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ARRAY PROCESSING IN SEMI-HOMOGENEOUS RANDOM FIELDS

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RESUME

TRAITEMENT D'UN RÉSEAU DANS DES CHAMPS ALÉATOIRES SEMI - HOMOGÈNES

Des modèles de bruit appropriés pour examiner un système Sonar passif pour des sondages goniométriques et pour des mesures du spectre d'une cible dans un milieu bruité, sont les champs aléatoires homogènes consistant d'ondes planes. Du fait que l'hypothèse de stationnarité est fréquemment irréaliste, nous examinons des généralisations selon les procédés semi-stationaires stochastiques et nous introduisons des champs aléatoires semi-homogènes. Nous démontrons comment les méthodes de Priestley, qui estiment la densité spectrale évolutive peuvent être modifiées pour le faisceau rayonnant et pour l'analyse spectrale des signaux rayonnants, par èxemple si en fait un réseau de lignes est utilisé.
Des échantillons des signaux peuvent être traités par l'intermédiaire des algorithmes rapides qui se basent sur les transformations de Fourier rapides et qui sont comparables aux algorithmes déjà connus dans le cas de stationnarité. Finalement, nous indiquons une méthode de poursuite pour des lignes moduleès dans un spectre évolutif.

SUMMARY

ARRAY PROCESSING IN SEMI-HOMOGENEOUS RANDOM FIELDS

Useful noise models for investigating a passive sonar system for taking the bearings and measuring the spectrum of a target in ambient noise are homogeneous random fields consisting of plane waves. Since the assumption of stationarity is frequently unrealistic, we investigate generalizations in the sense of Priestley's semi-stationary stochastic processes and introduce semi-homogeneous random fields. We show how Priestley's method for estimating the evolutionary spectral density can be modified for beam forming and spectral analysis of the beam signals, e.g. if a line array is used. Sampled signals can be processed by fast algorithms which base on the fast Fourier transform and which are similar to known algorithms in the case of stationarity. Finally, we indicate a method for tracking modulated lines in an evolutionary spectrum.

ARRAY PROCESSING IN SEMI-HOMOGENEOUS RANDOM FIELDS

Introduction

Passive array processing is, following Liggert's [3] definition, interpretation of noise and noise-like signals received by an array of sensors, i.e. hydrophons in the case of sonar. The paper deals with processing methods for taking the bearings and measuring the spectra of signals which are generated by distant targets and which are disturbed by ambient noise from the far field. Frequently used models for stationary ambient noise at the outputs of the sensors are homogeneous random fields consisting of plane waves, cf. for example [1], [3]. Stationary signals can be incorporated in such models as discrete components of the random field. General properties of homogeneous random fields are proved in Yaglom's book [13]. Spectral estimation and beamforming techniques for estimating the targets have been frequently investigated. Smoothing of short-period spectra of a beam signal is a well known method to estimate the spectrum of noise from a given direction. It was shown in [9], [12] that this can be approximately done for sampled data by a fast method basing on multivariate fast Fourier transform.

The assumption of stationarity of the signals and of noise and moreover of homogeneity of the random field is unrealistic in many applications, especially for fluctuating ambient noise and moving targets. In this paper, we generalize the concept of homogeneity. Homogeneous random fields are composed of uncorrelated plane, elementary waves. The random fields we are interested in are superpositions of similar, but slowly modulated elementary waves. This concept is a direct generalization of Priestley's [6] model of semi-stationary processes. Therefore, we call the corresponding random fields semi-homogeneous. A semi-stationary process possesses evolutionary, i.e. time dependent, spectra which describe the power over frequency and time and which can be estimated under suitable conditions. A semi-homogeneous random field is characterized by evolutionary frequency-wavenumber spectra. We indicate that these spectra can be estimated by careful smoothing of short-period frequency-wavenumber spectrograms. Then, the frequently implemented tracking of the evolution of short-period spectrograms is a reasonable method for analysing a long observation of a semi-homogeneous random field. Similar to the case of homogeneity, short-period spectrograms of the beam signals generated by weighting, delaying, and summing the sensor outputs can be approximately computed by a variant of the multidimensional FFT-method indicated above. Finally, we mention some special cases, e.g. models with a fixed velocity of propagation, line arrays of sensors in such a model in the plane, and a method for estimating of modulated lines in evolutionary frequency spectra.

