds of the error probability vs. SNR for several values of b.

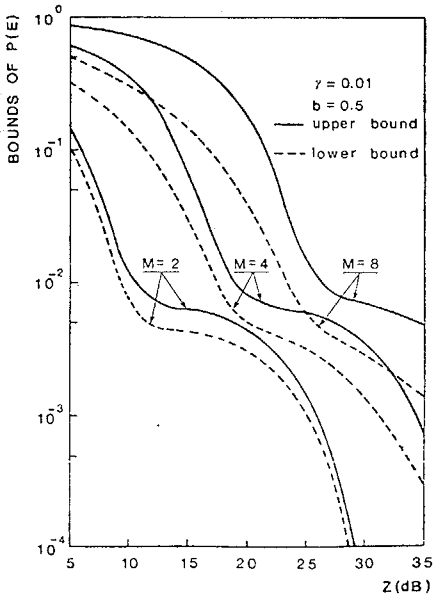

Fig.4 ASK systems: bounds of the error probability vs. SNR for several values of M.

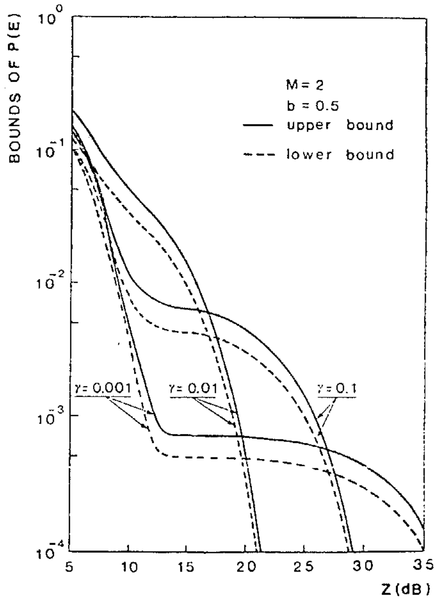

Fig.5 ASK systems: bounds of the error probability vs. SNR for several values of γ.

where σ_i^2 is the variance of both impulsive components at the outputs of the matched filters.

Fig.3 shows, for γ=.01 and M=2, the bounds of the error probability evaluated according to (13), (14), (15) and (17). The SNR Z and the parameter b are defined by:

$$Z = \sqrt{\frac{E}{\sigma_g^2 + \sigma_i^2}} \quad ; \quad b = \frac{\sigma_g^2}{\sigma_g^2 + \sigma_i^2} \tag{18}$$

The results are also presented for several values of M with γ=.01 and b=.5 (Fig.4) and for several values of γ with b=.5 and M=2 (Fig.5).

The following observations can be made:

a) For a given SNR, in correspondence of the range of practical values of P(E), the system performance is poorer (Fig.3) when also (or only) impulsive noise is present than when there is Gaussian noise alone (b=1). The particular case b=0 (purely impulsive noise) gives an upper bound for the error probability in the range of practical values of P(E). Moreover, in such range of SNR's, with Z, γ and M fixed, as the impulsive noise power increases, with respect to that of Gaussian (i.e., as b decreases from one to zero), the P(E) increases more and more slowly.

b) For a given SNR in the range of lowest values the system performance is better when also (or only) impulsive noise is present (Fig.3). In particular the case b=0 gives a lower bound for P(E).

c) For purely impulsive noise the error probability is practically expressed by γ(M-1)/M over a wide range of SNR's (Fig.3), whereas it is strongly dependent on the SNR when a Gaussian noise component is also present (b ≠0). This behaviour is readily explained considering that for purely impulsive noise the probability of occurrence of an impulse in the signaling duration is γ and that, for relatively weak SNR's, whenever such an impulse occurs, the decision can as well be made by chance. On the other hand, when the Gaussian noise is present, its influence on P(E) at weak SNR's is predominant and therefore the performance is strongly sensitive to the SNR.

d) From the previous considerations one deduces that there is a transition region (Figg.3,4 and 5) between the range of SNR's in which the performance is dominated by the Gaussian noise component and the range of high SNR's in which the impulsive noise dominates. Such transition region is located at values of the SNR more and more high for increasing values of M (Fig.4) and/or for decreasing values of γ (Fig.5).

e) The results show that the bounds are fairly tight though their tightness in the range of intermediate values of Z decreases as M increases (Fig.4). On the other hand no attempt has been made to optimize the aperture angle of the sectors of circle of Figg.1 and 2 in order to improve the tightness of the bounds. In fact this improvement is not easily performable because the optimum aperture angles are complicated functions of M, γ, Z and b. However we point out that for high SNR's only the set of the points inside the circle of radius $R_{i1} = \Delta/2$ gives significant contribution to the error probability.

