SEPTIEME COLLOQUE SUR LE TRAITEMENT DU

SIGNAL ET SES APPLICATIONS

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A New Version Of The RZI

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RESUME

La description des signaux par la virtue de leurs distributions de zeros a été l objet de profondes études durant les dernières années. La representation basée sur les zéros donne un modèle multiplicatrice du signal avec une bande limitée,i.e. après avoir transférer le signal à des zeros réels,il est representé par un polynôme.

Le problèm de reconstructions du signal quand les zéros sont connus,est possible en utilizant un systeme qui a été proposé et réalisé en premier lieu par Dr.H.B.Voelcker (1). Ce système nommé [1] Interpolation des Zéros Reels (Real Zero Interpolator)RZI. Ce systeme donne un minimum de largeur de bande de frequence de la forme des ondes due a un nombee fini de zéros.

Le RZI est un système très utile pour les systèmes de communications modernes parce que la plupart des systémes ont des signaux qui ont des zéros représentant la majorité des informations contenues par le signal. Les examples typiques sont: 2-niveaux fac-similaire, courant de donnée binaire,discours limité, etc.

Le RZI donnera un signal continu avec les même places des zéros pour chacune des signaux. Le contenu des informations de ces signaux est le même si les places

SUMMARY

Description of signals by virtue of their zero patterns was under extensive and rigorous study in recent years. The zero-based representation provides for a multiplicative model of the bandlimited signal,i.e.the signal after being converted to a wholly real zero signal is represented by a polynomial.

The problem of reconstruction of the signal when its zero crossings are known is possible using a device first suggested and realized by Dr.H.B.Voelcker. This device was called the Real Zero Interpolator RZI [1] . It provides a minimum bandwidth waveform to a finite set of zeros.

The RZI proves to be a useful device in modern communication systems because several of these systems involve signals whose zero crossings carry most of the information content of the signal. Typical examples are 2-level facsimile, binary data streams,etc. The RZI will provide a continuous signal with the same zero locations,which are the informational attributes of the signal,for each of these signals. The interpolated signa can then be transmitted at much reduced bandwidth. Further specialized applications of the RZI include polarity correlation,signal detection and parameter estimation based on zero crossings.

The recovery of the signal from its zeros is,in general,a complicated problem. The existing model attributed to Dr.H.B.Voelcker is built utilizing the amplitude-phse relationships of analytic signals.

The author here presents a second version of the RZI based on a wholly different approach,affecting multiplication of elementary signals,each associated ith a specific zero, in a "quasi" real time manner.

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des zéros qui ont toutes les informations sont preservées. Le signal interpolé pourrait être transmis à une largeur de bande de frequence réduit. D'autres applications spécialisées du RZI sont la correlation des polarités, detection des signaux et l'estimation des paramètres bases sur les placements des zéros.

Le problême d'obtenir le signal de ses zéros est, en général, très compliqué. Un modèle attribue au Dr. H.B. Voelcker a été construit utilisant les amplitude-phase relations des signaux analytiques.

L'auteur presente une seconde version du RZI basée sur une idée tout a fait différente, affectant la multiplication des signaux élémentaires associés avec les zéros du signal de la facon du temps quasi-réel.

SUMMARY

The zero based representation of signals,first described by Dr.H.B.Voelcker [1] ,being based on polynomial expansions of signals is more convenient for the field of modulation theory and for non linear systems which usually involve multiplicative models of signals than the well established representation comprising Fourier series and Integrals,and Laplace transforms. The latter representations,based on sums,are convenient for for linear systems where the superposition principle can be applied.

In the zero based model of representation of signals a band limited signal (which is the type of signal almost always encountered in practice) s(z) is represente by the polynomial:

$$s(z) = s(0) \cdot \prod_{n=1}^{N} (1 - z/z_n) \qquad (1)$$

where $\{z_n\}$ are the zeros of the signal and N is the zero count.

Each of the ramp signals $(1-z/z_n)$ constitutes an "elementary signal". All elementary signals have the same form and each one is associated with a zero of the signal.

The representation(1) was shown [2] to be unique, in the sense that all bandlimited signals having the same zero set can differ only by frequency translations and constant scale factors.

The problem of interpolating a min. bandwidth waveform to a finite set of zeros has proved to be a complicated problem. Mathematically,there are certain constraints on the assymptotic growth of the signal such that the interpolation might be guaranteed. These constraints obliges the signal to be of a certain type callee EFET$^+$of class B [2],which means an entire function of exponential type with strictly positive density. Signals used commonly in most of the communication systems conform with these conditions.

Dr.H.B.Voelcker proposed a design of the RZI[1]. Its block scheme is illustratee in Fig.1. It is,in principle, based on the amplitude-phase relationships of analytic signals. A critical block is the Hilbert transform network which can be only approximately realized.