Semi-Homogeneous Random Fields

Let us assume that the d-dimensional real vector x describes the position of a sensor in an array of a finite number of sensors and that the sensor outputs are sampled with a period τ. If we presume wide-sense stationarity in time and space which means homogeneity, the output of the receiver at position x can be described by a Cramer representation, cf. [1],

$$Y(m,x) = \int_{-\pi/\tau}^{\pi/\tau} \int_{R^d} e^{j(\omega\tau m+kx')} Z(d\omega,dk) \quad (|m| = 0,1,\ldots),$$

where $\omega/(2\pi)$ is a frequency in Hertz, k is a d-dimensional real vector-wavenumber and x' means the transposed vector of x. Z describes a zero mean orthogonal random field, i.e. $Z(\Delta\omega,\Delta k)$ is a random function of the intervals $\Delta\omega,\Delta k$ with expectation

$$EZ(\Delta\omega,\Delta k)=0,$$

for disjoint intervals $\Delta_1\omega,\Delta_2\omega$ and $\Delta_1 k,\Delta_2 k$,

$$Z(\Delta\omega,\Delta_1 k+\Delta_2 k) = Z(\Delta\omega,\Delta_1 k) + Z(\Delta\omega,\Delta_2 k) \text{ and}$$

$$Z(\Delta_1\omega+\Delta_2\omega,\Delta k) = Z(\Delta_1\omega,\Delta k) + Z(\Delta_2\omega,\Delta k) \text{ and}$$

$$EZ(\Delta_1\omega,\Delta_1 k)Z(\Delta_2\omega,\Delta_2 k)^* = 0,$$

$$E|Z(\Delta\omega,\Delta k)|^2 = F(\Delta\omega,\Delta k) = \int_{\Delta\omega}\int_{\Delta k} f(\omega,k)\,d\omega dk .$$

Herein, F is the frequency-wavenumber spectrum and the most right equation is correct if a spectral density f of F exists. Consequently, Y(m,x) can be thought as a superposition of plane waves $e^{j(\omega\tau m+kx')}$ with orthogonal weights $Z(d\omega,dk)$.

Generalizing Priestley's [6] method to define a class of non-stationary processes, we think of a random field which is a superposition of modulated waves $A_{\tau m,x}(\omega,k)\cdot e^{j(\omega\tau m+kx')}$ with orthogonal weights $Z(d\omega,dk)$, where the complex and deterministic modulation $A_{\tau m,x}(\omega,k)$ slowly oscillates in m and x in comparison with $e^{j(\omega\tau m+kx')}$ and does not overmodulate the elementary wave. The output of the receiver is therefore

$$(1) \quad Y(m,x) = \int_{-\pi/\tau}^{\pi/\tau}\int_{R^d} e^{j(\omega\tau m+kx')} A_{\tau m,x}(\omega,k)Z(d\omega,dk) .$$

We call a random field Y(m,x) <u>semi-homogeneous</u> if there exists a zero mean random field Z and a function A such that Y(m,x) can be represented by (1), where the complex modulation function A has the following properties. There exists a spectral representation

$$(2) \quad A_{t,x}(\omega,k) = \int_{-\pi/\tau}^{\pi/\tau}\int_{R^d} e^{j(\theta t+\ell x')} H_{\omega,k}(d\theta,d\ell),$$

where $A_{0,0}(\omega,k) = 1$ and $|H_{\omega,k}(d\theta,d\ell)|$ has its maximum for $\theta=0$, $\ell=0$ and

$$B_A = [\sup_{\omega,k} \int\int |(\theta,\ell)|\,|H_{\omega,k}(d\theta,d\ell)|]^{-1} > 0 .$$

A homogeneous random field is of course semi-homogeneous with A=1. If a function $A_{t,x}$ is independent of (ω,k) and satisfies the conditions (2), the product of $A_{m\tau,x}$ and a homogeneous random field Y(m,x) is semi-homogeneous.

For a special semi-homogeneous random field Y, we consider the class of all functions A as above and define $B_Y = \sup_A B_A$ as the <u>characteristic width</u> of the random field Y. The number $2\pi B_Y$ can be roughly interpreted as the (d+1)-th power of the diameter of a maximum sphere in which the field can be treated as approximately homogeneous. For the sake of simplicity, we assume in the following that there exists one and only one A with $B_A=B_Y$ and we consider the corresponding natural representation (1). We shall not discuss the problems if there does not exist a maximum or A is not unique which could be done similar to [6].

