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EFFICIENT ENCODING OF OCEAN REVERBERATIONS BY AUTO-REGRESSIVE MODELLING

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RESUME

Nous décrivons une procédure pour représenter efficacement la réverbération dans l'océan par un modèle auto-regressif (A-R). Le modèle A-R, réalisé sous forme d'un filtre digital qui ne comprend que des poles, représente le signal de réverbération par le biais d'un ensemble de paramètres variables avec le temps. Ces paramètres, qui sont les coefficients du filtre, sont obtenus en minimisant l'erreur quadratique moyenne entre les valeurs réelles et prédites des échantillons de la réverbération, et donc fournissent la meilleure estimation au sens du maximum d'entropie du signal de réverbération. Les méthodes analytiques et expérimentales utilisées dans l'analyse et la synthèse sont exposées en détail. Les procédures d'estimation adaptative et par bloc, utilisées pour obtenir l'ensemble optimum des coefficients, sont décrites, et leurs avantages respectifs pour une réalisation pratique sont mentionnés. Les propriétés statistiques de la fonction résidue sont analysées, et plusieurs fonctions d'excitation synthétiques sont suggérées pour la synthèse.

SUMMARY

We describe a procedure for efficient encoding of ocean reverberations by auto-regressive modelling. The AR model, realized as an all-pole digital filter represents the reverberation waveform in terms of a set of timevarying parameters. These parameters, which are the filter coefficients are determined by minimizing the mean-square-error between the actual and the predicted values of the reverberation samples, and thereby provide the best maximum entropy estimate of the reverberation waveform. Detailed analytical and experimental procedures for the analysis and synthesis are illustrated. Adaptive and Block estimation procedures used in obtaining the optimum set of coefficients are described and their relative advantages in hardware implementation are pointed out. Statistical properties of the residual function are analysed and several artificial excitation functions are suggested for the synthesizer.



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I. INTRODUCTION.

Mathematical analysis of the behaviour of general dynamic systems has been an area of active research and this problem has been pursued with accelerated vigour since the advent of electronic digital computers. The analysis of the outputs of dynamic systems was for the most part the concern of "time series analysis". This paper utilizes a similar approach to the problem of ocean reverberation analysis. This approach is generally referred to as "Linear Prediction". In a generalized form it is given by:

$$S_n = -\sum_{k=1}^p a_k s_{n-k} + G \sum_{l=0}^q b_l u_{n-l} \quad (1)$$

where $b_0 = 1$

Here $a_k, 1 \leq k \leq p, b_l, 1 \leq l \leq q$ and the gain G are the parameters of the hypothesized system, and S_n is the output at the n^{th} instant of time for a corresponding input of u_n . Equation (1) says that the output S_n is a linear function of past outputs and present and past inputs. That is, the signal S_n is predictable from linear combinations of past outputs and inputs. Hence the name "linear prediction". The parametric representation given in equation (1) is also referred to as a pole-zero model. For a tutorial review, the reader is referred to refs. [1] and [2]. There are two special cases of the model that are of interest :

- (1) all-zero model : $a_k = 0, 1 \leq k \leq p$
- (2) all-pole model : $b_l = 0, 1 \leq l \leq q$

The all-zero model is known in the statistical literature as the moving average (MA) model, and the all-pole model is known as the autoregressive (AR) model. The pole-zero model is then known as the autoregressive moving average (ARMA) model.

In this paper we will be concerned only with the all-pole (AR) model. In section II we formulate the model and discuss the methods for deriving it using computer algorithms. Section III deals primarily with the applications of this procedure to ocean reverberations and illustrates analysis and synthesis experiments conducted for verifying the model.

II. AUTO-REGRESSIVE MODEL PARAMETER ESTIMATION

A. AR Model

In the all-pole model, we assume that the signal S_n is given as a linear combination of past values and some input u_n .

$$S_n = -\sum_{k=1}^p a_k s_{n-k} + G u_n \quad (2)$$

where G is a gain factor

In the Frequency domain, this can be written as,

$$H(Z) = \frac{G}{1 + \sum_{k=1}^p a_k Z^{-k}} \quad (3)$$

Figs 1(a) and (b) illustrate the model in frequency and time domains.