IV.FSK performance analysis

In noncoherent FSK systems the received signal is

$$r(t) = F\sqrt{\frac{2E}{T}}\cos(\omega_i t + \alpha) + n(t) \qquad i=1,\dots,M \tag{19}$$

where: E is the energy content of the transmitted signals; T is the signaling duration; $\omega_i = 2\pi(n_o+i)/T$ with n_o some fixed integer; α is a rv uniformly distributed over a 2π interval; n(t) is the additive mixture of Gaussian and impulsive noise; F is a fluctuating path-transmission factor, modeled as a rv Rayleigh distributed, which takes into account a slow (in comparison with T) and non-selective signal fading. The presence of such fading is considered here because, as well-known, the incoherent FSK systems are in common use in applications where fading is expected and synchronous detection is not feasible.

With reference to the optimum (for equally likely messages) receiver for AWGN, when the ith signal is sent, a correct decision will be made if, and only if, for all j≠i one has:

$$A_j = \sqrt{x_j^2 + y_j^2} < \sqrt{x_i^2 + y_i^2} = A_i \qquad j\neq i \tag{20}$$

where x_j and y_j are given by

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$$x_j = \delta_{ij} F\sqrt{E}\cos\alpha + \int_0^T \sqrt{\frac{2}{T}}\, n(t)\cos\omega_j t\, dt \qquad (21)$$

$$j=1,2,\dots,M$$

$$y_j = -\delta_{ij} F\sqrt{E}\sin\alpha + \int_0^T \sqrt{\frac{2}{T}}\, n(t)\sin\omega_j t\, dt \qquad (22)$$

where δ_{ij} is the Kronecker delta.

The error probability $P_E(i)$ conditioned to the ith transmitted signal is given by

$$P_E(i) = \int_0^\infty p_F(v)\int_0^\infty P(\bigcup_{j\neq i} A_j > w/A_i=w)\, p_{A_i}(w/F=v)\,dw\,dv \qquad (23)$$

where $p_F(\cdot)$ is the pdf of the rv F and $p_{A_i}(\cdot/F=v)$ is the conditional pdf of A_i given F=v.

The evaluation of the probability $P(\bigcup_{j\neq i} A_j > w / A_i=w)$ requires the knowledge of the joint conditional pdf of the rv's A_j given $A_i=w$. Such pdf is very hard to calculate because the rv's A_j are not statistical independent of one another. Therefore a suitable bounding technique of the error probability must be found.

In the present case a geometric approach is not possible because the observation space is 2M-dimensional.

The union bound technique is considered here to obtain upper and lower estimations of the performance:

$$\max_j\left\{P(A_j>A_i)\right\} \le P_E(i) \le \sum_{\substack{j=1\\ j\neq i}}^{M} P(A_j>A_i) \qquad (24)$$

where

$$P(A_j>A_i) = \int_0^\infty\int_0^{t_2} p_{A_iA_j}(t_1,t_2)\,dt_1 dt_2 \qquad (25)$$

The joint pdf $p_{A_iA_j}(\cdot,\cdot)$, on the previous assumptions, is expressed [17] by

$$p_{A_iA_j}(t_1,t_2) = \frac{1-\gamma}{\sigma_g^2}\,\frac{2t_1t_2}{2\sigma_g^2+\sigma_F^2E}\exp\left(-\frac{t_1^2+t_2^2}{2\sigma_g^2}\right)\exp\left[\frac{t_1^2\sigma_F^2 E}{2\sigma_g^2(2\sigma_g^2+\sigma_F^2E)}\right] + 2\gamma t_1t_2\int_0^\infty\int_0^\infty\int_0^\infty p_a(a)\rho\rho_1\exp\left[-\frac{\sigma_g^2}{2}(\rho^2+\rho_1^2)\right]\exp\left(-\frac{\sigma_F^2E}{4}\rho^2\right)\cdot$$

$$\cdot\, J_0(t_1\rho)\,J_0(t_2\rho_1)\,J_0\left(a\sqrt{\frac{2}{T}}\rho\right)J_0\left(a\sqrt{\frac{2}{T}}\rho_1\right)d\rho\, d\rho_1\, da \qquad (26)$$

where σ_F^2 is the second order moment of the rv F.

Equation (26) shows that $p_{A_iA_j}(\cdot,\cdot)$ is independent on the values of i and j. Therefore from (24) and (25) it follows that the character error probability P(E) is bounded by

$$P_2(E) \le P(E) \le (M-1)P_2(E) \qquad (27)$$

where $P_2(E) \doteq P(A_j>A_i)$ is the error probability for binary FSK systems.