Since Y(m,x) has expectation zero and variance $\mathrm{Var}\, Y(m,x) = \int\int |A_{\tau m,x}(\omega,k)|^2 F(d\omega,dk)$, we define the evolutionary frequency-wavenumber spectrum with respect to A as

$$F_{m,x}(d\omega,dk) = |A_{\tau m,x}(\omega,k)|^2\, F(d\omega,dk)$$

and the corresponding evolutionary density in case F has a density f as

$$f_{m,x}(\omega,k) = |A_{\tau m,x}(\omega,k)|^2\, f(\omega,k) .$$

Spectral estimation

In this section, we sketch a method for estimating the evolutionary power of a semi-homogeneous random field in an (ω,k)-band or the evolutionary density, if it exists and is sufficiently smooth, from one observation. Priestley [6] showed for semi-stationary processes that careful time smoothing of the time evolution of short-period spectrograms yields reasonable estimates if the width of the data window is small in comparison with both the width of the smoothing window and the characteristic width of the process and if both are much smaller than the range of observation.

The complex demodulation which means the short-period time-space Fourier transform of Y(m,x) is first investigated. With respect to the intended applications, let us presume a finite time-space window g(m,x), where x ranges over the finite set of sensor positions. The complex demodulation of Y(m,x) is defined as

$$(3) \quad U(m,x;\omega,k) = \sum_{\bar m}\sum_{\bar x} g(m-\bar m,x-\bar x)Y(\bar m,\bar x)e^{-j(\omega\tau\bar m+k\bar x')}$$

for given frequency ω and wavenumber vector k. Using (1) and (2), the right hand side equals

$$\iiiint \sum_{\bar{m}}\sum_{\bar{x}} g(m-\bar{m},x-\bar{x})\, e^{j((\bar{\omega}+\theta-\omega)\tau\bar{m}+(\bar{k}+\ell-k)\bar{x}')} H_{\bar{\omega},\bar{k}}(d\theta,d\ell)Z(d\bar{\omega},d\bar{k})$$

resulting in

$$(4)\quad U(m,x;\omega,k) = \iiiint e^{j((\bar{\omega}-\omega)\tau m+(\bar{k}-k)x')} e^{j(\theta\tau m+\ell x')} G(\bar{\omega}+\theta-\omega,\bar{k}+\ell-k) H_{\bar{\omega},\bar{k}}(d\theta,d\ell)Z(d\bar{\omega},d\bar{k}) ,$$

where

$$(5)\quad G(\omega,k) = \sum_m\sum_x g(m,x)e^{-j(\omega\tau m+kx')} .$$

If roughly spoken $H_{\omega,k}(d\theta,d\ell)$ behaves as a δ-function with respect to $G(\theta,\ell)$, we obtain $U\approx\bar{U}$, where

$$\bar{U}(m,x;\omega,k) = \iint e^{j((\bar{\omega}-\omega)\tau m+(\bar{k}-k)x')} G(\bar{\omega}-\omega,\bar{k}-k)A_{\tau m,x}(\bar{\omega},\bar{k})Z(d\bar{\omega},d\bar{k}) .$$

The approximation is specified in the following sense.

Let $R_1 = \iint e^{j(\theta\tau m+\ell x')} (G(\bar{\omega}-\omega+\theta,\bar{k}-k+\ell)-G(\bar{\omega}-\omega,\bar{k}-k))H_{\bar{\omega},\bar{k}}(d\theta,d\ell)$.

Then,

$$|R_1| \leq \sup_{\bar{\omega},\bar{k}} |\mathrm{grad}\, G(\omega,k)| \iint |(\theta,\ell)|\, |H_{\omega,k}(d\theta,d\ell)| \leq B_g/B_Y ,$$

where B_g is a measure of the width of the data window, namely

$$B_g = \sum_m\sum_x |(\tau m,x)|\,|g(m,x)| .$$

If for an $\varepsilon>0$ $B_g<\varepsilon B_Y$, it can be shown similar to the proof of Theorem 8.1 in [6] that $E|U-\bar{U}|^2=O(\varepsilon)$ and then

$$(6)\quad E|U(m,x;\omega,k|^2 = \iint |G(\bar{\omega}-\omega,\bar{k}-k)|^2 F_{m,x}(d\bar{\omega},d\bar{k})+O(\varepsilon) .$$

Concluding, $|U(m,x;\omega,k|^2$ is an approximately unbiased estimator of the integral in (6). If we furthermore assume that $F_{m,x}$ has a wavenumber-bandlimited and smooth density $f_{m,x}$ and $|G(\omega,k)|^2$ behaves as a δ-function with respect to $f_{m,x}(\omega,k)$ in the usual sense, the estimator is approximately unbiased for $f_{m,x}(\omega,k)$.