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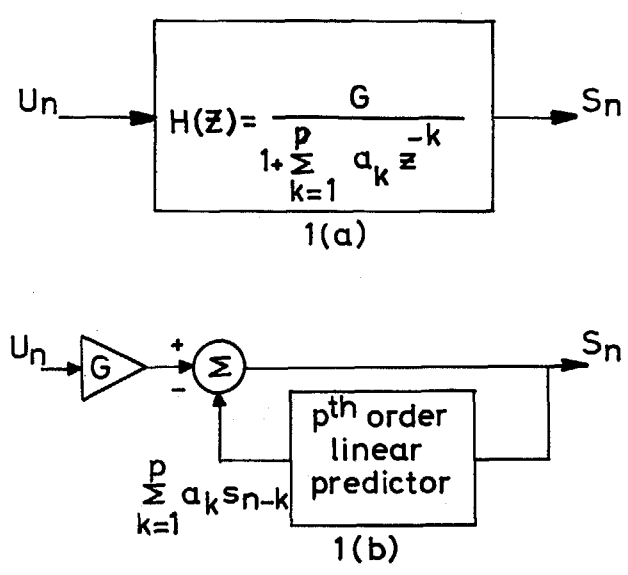


Fig.1 Discrete all-pole model representation.

(a) Frequency domain, (b) Time domain.

Once the Formulation has been done as above, the problem now is to determine the predictor coefficients a_k and the gain G in some manner. Two approaches have been attempted for the above solution. They are: (i) Least Squares Analysis(LSA), and (ii) Least-Mean-Square (LMS) approach.

B. Least Square Analysis

Here we assume that the input u_n is totally unknown. Therefore the signal S_n can be predicted only approximately from a linearly weighted summation of past samples. If \hat{S}_n is the estimated sample value at n^{th} instant of time.

$$\hat{S}_n = - \sum_{k=1}^p a_k s_{n-k} \tag{4}$$

Then the error between the actual value S_n and the predicted value \hat{S}_n is

$$e_n = S_n - \hat{S}_n = S_n + \sum_{k=1}^p a_k s_{n-k} \tag{5}$$

e_n is also known as the "residual". In the LSA method, the parameters a_k are obtained as a result of the minimisation of mean or total squared error with respect to each of the parameters. We will demonstrate LSA approach to both deterministic signals, and random signals. In the case of deterministic signals, the total squared error only needs to be minimized.

$$E_n = \sum_n e_n^2 = \sum_n (s_n + \sum_{k=1}^p a_k s_{n-k})^2 \tag{6}$$

For minimum E , $\frac{\partial E}{\partial a_k} = 0 ; 1 \leq k \leq p$ (7)

After some algebraic manipulations we will get,

$$\sum_{k=1}^p a_k \sum_n s_{n-k} s_{n-i} = - \sum_n s_n s_{n-i} \tag{8}$$

Equation (8) is called as the "normal equation". Minimum Total square error $E_p = \sum_n s_n^2 + \sum_{k=1}^p a_k \sum_n s_n s_{n-k}$ (9)

If we assume that the signal is non-zero only in a finite interval of analysis, the error can be minimized over an infinite duration. Then Eqn(8) reduces to

$$\sum_{k=1}^p a_k R(i-k) = -R(i), 1 \leq i \leq p \tag{10}$$

where $R(i) = \sum_{n=-\infty}^{\infty} s_n s_{n+i}$ (11)

is the autocorrelation function of signal S_n . The minimum squared error is given by

$$E_p = R(0) + \sum_{k=1}^p a_k R_k \tag{12}$$



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Equation (10) is called the autocorrelation method. Normally, the signals of interest will not satisfy the finite interval assumption. In order to cater to this, we have to window the analysis interval such that

$$S'_n = \begin{cases} S_n W_n & 0 \leq n \leq N-1 \\ 0 & \text{Otherwise} \end{cases} \quad (13)$$

where W_n is a window function. Then

$$R(i) = \sum_{n=0}^{N-1-i} S'_n S'_{n+i} \quad (14)$$

In contrast with the autocorrelation method, if we assume that the error is minimized over a finite interval $0 \leq n \leq N-1$ then we have

$$\sum_{k=1}^p \alpha_k \phi_{ki} = -\phi_{0i}, \quad 1 \leq i \leq p \quad (15)$$