Numerical results and discussion

The bounds stated are evaluated here on the assumption that the areas of noise impulses are distributed according to (16). Therefore from (25), (26) and (27) one obtains that:

$$P_2(E) = (1-\gamma)\frac{1}{2+\frac{Z_0^2}{2b}} + \frac{\gamma}{2}\left[1-\frac{1}{\sqrt{1+\frac{4(1-b)}{\gamma(b+\frac{Z_0^2}{2})}}}\right] \qquad (28)$$

where b has been previously defined in (18) and Z_0 is the mean (over fading) SNR expressed by

$$Z_0 = \sqrt{\frac{\sigma_F^2 E}{\sigma_g^2+\sigma_i^2}} \qquad (29)$$

The validity of (28) may be checked by comparing it with the results obtained for purely Gaussian noise [16] and with the limiting situations ($Z_0 \to 0$ and $Z_0 \to \infty$).

Equation (28) shows the error rate as a sum of two terms: the former takes into account the presence of Gaussian noise acting when the useful signal is not affected by any noise impulse; the latter provides the con-

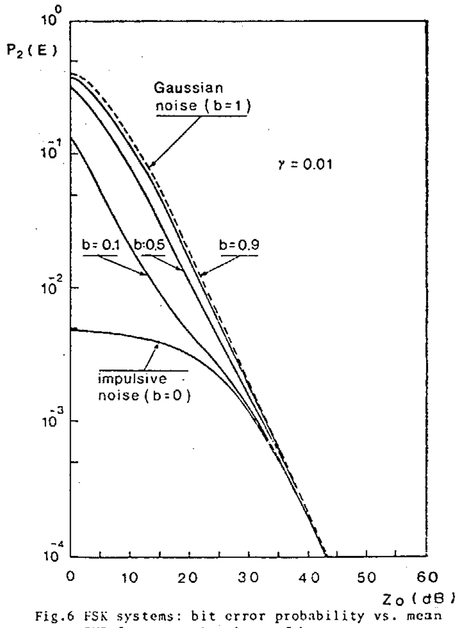

Fig.6 FSK systems: bit error probability vs. mean SNR for several values of b.

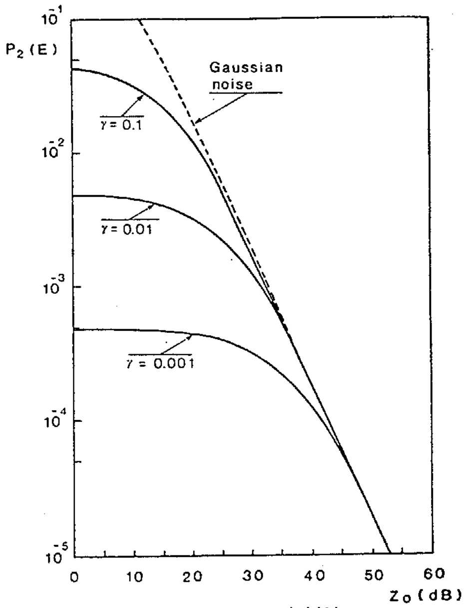

Fig.7 FSK systems: bit error probability vs. mean SNR for purely impulsive noise and for several values of γ.

tribution of the impulsive noise, modified by the presence of the Gaussian noise. Moreover for high SNR's ($Z_o^2 \gg 2b$) the Gaussian and the impulsive noise can be considered acting separately.

Fig.6 shows the error probability $P_2(E)$ for binary noncoherent FSK systems (for the M-ary case $P_2(E)$ allows to derive, according to (27), the bounds for the character error probability) as a function of the mean SNR Z_o for $\gamma = .01$ and for several values of b.

It is evident from the curves that for purely impulsive noise at low SNR's $P_2(E)$ is practically independent on Z_o over a range of SNR's which, on the other hand, is more and more wide for decreasing values of γ (Fig.7), as it can be derived from (28) considering that the system performance for b=0 is a function of γZ_o^2. In the same range of SNR's, whenever the Gaussian noise is present, the probability $P_2(E)$ is relatively sensitive to the SNR. Therefore, as in the ASK systems, in such range of Z_o the error characteristics are dominated primarily by the Gaussian noise component; in other words the error probability is largely independent on the parameter γ of the impulsive noise.

For high SNR's also the impulsive component gives a significant contribution to $P_2(E)$. Moreover, independently on the values of b and γ (Figg.6 and 7), in such region of Z_o, $P_2(E)$ is adeguately expressed (see eq.(28)) by $2/Z_o^2$.

V.Conclusions

The performance evaluation for M-ary noncoherent ASK and FSK systems in the simultaneous presence of Gaussian and impulsive noise is presented. Since the FSK systems are preferred in applications where signal fading is expected, for such systems also fading is taken into account.

An exact expression of the error rate for binary FSK systems is given, whereas, in the other examined cases, upper and lower bounds for the character error probability are derived.

The analysis assumes an unfiltered Poisson process to model the impulsive noise at the receiver's front, but it can be easily modified to include the effects of bandlimiting of the noise.

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