The variance of $|U|^2$ for a normally distributed random field with a smooth density $f_{m,x}$ can be evaluated similar to the treatment in [5] and [10]. We obtain for $\omega\neq 0 \bmod(\pi/\tau)$ and $k\neq 0$

$$\mathrm{Var}(|U(m,x;\omega,k)|^2) \approx [\iint |G(\bar{\omega}-\omega,\bar{k}-k)|^2 f_{m,x}(\bar{\omega},\bar{k})d\bar{\omega}d\bar{k}]^2$$

which is independent of the range of observations. Careful smoothing of $|U(m,x;\omega,k)|^2$ can reduce the variance. We therefore use

$$(7)\quad P(m,x;\omega,k) = \sum_{\bar{m}}\sum_{\bar{x}} w(m-\bar{m},x-\bar{x})\,|U(\bar{m},\bar{x};\omega,k|^2$$

as an estimate of the integral in (6), where $w(m,x)$ is a non-negative and finite window with $\sum_m\sum_x w(m,x) = 1$. The expectation of P is

$$(8)\quad EP(m,x;\omega,k) \approx \iint |G(\bar{\omega}-\omega,\bar{k}-k)|^2 \sum_{\bar{m}}\sum_{\bar{x}} w(m-\bar{m},x-\bar{x})F_{\bar{m},\bar{x}}(d\bar{\omega},d\bar{k}) .$$

Under the above assumptions and if the width of w is large in comparison with that of g, we shall find

$$(9)\quad \mathrm{Var}\, P(m,x;\omega,k) \approx O(\sum_m\sum_x w(m,x)^2)$$

which can be made small with increasing width of w.

Summarizing, we showed heuristically that careful smoothing of the time evolution of short time-space spectrograms is a reasonable method to estimate the evolutionary frequency wave-number spectrum if the width of the data window is small in comparison with both the width of the smoothing window and the characteristic width of the process, and if the latter are much smaller than the range of observation. We cannot obtain simultaneously a high resolution in both the time-space domain and the frequency-wavenumber domain.

Beamforming

Semi-homogeneous random fields may be interpreted as a superposition of approximately plane waves. Let us assume to be interested in those nearly plane waves propagating with velocity c and coming from a direction characterized by a unit vector e, i.e. travelling with a slowness vector $-\tau s=-e/c$. Intuitively, we could estimate the corresponding signal by weighting, delaying and summing the sensor outputs and then estimating the evolutionary frequency spectrum of the beam signal. We show in this section how this can be done and, similar to the case of homogeneity, that computing suitable traces of (7) yields approximately the same estimate.

We first investigate the beam signal generated by classical beamforming at point x if the direction of the beam is described by the vector s and which is

$$Y_s(m,x) = \sum_{\bar{x}}\gamma(x-\bar{x})Y(m-s\bar{x}',\bar{x}) .$$

Herein, we assume that the sensor data are sufficiently oversampled, the numbers $s\bar{x}'$ are integers, and γ is a finite window with a suitable width. The short-time Fourier transform of $Y_s(m,x)$ is

$$V_s(m,x;\omega) = \sum_{\bar{m}}\beta(m-\bar{m})Y_s(\bar{m},x)e^{-j\omega\tau\bar{m}} .$$

Defining $g(m,x)=\beta(m)\gamma(x)$ and using (1),(2),(4),and (5), we find

$$(10)\quad V_s(m,x;\omega) = \iiiint e^{j((\bar{\omega}+\theta-\omega)\tau m+(\bar{k}+\ell-(\bar{\omega}+\theta)\tau s)x')} G(\bar{\omega}+\theta-\omega,\bar{k}+\ell-(\bar{\omega}+\theta)\tau s)H_{\bar{\omega},\bar{k}}(d\theta,d\ell)Z(d\bar{\omega},d\bar{k}).$$

Assuming the same restrictions as in the last section and the existence of smooth densities $f_{m,x}$, one can show similar to the treatment for U that

$$E|V_s(m,x;\omega)|^2 \approx \iint |G(\bar{\omega}-\omega,\bar{k}-\bar{\omega}\tau s)|^2 f_{m,x}(\bar{\omega},\bar{k})d\bar{\omega}d\bar{k}$$

and for $\mathrm{Var}(|V_s(m,x;\omega)|^2)$ approximately the square of the right hand expression.