$$E_p = \phi_{00} + \sum_{k=1}^p \alpha_k \phi_{0k} \quad (16)$$

Where $\phi_{ik} = \sum_{n=0}^{N-1} S_{n-i} S_{n-k} \quad (17)$

is the covariance of the signal S_n in the given interval. This is called the covariance method. This reduces to the autocorrelation method if N goes to infinity. The covariance method is similar to the method of Prony where a signal is approximated by the summation of damped sinusoids [3]. In the case of random signals, the residual e_n is also random. Hence we have to minimize the expected value of the squared error to obtain the parameters of the

model. Thus

$$E_n = E[e_n^2] = E\left[\left(S_n + \sum_{k=1}^p \alpha_k S_{n-k}\right)^2\right] \quad (18)$$

where $E[\cdot]$ denotes Expected value.

Applying the same algebra as before we obtain the following normal equation:

$$\sum_{k=1}^p \alpha_k E[S_{n-k} S_{n-i}] = -E[S_n S_{n-i}], \quad 1 \leq i \leq p \quad (19)$$

The minimum average error is then given by

$$E_p = E[S_n^2] + \sum_{k=1}^p \alpha_k E[S_n S_{n-k}] \quad (20)$$

Taking the expectations depends on whether the process S_n is stationary or non-stationary. For a stationary process S_n ,

$$E[S_{n-k} S_{n-i}] = R(i-k) \quad (21)$$

where $R(i)$ is the autocorrelation of the process. For a stationary (and ergodic) process, the autocorrelation can be computed as a time average. Note that the stationary case is similar to the deterministic signal formulation.

For a non-stationary process,

$$E[S_{n-k} S_{n-i}] = R(n-k, n-i) \quad (22)$$

Where $R(t, t')$ is the non-stationary autocorrelation between times t and t' .

The Eqns. 19 and 20 reduce to.

$$\sum_{k=1}^p \alpha_k R(-k, -i) = -R(0, i) \quad (23)$$

$$E'_p = R(0, 0) + \sum_{k=1}^p \alpha_k R(0, k) \quad (24)$$

Note that non-stationary processes are not ergodic, and therefore one cannot

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substitute the ensemble average by a time average. However one can safely assume that the process is locally stationary over a short time interval.

If we substitute $\hat{\Phi}_k$ for $R(-k, -i)$ in Eqn (23), then we find that the non-stationary case is similar to the covariance method. The gain parameter can be calculated from the error E_p as

$$G^2 = E_p = R(0) + \sum_{k=1}^p a_k R(k) \quad (25)$$

The parameters a_k are calculated by solving the set of simultaneous equations derived from the auto-correlation or covariance normal equations. For the autocorrelation method, because of the symmetry of the matrix, an elegant recursive procedure can be formulated. Refer Levinson [4]. This leads to a stable all-pole digital filter. That is, all the poles of the digital filter lie inside the unit circle in the Z - domain. For the covariance method, a procedure developed by Crout can be used for solving the set of simultaneous equations. This need not however result in a stable filter. Extra computations have to be done to stabilize the filter coefficients [2].

C. Least-Mean-Square Approach.

This method was first proposed by Widrow [5], [6]. In this, the prediction filter coefficients are updated as each new data sample is received. This is based on the intuition that the expected value of the squared error being a

quadratic function has a unique minimum value, and hence by a gradient search, the minimum can be found. At each iteration the predictor parameters, namely the filter coefficients are updated and once the optimum point is reached, the iteration is terminated. The LMS algorithm is thus given by

$$a_k(i+1) = a_k(i) + \mu [S(i) - \hat{S}(i)] S(i-k) \quad (26)$$

where a_k denotes the k^{th} parameter or filter coefficient, $S(i)$ denotes the signal sample value at the i^{th} instant of time, $\hat{S}(i)$ is the estimated value of $S(i)$, and μ is the gradient step size. By proper choice of μ , the speed of convergence can be controlled. Convergence is guaranteed if $0 < \mu < 2/\lambda_{\max}$ where λ_{\max} is the maximum eigenvalue of the autocorrelation matrix. The estimated value of sample S_i is

$$\hat{S}(i) = \sum_{k=1}^p a_k S(i-k) \quad (27)$$

At the start of the iteration process, a_k 's can be conveniently assumed to be zero. The choice of proportionality constant μ determines both the rate of convergence of the processor and the steady-state mean-square error. If the input sequence has automatic gain control before processing, then the input signal power is held constant. For simplicity if signal power input=1, then the step size μ can be written as:

$\mu = \frac{\alpha}{p}$ where α is in the range of 0.1 and 1.0, and p is the length of the filter. Using this approximation excellent are



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obtained. This type of formulation is an adaptive procedure and is referred to in the literature as Adaptive Linear Prediction [7] and Stochastic Approximation method [8]. If the observation noise process is assumed to be non-zero with a certain probability density function, and if the error variance matrix is not necessarily an Identity matrix, then we have the generalized formulation of the filter widely known as the Kalman Filter Algorithm.

The LSA approach is a Batch procedure whereas the LMS approach is a Sequential procedure. We also refer to the Batch procedure as Block or Frame-by-Frame estimation and to the sequential procedure as Adaptive or Sample-by-Sample estimation.

For real time hardware implementation, using the LSA approach, the complexities are increased because of the fact that matrix inversions have to be done in real-time. Also, the optimum solution is reached only after processing all the samples in the analysis interval. However using the LMS approach, the above problems do not exist. The hardware operates on vectors instead of matrices, which considerably reduces the cost and complexity. Unlike the batch process, output is available at every sampling instant, which in certain cases becomes a great advantage.

III MODELLING OF OCEAN REVERBERATIONS

A. Analysis.

Reverberation process has been

studied extensively for understanding its statistical behaviour using Frequency domain or spectral analysis methods [9]. In this paper, the reverberation process is analysed in the time domain using the auto-regressive model developed in section II based on two assumptions: (i) as a stationary random process and (ii) as a non-stationary random process. Least Squares Analysis based on both the autocorrelation and covariance methods is done to model the process. Also, using the LMS approach the optimum filter coefficients are calculated. Goodness of the model is verified in the spectral domain as the AR model guarantees a maximum entropy estimate of the input spectrum [10].

In other words, the AR model whitens the input process in such a way that the error or residual becomes white noise. This aspect can also be verified by computing the probability density function estimate of the residual functions.

B. Synthesis:

One of the best means of experimentally verifying the accuracy of a mathematical model is to excite the model parameters and synthesize the dynamics of the process. By simple intuition, the optimum excitation function for the synthesis is the prediction error or residual. If the statistical properties of the residual are known a-priori, one could simulate these functions using random process of known probability density functions. Hence several artificial

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excitation functions can be found. Such a study was undertaken for the reverberation waveform and the efficiencies of the various AR model formulations were determined.

IV CONCLUSION

Ocean Reverberations have been successfully modelled using the concept of linear prediction employing Auto-Regressive (AR) models. Stationary as well as Non-stationary formulations have been studied for the reverberation waveform. Adaptive techniques for the estimation of the model have also been illustrated. Analytical methods and computer algorithms have been described for the analysis and synthesis.

REFERENCES

1. J. Makhoul, "Linear Prediction: A Tutorial Review", Proc IEEE, April, pp 561-580.
2. B.S. Atal and S.L. Hanauer, "Speech Analysis and Synthesis by linear prediction of speech wave", JASA, 1971, pp 637-655.
3. F.B. Hildebrand, Introduction to Numerical values, McGraw Hill 1956.
4. N. Levinson, "The Wiener RMS error criterion in filter design and prediction", J. Maths Phys., Vol.25, 1947, pp. 261-278.
5. B. Widrow and M.E. Hoff, "Adaptive switching circuits", IRE 1960 Wescon Conv. Rec., part 4, pp 96-104.
6. B. Widrow et al., "Adaptive Noise Cancelling: Principles and Applications", Proc IEEE, Dec 1975, pp 1692-1716.
7. L.J. Griffiths, "Rapid Measurement of Digital Instantaneous frequency", IEEE Trans. ASSP, April 1975, pp 207-221.
8. J.D. Gibson et al., "Digital Speech Analysis using Sequential Estimation Techniques", IEEE Trans. ASSP, August 1975, pp. 362-369.
9. T.D. Pleasmons, First & Second order Statistics of the envelope functions of Narrowband Reverberation Processes, ARL Report, 1972.
10. J.P. Burg, "The relationship between maximum entropy spectra and maximum likelihood spectra", Geophysics, April 1972, pp. 375-376.