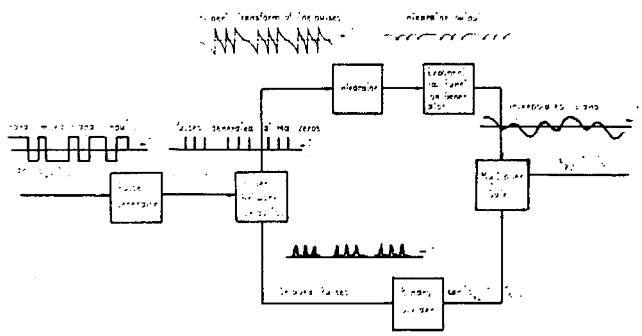

Fig.1 Block scheme of the RZI

In this paper,the author suggests a new version of the RZI.

Referring to the complex representation of a real-zero signal:

$$s_{RZ}(t) = \mathrm{Re}\{ |s_{RZ}(t)| e^{j\phi s_{RZ}(t)} \}$$
$$= |s_{RZ}(t)| \cos\phi s_{RZ}(t) \qquad (2)$$

where $\phi_{s_{RZ}}$ is the instantaneous phase of s(t),and since the latter is real, $\cos\phi_{s_{RZ}} = \pm 1$

Assume now that a sequence $\ldots, \tau_{i-1}, \tau_i, \tau_{i+1} \ldots$

on the real time axis can be rgarded as the zero srossings of a bandlimited real zero signal $s_{RZ}(t)$. The following equation then holds:[1] :

$$\phi' s_{RZ}(t) = \sum_{2n_R} \pi\delta(t-\tau_i) \qquad (3)$$

Since $s_{RZ}(t)$ is an entire function, the following Hilbert transform relationship holds true between its phase and modulus:

$$\ln|s_{RZ}(t)| = H_1[\phi' s_{RZ}(t)] = \sum \frac{\;}{(t-\tau_i)} \qquad (4)$$

The second half of the eqn. makes use of the well known Hilbert transform:

$$H\{\gamma(t-\tau_i)\} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\gamma(\tau-\tau_i)}{t-\tau_i}\,d\tau = \frac{1}{t-\tau_i} \qquad (5)$$

Solving(4)for the modulus and combining it with the phase, we obtain:

$$s_{RZ}(t) = |s_{RZ}(t)|\cos\phi\, s_{RZ}(t) \propto \frac{\pi}{2n_R}(t-\tau_i) \qquad (6$$

Utilizing eqn.(6)which states that the min. bandwidth signal can be viewed as being proportional to the product of all ramp signals of unit slope and intersecting the real time axis at the positions of the given real zeros; we suggest the following design of the RZI representee in block scheme in Fig.2 and Fig.3.

Note that we cannot affect multiplication in real time because all ramp signals, even those associated with future zero locations , must be simultaneously multiplied in real time.

The following is a brief description of the block scheme functions:

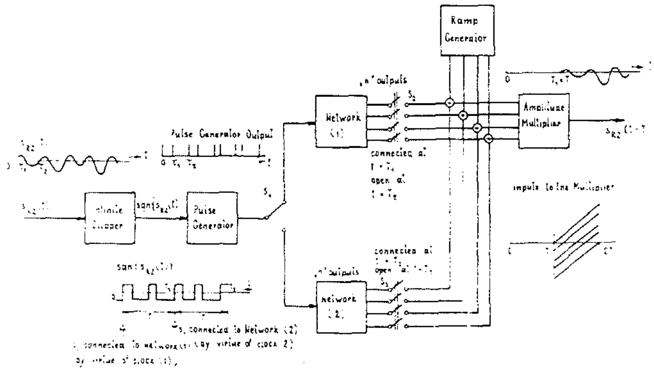

Fig.2. Block scheme of the suggested RZI

First the signal is converted to a wholly real zero signal [3] . Then it is passed through a hard limiter and a pulse generator to yield an output consisting of impulses at the zero locations. This train of impulses will be segmented each time period T which constitutes a suitable observation time window. Throughout the first time window the input is connected to the first network. This is achieved by means of the shown choice of clock 1 and the coincidence gate G_1. The train of impulses is lead to the n-stages counter K_1. To the outputs of this counter, flip-flops are connected followed by integrating networks which perform the function of "integrate and hold". Thus the output of these circuits will be DC voltages proportional to the excursions between consequitive zeros. Subtracting as shown, we obtain DC voltages proport. to the zero ordinates. These voltages are lead, each to one terminal of the adding network through the controlled switches $C_1, C_2, \ldots, C_n$ which are operat- e by an impulse generated from the trailing edge of clock by means of