We estimate the evolutionary frequency density $f_{x,m}(\omega,\omega\tau s)$ of the waves with slowness vector $-\tau s$ by smoothing the time evolution of the short-time spectrograms of the beam signal with a window w as in the last section,

$$(11)\quad P_s(m,x,\omega) = \sum_{\bar{m}}\sum_{\bar{x}} w(m-\bar{m},x-\bar{x})\,|V_s(\bar{m},\bar{x};\omega)|^2.$$

The estimator P_s has similar properties as P in (8) and (9). We only have to substitute $G(\bar{\omega}-\omega,\bar{k}-\bar{\omega}\tau s)$ for $G(\bar{\omega}-\omega,\bar{k}-k)$.

The short-time Fourier transform $V_s(m,x;\omega)$ of the beam signal can be approximated by $U(m,\bar{x};\omega,k)$ if $k=\omega\tau s$ is chosen, as motivated by (4) and (10). For that we set

$$\Delta_s(m,x;\omega) = V_s(m,x;\omega) - U(m,x;\omega,\omega\tau s)$$

and presume the above restrictions. We compute

$$(12)\quad E|\Delta_s|^2 \approx \iint |G(\bar{\omega}-\omega,\bar{k}-\omega\tau s+(\omega-\bar{\omega})\tau s)e^{j(\omega-\bar{\omega})\tau sx'} -G(\bar{\omega}-\omega,\bar{k}-\omega\tau s)|^2 f_{m,x}(\bar{\omega},\bar{k})d\bar{\omega}d\bar{k} .$$

The integral corresponds with the mean square error in the case of homogeneous random fields up to the replacement of f by $f_{m,x}$. Consequently, the approximation for wavenumber-bandlimited $f_{x,m}$ is the better the smaller the frequency bandwith of G is chosen. We conclude that $P_s(m,x;\omega)$ can be approximated by $P(m,x;\omega,\omega\tau s)$ since we know the following bounds

$$|E(P_s(m,x;\omega)-P(m,x;\omega,\omega\tau s))| \leq E|P_s-P| \leq \sum_{\bar{m}}\sum_{\bar{x}} w(m-\bar{m},x-\bar{x})E|\Delta_s(\bar{m},\bar{x};\omega)|^2 \approx \iint |\cdots|^2 \sum_{\bar{m}}\sum_{\bar{x}} w(m-\bar{m},x-\bar{x})f_{\bar{m},\bar{x}}(\bar{\omega},\bar{k})d\bar{\omega}d\bar{k},$$

where $|\cdots|^2$ means the corresponding term in (12). This bound is not greater than the supremum of $E|\Delta_s|^2$.

The remainder of this section deals with the computation of the estimate $P(m,x;\omega,\omega\tau s)$, where the point of observation is x=0. For the sake of simplicity, we assume that the sensors are placed at points $x=\eta n$ with $n=(n_1,\ldots,n_d)$ $(|n_i|=0,\ldots,N_o-1)$ and a sampling period η and that the time sampling points are τm $(m=0,\ldots,M_o-1)$. Let the support of the data window g be $(m=1-M,\ldots,0;$ $n_i=1-N,\ldots,0)$. Then generalizing [2], we obtain from (3)

$$U(m,\eta n;\omega,k) = e^{-j(\omega\tau m+k\eta n')} \sum_{\bar{m}=0}^{M-1}\sum_{\bar{n}_1,\ldots,\bar{n}_d=0}^{N-1} g(-\bar{m},-\eta\bar{n})Y(m+\bar{m},\eta(n+\bar{n}))e^{-j(\omega\tau\bar{m}+k\eta\bar{n}')}$$

for $m=0,\ldots,M_o-M$ and $n_i=1-N_o,\ldots,N_o-N$. This formula can be computed by (d+1)-dimensional FFT-algorithms if $\omega\tau=2\pi i/M$ $(i=0,\ldots,M-1)$ and $k=(k_1,\ldots,k_d)$ with $k_p\eta=2\pi i/N$ $(i=0,\ldots,N-1)$. The exponential factor can be automatically obtained if the transform of the windowed data piece suitable rotated is taken. Because of the band limitation of the window, $U(m,\eta n;\omega,k)$ is highly oversampled with respect to m and n. We conjecture that, as in Welch's method [11], overlappings of 50 per cent in each direction of the windowed data could suffice for smoothing of $|U(m,\eta n;\omega,k)|^2$. Then, the spectrogram is only computed for $m=\tilde{m}M/2$ and $n_i=\tilde{n}_iN/2$ with the above restrictions and even M and N. Since the smoothing window is much more frequency bandlimited than the data window, a similar argument shows that $P(m,0;\omega,k)$ should be computed only for $m=m^+LN/2$, where LN describes the time duration of w. We use formula (7) with the modification that the sum is only taken over points $m=\tilde{m}M/2$ and $x=\eta\tilde{n}N/2$. $P(m,0;\omega,k)$ is required for $\omega\tau=2\pi i/N$ $(i=0,\dots,N-1)$ and $k=\omega\tau s$ for given vectors $s=s^1,\dots,s^K$. As usual, we have to interpolate. Space does not permit a discussion.

Special Cases

1) Let us consider semi-homogeneous random fields consisting of plane waves with fixed velocity c of propagation. The model (1) simplifies to

$$Y(m,x)=\int_{-\pi/\tau}^{\pi/\tau}\int_S e^{j\omega\tau(m+sx')}A_{\tau(m+sx')}(\omega,\omega\tau s)Z(d\omega,\omega\tau ds), \tag{13}$$

where S is the perephery of the sphere with radius $|s|=1/(c\tau)$. $A_t(\omega,\omega\tau s)$ must have a spectral representation, $\int e^{j\theta t}H_{\omega,s}(d\theta)$, where $|H_{\omega,s}(d\theta)|$ is maximal at $\theta=0$. If we consider spectral distributions over S instead of R^d, the results of the paper can be transferred to this case. Especially, we can write (13) in Cartesian coordinates for d=2 and a point $x=(0,p)$

$$Y(m,x) = \int_{-\pi/\tau}^{\pi/\tau}\int_0^{2\pi} e^{j\omega(\tau m+p/c\sin\alpha)} A_{\tau m+p/c\sin\alpha}(\omega,\alpha)Z(d\omega,d\alpha),$$

where α is the angle between the x_1-axis and the vector s. The evolutionary frequency-angle spectral density is $f_{m,p}(\omega,\alpha)=|A_{\tau m+p/c\sin\alpha}(\omega,\alpha)|^2 f(\omega,\alpha)$ when f exists. If a long line array located on the x_2-axis is used for estimating that density, we can use the technique of the beforegoing sections with the restriction that the windows only use sensor positions on the line array. For example, we find

$$EP(m,0;\omega,\alpha) \simeq \iint|G(\bar{\omega}-\omega,(\bar{\omega}\sin\bar{\alpha}-\omega\sin\alpha)/c)|^2 \sum_{\bar{m}}\sum_p w(m-\bar{m},-p)f_{\bar{m},p}(\bar{\omega},\bar{\alpha})d\bar{\omega}d\bar{\alpha}$$

with the known problems induced by the argument of G.

2) A distant target radiating noise in a model (13) is characterized by a discrete point in the distribution over S. The noise generated by the target alone and received at position x can be described by

$$Y(m,x)=\int_{-\pi/\tau}^{\pi/\tau} e^{j\omega\tau(m+s_0x')}A_{\tau(m+s_0x')}(\omega,\omega\tau s_0)\cdot Z(d\omega,\omega\tau s_0), \tag{14}$$

where s_0 is the direction of the target. If s_0x' is an integer, $Y(m,x)=Y(m+s_0x',0)$. Defining $Z_x(d\omega) = e^{j\omega\tau s_0x'}Z(d\omega,\omega\tau s_0)$ and $A_{\tau m,x}(\omega)=A_{\tau(m+s_0x')}(\omega,\omega\tau s_0)$, we can interprete $Y(m,x)$ as a semi-stationary process in the sense of Tong, cf. [10],[8]. All results about evolutionary spectra and cross spectra of bivariate processes are directly applicable to an analysis of the two signals received at different sensor positions.

3) If the noise received from a point target is represented by (14) and Z contains a discrete part $Q(\omega_0)$ at frequency ω_0, then the discrete part of $Y(m,x)$ is

$$e^{j\omega_0\tau(m+s_0x')}A_{\tau(m+s_0x')}(\omega_0,\omega_0\tau s_0)Q(\omega_0).$$

Martin [4] called the corresponding part in the evolutionary frequency spectrum a modulated line and analysed the phase of the short-time Fourier transform at ω_0 by a regression with linear splines and unknown knots to estimate the instantaneous frequency of the modulated line.

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