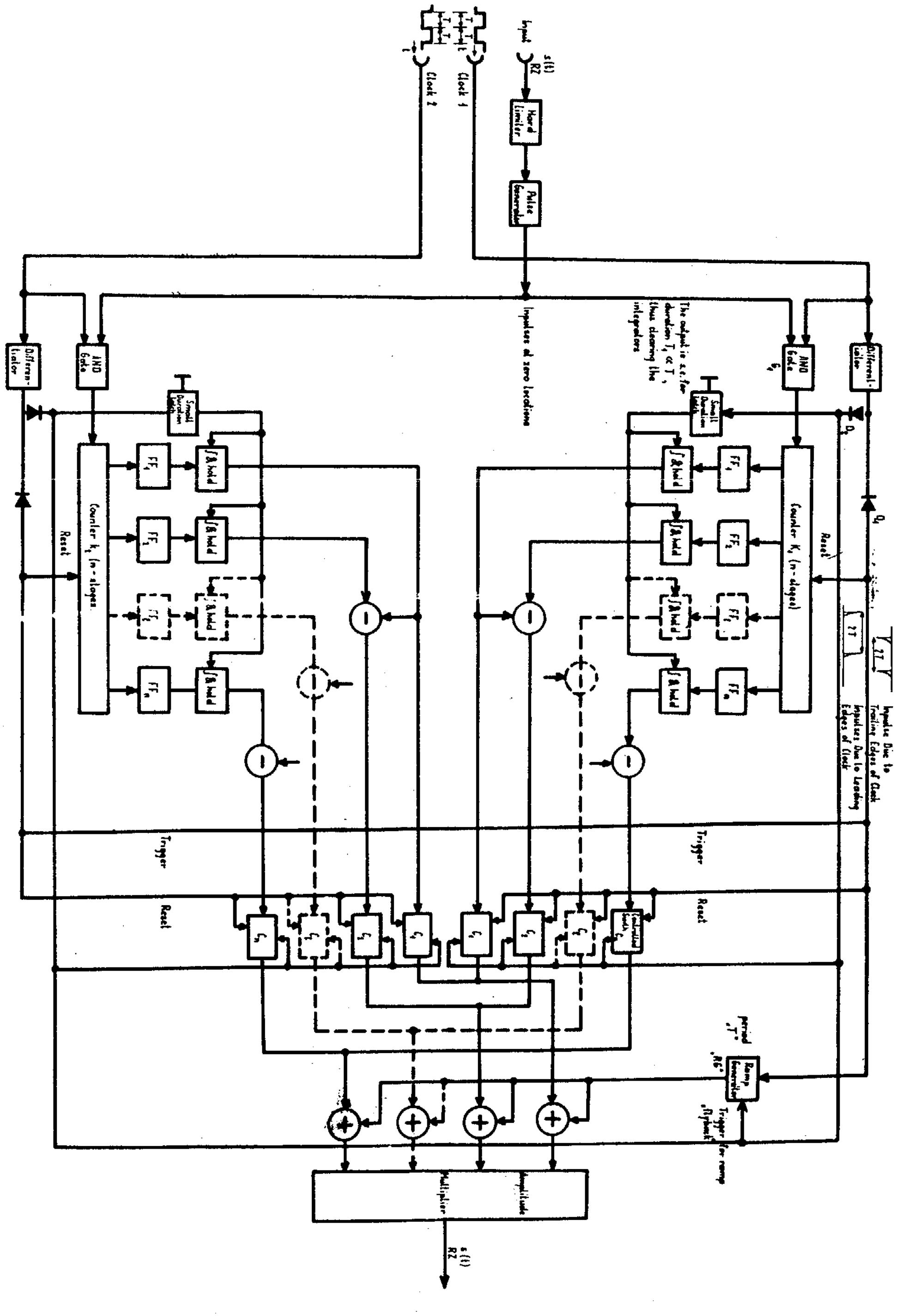

Fig. 3. BLOCK DIAGRAM

the differentiating network T_1 and the rectifier D_1. A ramp voltage generated by the ramp generator RG, which is also triggered into operation by the trailing edge of clock 1 is added to each of the mentioned DC voltages. All of these ramps, thus being situated at the zero locations are then, at the end of period T, fed to the analog multiplier an thus multiplied in real time.

So we see that multiplication is affected after elapse of a period T such that we might gain the picture of the future zeros. The integrators are cleared by means of the small duration latch L_1 which is triggered by the impulses generated from the leading edges of clock 1 by means of T_1 and D_2. The controlled switches are reset by the same impulses. During the consequitive time window, the second circuit is put into operation by means of clock 2 and so on.

The choice of the suitable time window T is governed by the average zero repetition period, the permissible distortion and the practi number of stages of the counter.

We note here that there are three sources of distortion of the reconstructed signal:

1. Due to the choice of a limited time window, the reconstructed signal will usually exhibit a certain error which is highest at the ends of the interval. This error decreases significantly by increasing "n".
2. Due to the segmentation error which will be small if the two networks are as symmetric as possible and if the clock pulses are identical in shape.
3. Due to the fact that the counters are triggered into operation by the first impulse corresponding to the first zero in each time window; a certain time jitter might occur. This jitter will prove to be exceeingly small with suitably chosen large time window.

In conclusion, we see that this version of the RZI may prove to be very useful due to its convenience in realization "as it uses exclusively digital IC 's"; and first of all, because it excludes the need for the Hilbert transform network which is the most critical part of the known type of the RZI.

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